Abstract

Under what conditions does, or should, a collective of rational individuals support the imposition of a binding constraint on their own collective action? Our innovation is to allow citizen-taxpayers in a standard political economy model to be risk-averse and uncertain about the future average cost of the collective good, their own future income or wealth, and the future distribution of the tax burden. We show that if citizen-taxpayers face such uncertainty, an agency problem — broadly defined as a flaw in the political process that produces collective decisions incongruent with majority preferences — is neither necessary nor sufficient to justify a binding tax ceiling. This is a striking result, because it contradicts existing theory and can help explain why, empirically, some tax systems are more subject to ceilings than others.

1 Introduction

The certainty of what each individual ought to pay is, in taxation, a matter of [such] great importance, that a very considerable degree of inequality [...] is not near so great an evil as a very small degree of uncertainty.

Adam Smith

The Wealth of Nations (Book V)

This paper provides a new and surprising answer to an important question in political economy and social choice theory: under what conditions does, or should, a collective of rational individuals support the imposition of a binding constraint on their own collective action. This is an important
question because such constraints are common in the United States. At the state level, explicit ceilings constrain the taxing and spending powers of 30 state governments; at the local level such tax and expenditure ceilings constrain local governments (i.e., municipalities and school districts) in 46 states. Further, Brooks, Halberstam, and Phillips (2012) show that as many as one in eight U.S. cities have tax ceilings imposed on them by their own voters. Over 30 years ago Brennan and Buchanan (1980) attributed the expansion in voter support for such ceilings during the 1970s tax revolt — e.g., California’s Proposition 13 and Massachusetts’s Proposition 2 & 1/2 — to agency-failure: “[t]he obvious implication is that a significant body of citizens . . . do not trust the in-period political process to produce results in accord with electoral will, for whatever reason.” Today it remains the consensus that, in the absence of externalities or something like time-inconsistent preferences, an agency problem that causes collection decisions to be incongruent with majority preferences is necessary and sufficient to create majority support among a collective of rational individuals for a binding constraint on their own collective action. Although we consider only an explicit constraint in the form a ceiling or floor on taxation or expenditure, agency problems have been shown to also motivate support for implicit and explicit constraints on government, e.g., the initiative process, term limits, super-majority requirements, and decentralization.

Our innovation is to allow citizen-taxpayers in a standard political economy model to be uncertain about the future average cost of the collective good, their own future wealth, and the future distribution of the tax burden. We show that if citizen-taxpayers face such uncertainty, an agency problem — broadly defined as a flaw in the political process that produces collective decisions

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3See, for example, Besley and Smart (2007), who write that “the reason that these restraints are being considered in the first place are failures in the political process . . . .” For time-inconsistent preferences, see, for example, Persson and Svensson (1989). For a concise discussion of agency problems, see Barro (1973).

4See, for example, Matsusaka (1995) on the initiative process, Besley and Smart (2002) on decentralization, Knight (2000) on super-majority requirements, and Besley and Case (1995) on term limits. For a broad review of agency problems and using political institutions to solve them, see Besley and Case (2003).
incongruent with majority preferences — is neither necessary nor sufficient to justify a binding tax ceiling. This is a striking result, one that contradicts existing theory.

We begin with a standard model in which members share the costs of providing a collective good via a uniform tax rate applied to each member’s tax base and each member has one vote in collective decisions. In any period, the collective determines via a majority vote the level of collective good it wants. If there is no agency problem (i.e., direct democracy), a majority vote determines the level of the collective good and the level of taxation necessary to finance it. If and only if there is an agency problem (i.e., representative democracy), the amount of taxes exceeds the majority-preferred amount.

Moving beyond the standard model, we allow each member of the collective to face three core uncertainties. Members do not know with certainty the future average cost of producing the collective good, the future across-taxpayer distribution of the aggregate tax burden necessary to finance the collective good, and each member’s future level of income or wealth. Following Alesina and Cukierman (1990) and Benabou and Ok (2001), we assume that citizen taxpayers are forward-looking agents with rational expectations about these uncertain future outcomes. Note that, under a uniform tax rate, each taxpayer’s share of the future tax burden (“tax share”) determines, in part, that agent’s marginal cost of an additional unit of the collective good. If each member’s demand for the collective good is a function of that member’s tax share and income/wealth, these expectations allow her to forecast the collective’s future demand for the collective good. Each member knows with certainty whether there exists a future agency problem. If there is no agency problem, each member expects to consume an amount of the collective good equal to her expectation of the collective’s future demand. If there is an agency problem, she expects her consumption to deviate from collective demand to the extent and direction commensurate with the agency problem.

We then ask under what conditions a majority of these forward-looking agents will support a binding ceiling or floor on the level of collective taxation. The alternative to imposing a ceiling, a floor, or both is to leave collective taxation unfettered. In other words, each member must decide
presently whether she prefers her expected unfettered future to her expected constrained future. If a majority of members prefers the expected unfettered future, a majority votes against any constitutional constraint and the future is unfettered. However, if a majority of members prefers the expected constrained future, a revenue floor, ceiling, or both constrain future revenue. Our two core findings are: (1) in the absence of an agency problem, uncertainty about the future distribution of the tax burden can produce majority support for a binding tax ceiling but never a binding tax floor; and (2) if there is an agency problem, uncertainty about future average costs can prevent a binding tax ceiling from obtaining majority support.

These findings represent a substantive contribution to political economy and social choice theory because together they contradict the consensus that an agency problem is necessary and sufficient to produce voter support for binding constraints on collective action. Our contribution to the literature on uncertainty and social choice are also methodological. This paper is similar in spirit to Benabou and Ok (2001) who considered whether voter uncertainty about future income affects voter support for redistributive taxation. Unlike Benabou and Ok (2001), we solve analytically, rather than numerically, for voting equilibria with risk-averse rather than risk-neutral voters.

What is the intuition behind these results? Our first and most surprising result is that uncertainty about the future distribution of the total tax burden can cause a majority of rational risk-averse individuals to support a binding constraint on future collective action in the absence of an agency problem. To see this, assume there is no agency problem. The uncertainty in tax share implies that the members of the collective each face idiosyncratic risk in the cost (to him or her) of acquiring the collective good. We know from Turnovsky, Shalit, and Shmitz (1980), that no rational individual would choose to constrain her own future buying choices when there is variance in the cost of acquiring a private good. On the contrary, individuals who expect volatile prices prefer to delay consumption choices, so as to respond optimally to the revealed price. Voting for a binding constraint on collective choice is the opposite of delaying choice: it is committing earlier. The absence of agency failure implies that the collective, like a rational individual, gets to determine
its own consumption choices. Thus, if we apply the individual choice intuition, it appears that a majority of the collective will not support committing today — via a ceiling it expects to bind — to tomorrow’s collective consumption level.

However, similar to Arrow (1950) and others, we show here that behavior that is irrational for a single agent can arise from a collective of rational agents. Unlike in the individual choice framework, if a member of the collective has a higher cost of purchasing the collective good than she expected, delaying her consumption choice is of limited value. This is because her consumption is determined by collective choice and not individual choice. That is, she cannot respond to an idiosyncratic cost shock by adjusting her consumption of the collective good unless she is part of a majority that wants to purchase less of the collective good. We show voters support a binding revenue ceiling — but not a floor — because a ceiling reduces their idiosyncratic tax-payment risk, which allows them to better smooth private consumption. We show that this result is strongest when distributional uncertainty and income/wealth uncertainty are not too positively correlated.

Our second result is that in the presence of uncertainty, the existence of an agency problem is not a sufficient condition for the existence of majority support for a binding constraint. Note that uncertainty about the future average cost of producing the collective good (“average-cost risk”) is equivalent to common uncertainty about the cost of acquiring the collective good. In other words, unlike tax share uncertainty, average-cost risk is shared or common among members of the collective. Because, in contrast to idiosyncratic risk, collective demand responds to common risk, average-cost risk does not produce majority support a ceiling in the absence of an agency problem. Further, we show that average-cost risk creates a demand for budget flexibility. For example, if voters cannot index perfectly a tax ceiling to future average cost, voters may not support any binding ceiling on future taxes.

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5That the results depends on the correlation between wealth or income and distributional uncertainty echoes the Varian (1980) result that the desire to smooth after-tax consumption makes a progressive income tax optimal when there is income uncertainty. In our model, a ceiling is most desirable when the tax system fails to smooth after-tax consumption.

6This result echoes Courant and Rubinfeld (1981) who argue that the tax ceilings have “obvious costs in lost
The uncertainty that drives our results is more than a theoretical curiosity. As Benabou and Ok (2001) note, individuals face uncertainty about their future incomes and their future place in the income distribution. Importantly, however, our model is flexible in that it does not assume that the collective uses an income tax to finance the collective good. In general, we believe that the magnitude and character of tax share (or distributional) uncertainty varies across tax systems. The future distribution of the tax burden is uncertain for two primary reasons. First, there is the uncertainty inherent in the current tax base definition. For example, under an income tax, a person is uncertain about her future (relative) income and its composition; under an ad valorem property tax a person faces uncertainty about her property’s future (relative) market value. Second, there is uncertainty as to the future tax system. This uncertainty relates to the possibility of future tax reforms that alter the definition of the tax base by eliminating or expending deductions or credits. For example, in the United States a revenue-neutral elimination of the mortgage interest deduction would produce substantial changes in the distribution of the income tax burden.

That distributional uncertainty varies across tax systems has interesting implications. For example, our model predicts that, for a given level of agency failure, voters will support imposing binding ceilings on some types of taxes more than on others. Thus, unlike Brennan and Buchanan (1980), who focus placing constitutional constraints on the tax system as a solution to agency problems, our results imply that the tax system may by itself – in the absence of an agency problem – justify the imposition of constitutional constraints. Our results imply that tax systems that inherently subject taxpayers to relatively more distributional risk — especially when that risk is not too positively correlated with wealth/income risk — and tax systems more likely to be reformed will produce relatively more support amongst voters for tax ceilings.

This result can potentially explain why, in the United States, tax ceilings are more prevalent

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7 For example, Anderson (2012) shows that the administration of the ad valorem property tax as a tax on market value creates substantial uncertainty as to future distribution of the property tax burden across taxpayers.

8 See, for example, Poterba and Sinai (2008).
on local governments (and their property taxes) and much less common on state government income taxes and sales taxes. Previous studies have focused on explaining their prevalence in terms of agency problems. For example, Romer and Rosenthal (1979) posit that agenda-setting power allows the government to achieve an amount of taxation in excess of that preferred by the median voter. Brooks, Halberstam, and Phillips (2012) posit that a local government derives its leviathan power from a combination of voter uncertainty about the preferences of politicians and the ability of elected politicians to enact policies that, in the short run, deviate from majority will. Generally, as discussed by Cutler, Elmendorf, and Zeckhauser (1999), the costs of monitoring and replacing elected officials make it difficult to eliminate or reduce special interest capture, excessive public employee salaries, spending on disfavored minorities, and other policies a majority of voters disfavors.

To our knowledge, ours is the only paper to show that rational voters support, or would benefit from, a tax ceiling on their government in the absence of an agency problem. Although we are not aware of any studies that measure the degree of distributional uncertainty present under the state income and sales taxes, for the local property tax, Anderson (2012) demonstrates substantial distributional uncertainty and shows that it is relatively uncorrelated with wealth uncertainty. Thus, if this type of uncertainty is relatively higher for local property taxes, our model would predict the higher prevalence of tax limitations on local government.

The paper proceeds as follows.

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9 See Inman (1982), Vigdor (2004), however, shows that voters support constraints on communities other than their own in the absence of an agency problem. Cutler, Elmendorf, and Zeckhauser (1999) argue that voter support for a revenue ceiling may arise from a mistaken belief that government has Leviathan Power. Thus, the revenue ceiling is something voters later regret. Fischel (1989) argues that court-ordered school finance reform led to voter support for ceilings on property tax revenues. Figlio and O’Sullivan (2001) examine the case when the agency problem is characterized by waste rather than excessive revenue and show that only under highly unrealistic assumptions will voters support a tax revenue ceiling. Nechyba (1997) argues that, given that popular opposition to the local property tax exists, the introduction of uniform income taxes by state government can reduce property taxes and increase welfare. He speculates that opposition to the property tax could arise without an agency problem if the tax was poorly administered.
2 The Model

Our model includes two stages. Stage one represents the present and stage two represents the uncertain future. In stage one, members vote on whether to impose constitutional constraints on the collective’s stage-two decision. In stage two, the collective decides via majority vote the level of taxation and collective good provision. This second stage vote is subject to binding constitutional constraints, if any. In sum, member’s face uncertainty in stage one but not in stage two.

Introducing uncertainty requires that we specify members’ attitudes toward risk. Members consume $x$, a private numeraire consumption good, and $G$, a collective good. Members value $x$ and $G$ according to a utility function $u(x, G)$ under which $x$ and $G$ are normal goods and weak complements. We assume strict risk-aversion and weak prudence in both goods. Formally, if we let $U_z$ be the derivative of $U$ with respect to $z$ and so forth, these assumptions are:

\[
\begin{align*}
    u_x &> 0 & u_{xx} &< 0 & (1) \\
    u_G &> 0 & u_{GG} &< 0 & u_{Gx} \geq 0 & (2) \\
    u_{Gxx} &\leq 0 & u_{GGx} &\leq 0 & (3) \\
    u_{xxx} &\geq 0 & u_{GGG} &\geq 0 & (4)
\end{align*}
\]

In the next subsection we describe how the collective sets $R$ in the second stage. If the member uses backward induction, her analysis starts in stage two.

2.1 Post-Constitutional Stage: No Uncertainty

In the post-constitutional stage there is no uncertainty. The collective sets a level of $G$ subject to members’ individual budget constraints, the collective’s budget constraint, the ability, if any, to subvert majority will, and any constitutional constraints on tax revenue enacted in the constitutional stage. In this subsection we define these constraints, define majority will and its subversion, and
define post-constitutional outcomes with and without constitutional constraints.

First, we define the budget constraints. There is a continuum of members, each with an index \( \theta \in \Theta = [0, 1] \). Each member faces an individual budget constraint. A member’s total expenditures must not exceed her wealth, \( w_\theta \). The price of \( x \) is one and members pay taxes to finance the collective good. A member’s tax payment equals the product of a uniform tax rate \( \tau \) and the member’s tax base \( a_\theta \).

Members also face a collective budget constraint. The collective must balance its budget so that total tax revenue equals total cost \( G + d \). This cost function implies a marginal cost of one, a fixed cost of \( d \), and an average cost of \( 1 + \frac{d}{G} \). Total tax revenue, \( R \) is the sum of tax payments collected from the collective’s continuum of members.

We assume that a member’s wealth and tax base are exogenous and that a member’s tax base is not necessarily a function of wealth. If we model wealth and tax base as choice variables, our model becomes more complex but our result is preserved. We discuss this more in Section Discussion.

Allowing tax base to not be a function of wealth allows us to consider a broad range of relationships between uncertainty in wealth and tax base. This is important because the direction and degree of correlation between uncertainty in tax base and wealth drive our main results. If instead a specific function defines their relationship, we are tied to only one direction and degree of correlation between wealth and tax base.

The preferences of members define majority will. A maximization problem defines each member’s preferred levels of \( x \) and \( G \):
\[
\max_{G \geq 0, x_\theta} u(x_\theta, G)
\]  
(5)

subject to:

member budget: \( w_\theta = x_\theta + t_\theta \)  
(6)

tax payment: \( t_\theta = \tau a_\theta \geq 0 \)  
(7)

collective budget: \( G + d = R \)  
(8)

tax revenue: \( R = \tau \int_{\theta \in \Theta} a_\theta \, d\theta \)  
(9)

The first order condition implies that the relative price of \( G \) determines each member’s preferred level of \( x \) and \( G \). If \( p_\theta \) is the relative price of \( G \), the first order condition is

\[
\frac{U_G}{U_x} = p_\theta
\]  
(10)

A member’s relative price of the collective good, \( p_\theta \), equals her share of the total tax base \( A = \int_{\theta \in \Theta} a_\theta \, d\theta \). To see this, note that \( \tau = \frac{R}{A} \) and substitute for \( \tau \) in a member’s budget constraint

\[
w_\theta = x_\theta + \left( \frac{a_\theta}{A} \right) R = x_\theta + (p_\theta) R = x_\theta + (p_\theta) (G + d)
\]  
(11)

Each member’s maximization problem is equivalent to an unconstrained problem of maximizing utility by choosing \( R \). The preferred revenue of member \( \theta \) is the solution to:

\[
\max_{R \geq 0} u(w_\theta - p_\theta R, R - d)
\]  
(12)

We call the maximand of this function \( r^*(\theta; d) \), i.e., the preferred revenue of member \( \theta \) under fixed costs \( d \).
The median \( r^*(\theta; d) \), \( R^*(d) \), represents the pivotal voter and thus majority will.

\[
R^*(d) = \text{Median}_{\theta \in \Theta} \{ r^*(\theta; d) \} \tag{13}
\]

We allow majority will to be subverted. \( \gamma \) is the magnitude of subversion. It represents the cents on the dollar by which revenue raised departs from the revenue that would be raised under the median voter theorem. \( \gamma \) is determined by constraints outside of model and is thus exogenous. [leviathan literature or note that we will talk about it later ... \( \gamma < \infty \) reflects electoral and other fiscal constraints on revenue-maximizing behavior] The exact mechanism creating leviathan power or hindering power is immaterial to our results.

Decision making in the post-constitutional stage is either unfettered or subject to constitutional constraints. If there are no constitutional constraints, the collective’s decision as to the level of revenue and collective good is unfettered. Unfettered revenue \( (R_{unf}) \) and unfettered collective good \( (G_{unf}) \) equal

\[
R_{unf} = (1 + \gamma)R^*(d) \\
\implies G_{unf} = (1 + \gamma)R^*(d) - d \tag{14}
\]

If \( \gamma > 0 \), the collective is subject to leviathan power because its unfettered revenue is greater than the majority prefers. If \( \gamma < 0 \), the collective is subject to a hindering power because its unfettered revenue is less than the majority prefers. If \( \gamma = 0 \), unfettered revenue reflects exactly majority will.

If there is a constitutional constraint, the revenue level actually implemented may not equal unfettered revenue. Suppose that in the first stage members impose a constitutional constraint on revenue \([Z, C] \), where \( Z < C \) and \( Z, C \in \mathbb{R}_+ \cup \{\infty\} \). \( Z \) represents the minimum allowable revenue and \( C \) the maximum allowable revenue. If \( Z = 0 \) and \( C < 0 \), there is a revenue ceiling. If \( C = \infty \) and \( Z > 0 \), there is a revenue floor. \( C \) either binds or does not bind. If it binds, post-constitutional
implemented revenue equals \( C \). If \( C \) does not bind, implemented revenue equals \((1 + \gamma) R^*(d)\).

Thus, the implemented level of post-constitutional revenue, \( R_{\text{imp}} \), depends the presence of constraints and whether those constraints are binding. If members impose a revenue floor \( Z > 0 \) and a ceiling \( C < \infty \), equation [15] defines the the post-constitutional stage implemented revenue as

\[
R_{\text{imp}}(Z, C) = \max \left[ Z, \min \left[ C, R_{\text{unf}} \right] \right] \tag{15}
\]

Although we consider the circumstances under which members support a revenue floor, in the remainder of the paper we focus on tax revenue ceilings. If there is no revenue floor \((Z = 0)\) but a revenue ceiling, implemented revenue and the implemented collective good equal

\[
R_{\text{imp}}(C) = \min \left[ C, R_{\text{unf}} \right] \tag{16}
\]

\[\implies G_{\text{imp}} = R_{\text{imp}} - d = \min \left[ C, R_{\text{unf}} \right] - d \]

Each member’s post-constitutional utility is a function of this implemented revenue,

\[
u(x_{\theta}, G_{\text{imp}}) \tag{17}\]

where: \( x_{\theta} = w_{\theta} - t_{\theta} \) \tag{18}

\[
t_{\theta} = p_{\theta} R_{\text{imp}} = \tau a_{\theta} \tag{19}\]

\[
\tau = \frac{R_{\text{imp}}}{A} \tag{20}\]

\[
G_{\text{imp}} = R_{\text{imp}} - d \tag{21}\]

In sum, the level of collective good and revenue set in the post-constitutional stage depends on whether members imposed a binding constitutional constraint in the constitutional stage. If there is no constitutional constraint or if there is a non-binding constraint, \( R_{\text{imp}} \) is set according to majority
will, subverted by any leviathan or hindering power, i.e., \( R_{imp} = R_{unf} \). On the other hand, if there is a binding constitutional constraint, it prevents the collective from setting \( R_{imp} \) according to its (subverted) majority will, i.e., \( R_{imp} \neq R_{unf} \).

In the next subsection we describe how, in the constitutional stage, members vote on whether to impose a constitutional constraint (i.e., revenue floor or ceiling) on themselves in the postconstitutional stage. Because there is uncertainty members in the constitutional stage cannot know with certainty whether a proposed constraint will in fact make them better or worse off in the post-constitutional stage. When voting in the constitutional stage, a member bases her vote on her expectations of the her post-constitutional utility. Once the post-constitutional stage arrives she cannot undo any constraints. Thus, a member may face some risk that a proposed constraint will make her post-constitutional outcome worse. Below we discuss how the presence of leviathan or hindering power affects whether a constraint makes a majority of members better or worse off — in the post-constitutional stage — than they will be under an unfettered outcome.

A constitutional constraint makes a majority of members better off — in the post-constitutional stage — when it brings \( R_{imp} \) closer to majority will \( (R^*(d)) \) than is the unfettered outcome, \( R_{unf} \). If leviathan power subverts majority will, \( R^*(d) < R_{unf} \), a binding ceiling may bring \( R_{imp} \) closer to majority will, e.g., when \( R^*(d) \leq R_{imp} < R_{unf} \). If hindering power subverts majority will, \( R^*(d) > R_{unf} \), a binding floor may bring \( R_{imp} \) closer to majority will, e.g., when \( R_{unf} < R_{imp} \leq R^*(d) \).

A constitutional constraint makes a majority of members worse off — in the post-constitutional stage — when it pushes \( R_{imp} \) further away from majority will than is the unfettered income. For example, once the post-constitutional stage arrives a ceiling may have been set low enough that a majority prefers to push past the ceiling but cannot, i.e., \( R_{imp} < R^*(d) < R_{unf} \). In this case, a majority in the post-constitutional stage may prefer unfettered leviathan power \( (R_{unf}) \) to the ceiling \((R_{imp})\). Thus, in our model, a majority of members may regret the ceiling they themselves earlier imposed.
If there is agency problem — i.e., no leviathan or hindering power — a binding constraint makes a majority of members worse off in the post-constitutional stage than they will be without the constraint. Yet, as we show in the next section, if there is uncertainty, a majority of members in the constitutional stage may support a revenue ceiling they expect to bind even if they know there is no leviathan power.

2.2 Constitutional Stage: Revenue Ceilings and Uncertainty

In this subsection we describe in detail the constitutional stage. In the constitutional stage members vote on whether to impose a constitutional constraint on themselves in the post-constitutional stage. When voting, a member in the constitutional stage is uncertain about the future cost of collective good provision and her future wealth. As a result, a member does not know with certainty the unfettered post-constitutional levels of revenue and the collective good. We demonstrate the ways in which this uncertainty can affect members’ support for constitutional constraints.

To do this, we construct a model of collective choice under uncertainty. Our model has the following core features: collective choice, risk-averse voters, uncertainty in the future cost of collective goods, uncertainty in future wealth, rational expectations of future cost and wealth, heterogeneous degrees of uncertainty across members, and flexibility to vary the degree and direction of correlation between wealth uncertainty and cost uncertainty.

For members of the collective there are two broad types of uncertainty about the future costs of the collective good: common uncertainty and idiosyncratic uncertainty. Common uncertainty is shared by all members of the collective. Idiosyncratic uncertainty is uncertainty not shared by all members of the collective. To emphasize the difference between common and idiosyncratic uncertainty we expose all members to only common uncertainty and an extreme form of idiosyncratic uncertainty that affects each member but is not shared with other members. Our central contribution is to show that these types of uncertainty have different effects on members’s support of constitutional constraints.
We now present the assumptions we use to model this uncertainty. In another section, we compare our assumptions with those in the prior literature and also discuss one alternative approach to modeling uncertainty that retains the same core features as above.

As mentioned, we model both common and idiosyncratic uncertainty. We model common uncertainty as uncertainty over the fixed cost parameter $d$. For simplicity, we assume $d$ can take on two levels: 0 or some value $D > 0$. We let $\pi_d$ be the probability that fixed costs are some level $d$; this implies $\pi_0 = 1 - \pi_D$. By definition, uncertainty in $d$ is shared by all members of the collective. For individual members, an uncertain $d$ creates no uncertainty about the relative price of $G$. Instead it creates uncertainty in the collective’s average cost of the collective good. This, in turn, creates uncertainty about post-constitutional majority will $R^*(d)$.

We model idiosyncratic uncertainty as each member’s uncertainty over her post-constitutional tax price and wealth. Each member is certain about the post-constitutional distribution of tax price and wealth, $F(\theta)$, but is uncertain about her place in that distribution. The fixed post-constitutional distribution of tax price and wealth, however, implies that our idiosyncratic uncertainty does not create uncertainty in $R^*(d)$.

Specifically, in the constitutional stage each member is assigned a unique index $i \in I = [0, 1]$ and is uncertain about her identity $\theta_i \in \Theta$ in the post-constitutional $\theta$ distribution. Recall that in the post-constitutional stage $\theta$ defines $p_{\theta}$ and $w_{\theta}$. Each member $i$ has belief distribution over $\Theta$ that represents her idiosyncratic uncertainty about her future $\theta$. Differences in belief distributions across members allow different members to have different levels of expectations and different degrees of idiosyncratic uncertainty.

Below we demonstrate how the degree of idiosyncratic uncertainty in the population affects majority support for constitutional constraints. We describe the degree of idiosyncratic uncertainty in the population in terms of belief profiles. A belief profile is a set that contains each member’s belief distribution. We denote belief profile $b$ as $F_b(\theta)$. The belief distribution of a member $i$ in belief profile $b$ is $F_b(\theta|i)$. The degree of population risk or idiosyncratic uncertainty varies across
belief profiles. If, for all \( i \neq 0,1 \), \( F_b(\theta|i) \) is a mean-preserving spread of \( F_{b'}(\theta|i) \), we say that belief profile \( b' \) is more risky than profile \( b \).[10]

We allow \( i \) and \( \theta \) to have an intuitive relation by assuming that a higher index in the constitutional stage is associated with a higher \( \theta \) in the post-constitutional stage. Specifically, we assume that belief profiles are positively ordered. A belief profile \( b \) is positive ordered when for all indices \( i \) and \( j \), \( i < j \) implies \( F(\theta|j) \) first-order stochastically dominates \( F(\theta|i) \).[11] That is, \( F(\theta|i) \) puts less probability weight on higher \( \theta \) than \( F(\theta|j) \).

Finally, we assume that all post-constitutional implemented revenue levels bankrupt no-one. That is, for all \( \theta \in \Theta \), we consider only \( R_{imp} \) which satisfy:

\[
w(\theta) > p(\theta)R_{imp}
\]

### 3 The Existence of Majority Support for a Constitutional Constraint

In this section we describe the conditions under which there exists some revenue ceiling \( C \) that receives majority support in the constitutional stage.

If no such \( C \) exists, post-constitutional revenue is unfettered. Unfettered post-constitutional revenue, \((1 + \gamma)R^*(d)\), depends on the level of \( d \). It equals either \((1 + \gamma)R^*(0)\), which we call \( R_0 \), or \((1 + \gamma)R^*(D)\), which we call \( R_D \). Note that \( R^*(0) < R^*(D) \).[12] A member may prefer a revenue ceiling, \( C \), to an unfettered post-constitutional outcome.

If there does exist a \( C \) that receives majority support, post-constitutional revenue depends on...
whether such a ceiling never binds, sometimes binds, or always binds the post-constitutional outcome. We focus on when there exists a sometimes-binding or always-binding \( C \) that receives majority support. If \( C > R_D \), it never binds. Thus, the existence of a never-binding \( C \) that receives majority support is equivalent to majority preference for the unfettered outcome.

Consider when there exists a sometimes-binding \( C \) that receives majority support. A \( C \) sometimes binds when \( R_0 \leq C \leq R_D \). Such a \( C \) binds when \( d = D \) and does not bind when \( d = 0 \). We call a sometimes-binding ceiling a \( \pi \)-binding ceiling because the likelihood that such a \( C \) binds equals the probability that \( d = D \), i.e., \( \pi \). A member prefers an \( \pi \)-binding ceiling to the unfettered outcome when she prefers \( C \) to \( R^*(D) \). To see when this is the case, let \( c^*_\pi(i) \) be the \( \pi \)-binding ceiling that generates the greatest utility for member \( i \).

\[
\max_{c \in [R_0, R_D]} \pi_0 \cdot \int_{\theta \in \Theta} u\left(w_\theta - p_\theta R_0, R_0 - 0\right) dF_b(\theta|i) \\
+ \pi_D \cdot \int_{\theta \in \Theta} u\left(w_\theta - p_\theta c, c - D\right) dF_b(\theta|i) \tag{23}
\]

The first-order condition which characterizes \( c^*_\pi(i) \) is:

\[
\text{FOC}_c : \int_{\theta \in \Theta} (u_g - p_\theta u_x) dF_b(\theta|i) = 0 \tag{24}
\]

where \( u_g \) and \( u_x \) are functions of \( c^*_\pi(i) \).

Next, consider when there exists majority support for an always-binding ceiling. A ceiling is always-binding when \( C < R_0 \). Let \( c^*_1(i) \) be the always-binding ceiling that generates the greatest constitutional maximization problem in equation 12 and apply the implicit function theorem to solve for \( \frac{dR}{dd} \):

\[
\frac{dR}{dd} = -\frac{pu_{gx} + u_{gg} + (-p)^2u_{xx} - pu_{xg}}{u_{gg} + pu_{xg}} \\
= -\frac{u_{gg} - 2pu_{gx} + p^2u_{xx}}{u_{gg} - pu_{xg}}
\]

The numerator and denominator are both negative, which implies \( \frac{dR}{dd} > 0 \). Since \( 0 < D \), this implies \( R^*(0) < R^*(D) \).
utility for member $i$.

\[
\max_{c \in [0,R_0]} \pi_0 \cdot \int_{\theta \in \Theta} u\left(w_{\theta} - p_{\theta} c, c - 0\right) dF_b(\theta|i) \\
+ \pi_D \cdot \int_{\theta \in \Theta} u\left(w_{\theta} - p_{\theta} c, c - D\right) dF_b(\theta|i)
\]  

(25)

Setting the first derivative to zero defines $c^*_1(i)$:

\[
\text{FOC}_c : \pi_0 \cdot \int_{\theta \in \Theta} (u_g - p_{\theta} u_x) dF_b(\theta|i) \\
+ \pi_D \cdot \int_{\theta \in \Theta} (u_g - p_{\theta} u_x) dF_b(\theta|i) = 0
\]  

(26)

where $u_g$ and $u_x$ are functions of $c^*_1(i)$.

A member’s constitutional preferences can be summarized $c^*(i)$, which defines a member’s most-preferred post-constitutional outcome. Of the three types of preferred post-constitutional options — no ceiling ($\infty$), a sometimes-binding ceiling ($c^*_\pi(i)$), and an always-binding ceiling ($c^*_1(i)$) — $c^*(i)$ is the option that yields the highest utility to member $i$. Formally,

\[
c^*(i) = \arg \max_{c \in \{c^*_1(i), c^*_\pi(i), \infty\}} \sum_{d \in \{0,D\}} \left[ \pi_d \cdot \int_{\theta \in \Theta} u\left(w_{\theta} - p_{\theta} r_d, r_d - d\right) dF_b(\theta|i) \right]
\]  

(27)

where $r_d = \min\left[c, (1 + \gamma) R^*(d)\right]$

(28)

We now define the outcome — no ceiling, a sometimes-binding, or an always-binding ceiling that wins a majority vote. Let the most-preferred ceiling of the pivotal or median member be $C^*$. Remember that as $C^*$ increases it eventually becomes never-binding and equivalent to the no
ceiling outcome.

\[ C^* = \text{Median}_{i \in I} \{ c^*(i) \} \]  

(29)

Denote this pivotal member \( i_m \), where \( i_m \) satisfies \( c^*(i_m) = C^* \). Member \( i_m \) is the \textit{pivotal constitutional voter} and ceiling \( C^* \) is the \textit{pivotal ceiling}.

The level of \( C^* \) determines whether there exists either an always-binding or sometimes-binding ceiling that receives majority support. If the pivotal ceiling is less than low-cost unfettered revenue — \( C^* < R_0 \) — then there exists an always-binding ceiling that receives majority support. Similarly, if the pivotal ceiling is between low-cost and high-cost unfettered revenue — \( R_0 \leq C^* \leq R_D \), there exists a sometimes-binding ceiling that receives majority support.

First, consider under what conditions majority support exists when there is no cost-uncertainty. These results echo those in the prior literature.

\textbf{Theorem 1.} If there is no cost-uncertainty, majority support for always-binding ceiling exists if and only if there is leviathan power.

Consider the case of no ceiling, or, equivalently, a ceiling strictly larger than \((1 + \gamma)R^*(d)\). Trivially, \( R^*(0) < R_{\text{imp}}(0) \) if and only if \( \gamma > 0 \); that is, the level of revenue raised exceeds majority will only in the presence of leviathan power. Given single-peaked preferences, any ceiling less than \( R_{\text{imp}}(0) \), but (weakly) larger than \( R^*(0) \) makes the pivotal voter better off. In this case, therefore, a binding ceiling is supported by the pivotal voter. On the other hand, if \( \gamma = 0 \), then \( R^*(d) = R_{\text{imp}}(d) \) and there is no binding ceiling which makes the pivotal voter better off, and therefore, there is no binding ceiling which is supported by the pivotal voter.

\textbf{Theorem 2. Corollary:} If there is no uncertainty, majority support an always-binding floor exists if and only if there is hindering power.

Next, we consider the implications uncertainty.
Theorem 3. The pivotal ceiling is increasing in common risk. That is, given a pivotal ceiling \( C^* \), if common risk increases—i.e., \( p_iD \) increases to \( \pi_D' \)—then

\[
C^{*'} \geq C^*
\] (30)

where the relationship is strict if \( C^* < R_D \).

(See proof [5.1])

The effect of increases in idiosyncratic risk on the pivotal ceiling depends on the relationship between uncertainty in wealth and uncertainty in tax price.

Definition: weak price-wealth relationship

We define the following relationship between uncertainty in wealth and uncertainty in tax price. The percent increase in wealth across \( \Theta \) is no larger than the percent increase in the tax price, and the same applies to the second derivative:

\[
\frac{w'}{w} \leq \frac{p'}{p} \quad \text{and} \quad \frac{w''}{w} \leq \frac{p''}{p}
\] (31)

(Noe that if \( w' < 0 \) and \( w'' < 0 \), these relationships hold automatically.)

Theorem 4. If the weak price-wealth relationship holds, the pivotal ceiling is decreasing in idiosyncratic risk. Suppose that there is less idiosyncratic risk in belief profile \( b \) than belief profile \( b' \) then

\[
C^{*'} \leq C^*
\] (32)

where the relationship is strict if \( C^{*'} < R_D \).

Explain. (See proof [5.2])
Theorem 5. Corollary: if the weak price-wealth relationship is violated by a sufficiently large amount, than the pivotal ceiling is increasing in idiosyncratic risk.

3.1 Discussion

In this subsection we discuss the intuition central to the result that tax-price risk — i.e., uncertainty about the future distribution of the tax burden — creates majority support for a tax ceiling. Suppose that \( R \) is known and only \( p \) is a random variable. Agents’ tax payments are given by \( pR \). and thus the variance in the tax payment is \( R^2 \times \text{var}(p) \) and tax-payment uncertainty is increasing in both \( R \) and \( \text{var}(p) \). Now compare two cases: one in which \( \text{var}(p) \) is strictly positive and the second in which \( \text{var}(p) \) is zero. In the first case, the agent will demand less \( R \) than the second, because in the first case, each dollar of revenue increases tax payment uncertainty, while in the second case, revenue does not increase tax payment uncertainty (which remains fixed at zero). The first case corresponds to the ex-ante stage, in which tax price is unknown so \( \text{var}(p) \) is positive, and the second case corresponds to the ex-post stage, in which tax price is known so \( \text{var}(p) \) is zero. This implies that each agent demands less Revenue ex-ante than she does (on average) ex-post. Therefore, ex-ante agents wish to put a ceiling on ex-post agents’ revenue choices.

Next, consider an example.

Suppose that agent type \( \theta \) has some tax price \( p(\theta) \), with \( p'(\theta) > 0 \), and that all types \( \theta \) same wealth \( w \). Moreover, suppose that the agent’s utility function is additively separable: \( u(c, g) = u_c(c) + u_g(g) \). Figure 1 depicts the ex-post optimum for any type \( \theta \). Revenue is on the \( x \)-axis and marginal utility is on the \( y \)-axis. The marginal benefit of a unit of revenue is an increase in one unit of \( g \): therefore, the marginal benefit is \( u'_g(R - d) \); therefore \( MB(d = D) > MB(d = 0) \). 

The marginal cost of a unit of revenue is the decrease in consumption due to the higher tax bill: \( MC = u'_c(w - p \cdot R) \cdot p \). (The fact that increased revenue reduces consumption causes this curve to be upward-sloping). As noted on the diagram, if \( p \) were uncertain, that uncertainty would raise the marginal cost curve (without changing the marginal benefit curve, due to additive separability.)
Figure 1: The ex-post optimal revenue levels for agent type $\theta$. Marginal benefit is derived from the value of the collective good and marginal cost is derived from the tax bill. Note that $\sigma^2 = 0$ because uncertainty is resolved.

Figure 2: The ex-post population $\Theta$ of ex-post optimal revenue. Leviathan power $\gamma$ increases revenue over desired levels. The median ex-post optimal revenue is at $\theta = \frac{1}{2}$.

Figure 3: Under a large variance $\sigma^2$ and small likelihood of damages $\pi_D$, the ideal ceiling $c^*_i$ for agent $i$ always binds. Since this ceiling binds when $d = 0$ and $d = D$, it is found where expected marginal benefit intersects marginal cost. Here, agent $i$’s ex-post optimal revenue levels $r^*$ exceed the median; so $i$ must be larger than $\frac{1}{2}$. If $i$ were $\frac{1}{2}$ and there were no leviathan power, then $r^*(d = 0)$ would overlap with $R_0$.

Figure 4: Under a smaller variance $\sigma^2$ and larger likelihood of damages $\pi'_D$, the ideal ceiling $c^*_i$ for agent $i$ sometimes binds. Since this ceiling only binds when $d = D$, it is found where marginal benefit under $d = D$ intersects marginal cost.
Therefore, the basic intuition can be seen in this diagram: That the ex-ante individual would prefer lower revenue on average.

Figure 2 depicts the population of types $\Theta$. Since price is increasing in $\theta$ and wealth is unchanging, marginal cost is increasing in $\theta$, and therefore optimal revenue is decreasing in $\theta$. The curves $r_\theta^*(1+\gamma)$ denote the optimal revenue increased by leviathan power. The median individual, by construction, is the agent with type $\theta = \frac{1}{2}$, and $R_0$ and $R_D$ are read off the $y$ axis at that point. Note that the assumption above assure us that, even if wealth is increasing, it is not increasing ‘fast enough’ to overturn the direction of revenue. This can be best thought of as an association in the beliefs of the individual: that, although higher types result in higher wealth, that higher wealth is not so much higher as to offset the increase in the tax price.

Consider Figure 3. In this figure, there is a large variance $\sigma^2$ and a small probability of damages $\pi_D$. The levels of ex-post revenue $R_0$ and $R_D$, derived from Figure 2, are depicted as vertical lines. These are the levels of revenue that the agent will expect, if there is no limit in place. $r_{\theta=1}^*$ represents the revenue levels that this agent will desire on average in the ex-post period; however, due to the tax price risk, the agent desires less in the ex-ante period. In particular, point $A$, contrasted with $r_{\theta=1}(d = 0)$, represents the value that the agent desires, ex-ante, in the case of zero damages; it is the intersection of the marginal benefit under zero damages and the marginal cost under positive risk. Similarly, point $B$ represents the value that the agent desires, ex-ante, in the case of damages $D$. Ideally, the agent would support a policy ex-ante which provides revenue from point $A$ in the case of zero damages and revenue from point $B$ in the case of damages $D$. This policy is unavailable to her, however. Instead, she chooses a policy of the ceiling $c_i^*$, which is the intersection of marginal cost and expected marginal benefit. Since $c_i^*$ is to the left of both $R_0$ and $R_D$, it always binds.

Now consider Figure 4. In this figure, the variance $\sigma^2$ has fallen and and a probability of damages $\pi_D'$ has risen. This causes the intersection of expected marginal benefit and marginal cost under variance to intersect to the right of $R_0$, at point $C$. This means that this agent does not prefer
a policy which always binds, as is the case in Figure 3. Instead, her optimal ceiling binds when damages are $D$, but not when damages are zero. Therefore, she chooses a policy of the ceiling $c_i^\ast$, which is the intersection of marginal cost under variance $\sigma^2$ and marginal benefit when damages are $D$, which binds when damages are $D$.

In both Figures 3 and 4, if the agent is $i = .5$, then she is the pivotal voter in the ex-ante stage, and her optimal ceiling is the one supported in the ex-ante stage. These figures could represent this case. Moreover, if leviathan power were zero, then $R_0$ would overlap with $r_{\theta=i=.5}(d = 0)$ and $R_D$ would overlap with $r_{\theta=i=.5}(d = D)$.

It can be seen from this graph that, so long as $\sigma^2 > 0$, the ex-ante pivotal voter will never select a ceiling which never binds.

4 Conclusion

This paper concerns the conditions under which a collective of rational individuals would impose a binding constraint on their own collective choices. Surprisingly, we show that uncertainty as to the future distribution of the tax burden (“distributional uncertainty”) can cause a majority of rational members in a collective to support such constraints even in the absence of an agency problem. This contradicts the prior literature which has thus far demonstrates that an agency problem is a necessary condition for such support to exist. Further, we also show that uncertainty as to the average cost of public services causes agency problem to no longer be a sufficient condition for such support to exist.

As we argued in the introduction, because the degree of distributional (idiosyncratic) uncertainty varies across tax systems, our results can help explain why in the United States tax ceilings are more common imposed on local governments that raise revenue via property taxes than on state government that raise revenue via sales and income taxes.

We also take the time here to note how our results relate to other economics research on idiosyn-
ocratic risk and collective action. For example, the macroeconomics literature has also explored the effects of idiosyncratic risk and implications for collective action. Aiyagari (1994) builds on the Brock and Mirman (1972) model to investigate the quantitative importance of individual risk for aggregate saving: Aiyagari points out the incompatibility of this with representative agent models, which is analogous to our finding that the ex-ante and ex-post median voter are time-inconsistent. Kimball and Mankiw (1989) and related papers discuss the insurance aspects of income taxes, but use representative agent models which cannot capture idiosyncratic risk.

There are also a variety of time inconsistent preference models that have implications for collective action. Typically these are present-biased preferences, for example hyperbolic preferences. This means that at time $t$ the agent chooses some level $x$ to consume at $t + 1$, but in time $t + 1$, the agent prefers some $y > x$. For this reason, the agent at time $t$ would be willing to pay for a means to restrict her future consumption. If one interprets the community as choosing $x$ then it appears that these models have quite a lot in common. However, although the apparent behavior in our model is the same on a population level as the individual behavior in these self-control models, none of these agents are time-inconsistent individuals. For example, none of these agents individually would support a resolution that would restrict their own behavior per se; for example, in terms of the model, no agent $i$ would support a resolution that would restrict their own future consumption $c_i$. The time inconsistency arises from the aggregation mechanism, that they must all choose the same level of public good and are concerned that their fellow agents will choose too high a level in the future.

Arrow’s Possibility Theorem (Arrow 1950) also deals with social choice. Arrow famously finds that, when seeking to aggregate individual preferences into a social preference, a small selection of ‘reasonable’ principles—that social preferences choose Pareto improvements whenever possible, that society not have a unilateral dictator, and that the set of allowable individual preferences

\footnote{There are too many time-inconsistent preference models to discuss here, since they are beyond the scope of this paper. Rabin (1998) has a section on time-variant preferences which provides a thorough overview.}
are unrestricted—imply that social choices must fail transitivity; i.e. that there is some selection of options \( z_1, z_2, \) and \( z_3 \) such that \( z_1 \succ z_2 \succ z_3 \succ z_1 \). In other words, Arrow tells us that rational individuals may necessarily aggregate into an irrational society. If time inconsistency is considered irrational, then one might say that the Anderson-Pape effect is also a case of rational (time consistent) individuals aggregating into an irrational (time inconsistent) society. This can be thought of as an example of time-inconsistent social choice due to idiosyncratic risk.

5 Proofs

5.1 Proof: More Common Risk Leads to a Less Restrictive Ceiling

The proof of Theorem 3 that more common risk (higher \( D \)) leads to a less restrictive limit:

We first show that for all \( i \), \( c^*_\pi(i) \) and \( c^*_1(i) \) both fall if they are not at corner solutions. Then we note that if, if all elements of a population falls, than the median also falls. This implies that \( C^*_\pi \) and \( C^*_1 \) both fall if they are not corner solutions. The result then follows.

The first order condition which characterizes \( c^*_\pi(i) \) is:

\[
\text{FOC}_{c^*_\pi} : \int_{\theta \in \Theta} (u_g - p_\theta u_x) dF_b(\theta|i) = 0 \tag{33}
\]

The first order condition which characterizes \( c^*_1(i) \) is:

\[
\text{FOC}_{c^*_1} : \sum_{d \in \{0, D\}} \pi_d \int_{\theta \in \Theta} (u_g - p_\theta u_x) dF_b(\theta|i) = 0 \tag{34}
\]

Where in both cases, \( u_g \) and \( u_x \) are functions of the ceiling.
Define the function within the integral as \( h(D; c) \). Then:

\[
h(D; c) = u_g \left( w - pc, c - D \right) - pu_x \left( w - pc, c - D \right)
\]

\[
\Rightarrow h'(D; c) = -u_{gg} + pu_{xg} > 0
\] (35)

Now consider the value of the integral under changes in \( c \).

\[
\frac{\partial h(D; c)}{\partial c} = u_{gx}(-p) + u_{gg} - pu_{xx}(-p) - pu_{xg}
\]

\[
= -pu_{gx} + u_{gg} + p^2u_{xx} - pu_{xg}
\] (37)

\[
\Rightarrow \frac{\partial h(D; c)}{\partial c} < 0
\] (38)

Which in turn implies that as \( c \) increases, the value of the integral falls.

Applying the implicit function theorem, and supposing that one of the FOCs hold with equality (i.e. not a corner solution), this implies that an increase in \( d \) leads to a fall in \( c \).

### 5.2 Proof: More Idiosyncratic Risk Leads to a More Restrictive Limit

The proof of Theorem 4 that more idiosyncratic risk leads to a more restrictive limit:

We first show that for all \( i \), \( c^*_n(i) \) and \( c^*_1(i) \) both fall. Then we note that if, if all elements of a population falls, than the median also falls. This implies that \( C^*_n \) and \( C^*_1 \) both fall. The result than follows.

The first order condition which characterizes \( c^*_n(i) \) is:

\[
\text{FOC}_c : \quad \int_{\theta \in \Theta} (u_g - p_\theta u_x) dF_b(\theta | i) = 0
\] (40)
The first order condition which characterizes $c_1^*(i)$ is:

$$F OC_c: \int_{\theta \in \Theta} \left[ \sum_{d \in \{0, D\}} \pi_d \left( u_g - p_u u_x \right) \right] dF_b(\theta|i) = 0$$  \hspace{1cm} (41)$$

Where in both cases, $u_g$ and $u_x$ are functions of the limit.

Define the following function:

$$g(\theta; c) = u_g \left( w - pc, c - d \right) - pu_x \left( w - pc, c - d \right)$$  \hspace{1cm} (42)$$

We wish to show that $g$ and $\sum_{d \in \{0, D\}} \pi_d g$ are concave in $\theta$. In the following text, we show the result for $g$, and the result for $\sum_{d \in \{0, D\}} \pi_d g$ is virtually identical, so is omitted. Also, for notational simplicity, let $w$ represent $w_g$ conceived of as a function from $\theta$ to $\mathbb{R}$, and $p$ the same with respect to $p_u$, and the derivatives $w'$, $p'$, $w''$, and $p''$ represent the derivatives of those functions with respect to $\theta$.

Note that the ideal ceiling $c$ which appears in the equation is a also function of $\theta$, but, due to the envelope theorem, we can ignore that fact when considering the derivatives of $g$. Continuing:

$$g'(\theta; c) = u_{gx} \cdot (w' - p'c) - pu_{xx} \cdot (w' - p'c) - p'u_x$$  \hspace{1cm} (43)$$

$$= (u_{gx} - pu_{xx}) \cdot (w' - p'c) - p'u_x$$  \hspace{1cm} (44)$$

$$\Rightarrow g''(\theta; c) = (u_{gx} - pu_{xx}) \cdot (w'' - p''c) + (u_{gxx} - pu_{xxx}) \cdot (w' - p'c)^2$$  \hspace{1cm} (45)$$

$$- p'u_{xx} \cdot (w' - p'c) - p''u_x - p'u_{xx} \cdot (w' - p'c)$$

$$= (u_{gx} - pu_{xx}) \cdot (w'' - p''c) + (u_{gxx} - pu_{xxx}) \cdot (w' - p'c)^2 - 2p'u_{xx} \cdot (w' - p'c) - p''u_x$$  \hspace{1cm} (46)$$

For concavity of $g$, we need $g'' < 0$, so we seek to sign these terms.

First, we need to briefly establish that $(w' - p'c) < 0$ and $(w'' - p''c) < 0$. Assumption 22 requires that $w(\theta) > p(\theta)R$ for all $R$ under consideration, which implies that $w(\theta) > p(\theta)c$. 28
Assumption 31 requires that \( \frac{w'}{w} \leq \frac{p'}{p} \implies \frac{w'}{w} \leq \frac{p'c}{pc} \), which, in conjunction with the claim of the previous statement, implies that \( \frac{w'}{w} < \frac{p'c}{p} \), which implies that \( w' < p'c \), which implies that \( (w' - p'c) < 0 \). A similar exercise establishes that \( (w'' - p''c) < 0 \).

Returning to signing the terms of \( g'' \): Under our utility assumptions, \( u_{gx} - pu_{xx} > 0 \), and as per the previous paragraph, \( (w'' - p''c) < 0 \) which implies the first term is strictly negative. Under our utility assumptions, \( u_{gx} - pu_{xx} \leq 0 \), and \( (w' - p'c)^2 \geq 0 \), so the second term is weakly negative. Under our assumptions, \( p' > 0 \) and \( u_{xx} < 0 \) and as per the previous paragraph, \( (w' - p'c) \leq 0 \), so the third term is weakly negative. Finally, \( p'' \geq 0 \) implies that the last term is weakly negative. Therefore the whole expression is strictly negative.

Now define \( h(\delta; c) \) as the function \( g(1 - \delta; c) \). Then, \( h' = -g' \) and \( h'' = g'' \). It can also be shown that \( g' \) is negative, and therefore \( h' \) is positive. Therefore, \( h \) is an increasing, concave function.

Now suppose that \( b' \) is more risky than \( b \) for all \( i \neq 0, 1 \). That means that \( F_{b'}(\theta|i) \) is a mean-preserving spread of \( F_b(\theta|i) \) for all \( i \neq 0, 1 \). Define \( \hat{F} \) as the survivor function of \( F \); i.e. that \( \hat{F}(\delta) = F(1 - \delta) \). Then \( \hat{F}_b(\delta|i) \) is a mean-preserving spread of \( \hat{F}_b(\delta|i) \). Since \( h \) is increasing and concave, then by the principle of second-order stochastic dominance, if \( b \) second-order stochastically dominates \( b' \), then \( \int h(\delta; c)d\hat{F}_{b'}(\delta|i) < \int h(\delta; c)d\hat{F}_b(\delta|i) \). So, holding \( c \) constant, increasing the riskiness decreases the value of this integral.

Now consider the value of the integral under changes in \( c \).

\[
\frac{\partial h(\delta; c)}{\partial c} = u_{gx}(-p) + u_{gg} - pu_{xx}(-p) - pu_{xg} \tag{47}
\]

\[
= -pu_{gx} + u_{gg} + p^2u_{xx} - pu_{xg} \tag{48}
\]

\[
\Rightarrow \frac{\partial h(\delta; c)}{\partial c} < 0 \tag{49}
\]

Which in turn implies that as \( c \) increases, the value of the integral falls.

Applying the implicit function theorem, and supposing that one of the FOCs hold with equality.
(i.e. not a corner solution, which occurs when $c^* < R_D$), we find that the change in $c$ due to an increase in riskiness is equal to the negative of the change in the integral of $h$ due to riskiness divided by the change in the integral of $h$ due to $c$. Since the signs of both of those terms are negative, then we find an increase in idiosyncratic riskiness leads to a fall in $c$. If it is a corner solution, then this is a weak relationship. QED.

An almost identical argument also serves to prove that $\sum_{d \in \{0, D\}} \pi_d h$ is concave. Given its similarity, it is omitted.

Proof of Corollary: Now suppose that Assumption [51] is violated by a sufficiently large amount; enough to cause $g'' > 0$. Now define the function $\tilde{g}(\theta; c) = -g(\theta; c)$. Note that $\tilde{g}' = -g' > 0$ and $\tilde{g}'' = -g'' < 0$, so $\tilde{g}$ is an increasing, concave function. Now suppose that $b'$ is more risky than $b$ for all $i \neq 0, 1$, so $F_{b'}(\theta|i)$ is a mean-preserving spread of $F_b(\theta|i)$ for all $i \neq 0, 1$. By the principle of second-order stochastic dominance, $\int_{\theta} \tilde{g}(\theta; c)d\hat{F}_b(\theta|i) > \int_{\theta} \tilde{g}(\theta; c)dF_{b'}(\theta|i)$ which implies $\int_{\theta} g(\theta; c)d\hat{F}_b(\theta|i) < \int_{\theta} g(\theta; c)dF_{b'}(\theta|i)$. This means, as riskiness increases, the value of this integral increases. Also, consider that, following 47, $\frac{\partial g(\theta; c)}{\partial c} < 0$. In this case, the implicit function theorem implies that an increase in idiosyncratic riskiness leads to an increase in $c$.

References


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