APPENDIX

The Simple Algebra of the *IS-LM* Model and the Aggregate Demand Curve

The chapter analyzes the *IS-LM* model with graphs of the *IS* and *LM* curves. Here we analyze the model with algebra. This alternative presentation offers additional insight into how monetary and fiscal policy influence aggregate demand.

The *IS* Curve

One way to think about the *IS* curve is that it describes the combinations of income Y and the interest rate r that satisfy an equation we first saw in Chapter 3:

$$Y = C(Y - T) + I(r) + G.$$

This equation combines the national income accounts identity, the consumption function, and the investment function. It states that the quantity of goods produced, Y, must equal the quantity of goods demanded, $C + I + G$.

We can learn more about the *IS* curve by considering the special case in which the consumption function and investment function are linear. That is, suppose that the consumption function is

$$C(Y - T) = a + b(Y - T),$$

where a and b are numbers greater than zero. The parameter b is the marginal propensity to consume, so we expect b to be between zero and one. The parameter a influences the level of consumption; it captures everything that affects consumer spending other than disposable income. Similarly, suppose the investment function is

$$I(r) = c - dr,$$

where c and d also are numbers greater than zero. The parameter d determines how much investment responds to the interest rate; because investment rises when the interest rate falls, there is a minus sign in front of d. The parameter c influences the level of investment; it captures everything that affects investment spending other than the interest rate.

We can now derive an algebraic expression for the *IS* curve and see what influences the *IS* curve's position and slope. If we substitute the consumption and investment functions into the goods-market equilibrium condition, we obtain

$$Y = [a + b(Y - T)] + (c - dr) + G.$$

Note that Y shows up on both sides of this equation. We can simplify this equation by bringing all the Y terms to the left-hand side and rearranging the terms on the right-hand side:

$$Y - bY = (a + c) + (G - bT) - dr.$$

We solve for Y to get

$$Y = \frac{a + c}{1 - b} + \frac{1}{1 - b}G + \frac{-b}{1 - b}T + \frac{-d}{1 - b}r.$$

This equation expresses the *IS* curve algebraically. It tells us the level of income Y for any given interest rate r and fiscal policy G and T. Holding fiscal policy fixed, the equation gives us a relationship between the interest rate and the level of income: the higher the interest rate, the lower the level of income. The *IS* curve graphs this equation for different values of Y and r given fixed values of G and T.

Using this last equation, we can verify our previous conclusions about the *IS* curve. First, because the coefficient of the interest rate is negative, the *IS* curve slopes downward: higher interest rates reduce income. Second, because the coefficient of government purchases is positive, an increase in government purchases shifts the *IS* curve to the right. Third, because the coefficient of taxes is negative, an increase in taxes shifts the *IS* curve to the left.

The coefficient of the interest rate, $-d/(1 - b)$, tells us what determines whether the *IS* curve is steep or flat. If investment is highly sensitive to the interest rate, then d is large, and income is highly sensitive to the interest rate as well. In this case, small changes in the interest rate lead to large changes in income: the *IS* curve is relatively flat. Conversely, if investment is not very sensitive to the interest rate, then d is small, and income is also not very sensitive to the interest rate. In this case, large changes in interest rates lead to small changes in income: the *IS* curve is relatively steep.

Similarly, the slope of the *IS* curve depends on the marginal propensity to consume b. The larger the marginal propensity to consume, the larger the change in income resulting from a given change in the interest rate. The reason is that a large marginal propensity to consume leads to a large multiplier for changes in investment. The larger the multiplier, the larger the impact of a change in investment on income, and the flatter the *IS* curve.

The marginal propensity to consume b also determines how much changes in fiscal policy shift the *IS* curve. The coefficient of G, $1/(1 - b)$, is the government-purchases multiplier in the Keynesian cross. Similarly, the coefficient of T, $-b/(1 - b)$, is the tax multiplier in the Keynesian cross. The larger the marginal propensity to consume, the greater the multiplier, and thus the greater the shift in the *IS* curve that arises from a change in fiscal policy.

The *LM* Curve

The *LM* curve describes the combinations of income Y and the interest rate r that satisfy the money-market equilibrium condition

$$M/P = L(r, Y).$$

This equation simply equates money supply and money demand.

We can learn more about the *LM* curve by considering the case in which the money demand function is linear—that is,

$$L(r, Y) = eY - fr,$$

where e and f are numbers greater than zero. The value of e determines how much the demand for money rises when income rises. The value of f determines how much the demand for money falls when the interest rate rises. There is a minus sign in front of the interest rate term because money demand is inversely related to the interest rate.

The equilibrium in the money market is now described by

$$M/P = eY - fr.$$

To see what this equation implies, rearrange the terms so that r is on the left-hand side. We obtain

$$r = (e/f)Y - (1/f)M/P.$$

This equation gives us the interest rate that equilibrates the money market for any values of income and real money balances. The *LM* curve graphs this equation for different values of Y and r given a fixed value of M/P.

From this last equation, we can verify some of our conclusions about the *LM* curve. First, because the coefficient of income is positive, the *LM* curve slopes upward: higher income requires a higher interest rate to equilibrate the money market. Second, because the coefficient of real money balances is negative, decreases in real balances shift the *LM* curve upward, and increases in real balances shift the *LM* curve downward.

From the coefficient of income, e/f, we can see what determines whether the *LM* curve is steep or flat. If money demand is not very sensitive to the level of income, then e is small. In this case, only a small change in the interest rate is necessary to offset the small increase in money demand caused by a change in income: the *LM* curve is relatively flat. Similarly, if the quantity of money demanded is not very sensitive to the interest rate, then f is small. In this case, a shift in money demand due to a change in income leads to a large change in the equilibrium interest rate: the *LM* curve is relatively steep.

The Aggregate Demand Curve

To find the aggregate demand equation, we must find the level of income that satisfies both the *IS* equation and the *LM* equation. To do this, substitute the *LM* equation for the interest rate r into the *IS* equation to obtain

$$Y = \frac{a + c}{1 - b} + \frac{1}{1 - b}G + \frac{-b}{1 - b}T + \frac{-d}{1 - b}\left(\frac{e}{f}Y - \frac{1}{f}\frac{M}{P}\right).$$

With some algebraic manipulation, we can solve for Y. The final equation for Y is

$$Y = \frac{z(a + c)}{1 - b} + \frac{z}{1 - b}G + \frac{-zb}{1 - b}T + \frac{d}{(1 - b)[f + de/(1 - b)]}\frac{M}{P},$$

where $z = f/[f + de/(1 - b)]$ is a composite of some of the parameters and is between zero and one.

This last equation expresses the aggregate demand curve algebraically. It says that income depends on fiscal policy, G and T, monetary policy M, and the price

level P. The aggregate demand curve graphs this equation for different values of Y and P, given fixed values of G, T, and M.

We can explain the slope and position of the aggregate demand curve with this equation. First, the aggregate demand curve slopes downward, because an increase in P lowers M/P and thus lowers Y. Second, increases in the money supply raise income and shift the aggregate demand curve to the right. Third, increases in government purchases or decreases in taxes also raise income and shift the aggregate demand curve to the right. Note that, because z is less than one, the multipliers for fiscal policy are smaller in the *IS-LM* model than in the Keynesian cross. Hence, the parameter z reflects the crowding out of investment discussed earlier.

Finally, this equation shows the relationship between the aggregate demand curve derived in this chapter from the *IS-LM* model and the aggregate demand curve derived in Chapter 9 from the quantity theory of money. The quantity theory assumes that the interest rate does not influence the quantity of real money balances demanded. Put differently, the quantity theory assumes that the parameter f equals zero. If f equals zero, then the composite parameter z also equals zero, so fiscal policy does not influence aggregate demand. Thus, the aggregate demand curve derived in Chapter 9 is a special case of the aggregate demand curve derived here.

CASE STUDY

The Effectiveness of Monetary and Fiscal Policy

Economists have long debated whether monetary or fiscal policy exerts a more powerful influence on aggregate demand. According to the *IS-LM* model, the answer to this question depends on the parameters of the *IS* and *LM* curves. Therefore, economists have spent much energy arguing about the size of these parameters. The most hotly contested parameters are those that describe the influence of the interest rate on economic decisions.

Those economists who believe that fiscal policy is more potent than monetary policy argue that the responsiveness of investment to the interest rate—measured by the parameter d—is small. If you look at the algebraic equation for aggregate demand, you will see that a small value of d implies a small effect of the money supply on income. The reason is that when d is small, the *IS* curve is nearly vertical, and shifts in the *LM* curve do not cause much of a change in income. In addition, a small value of d implies a large value of z, which in turn implies that fiscal policy has a large effect on income. The reason for this large effect is that when investment is not very responsive to the interest rate, there is little crowding out.

Those economists who believe that monetary policy is more potent than fiscal policy argue that the responsiveness of money demand to the interest rate—measured by the parameter f—is small. When f is small, z is small, and fiscal policy has a small effect on income; in this case, the *LM* curve is nearly vertical. In addition, when f is small, changes in the money supply have a large effect on income.

Few economists today endorse either of these extreme views. The evidence indicates that the interest rate affects both investment and money demand. This finding implies that both monetary and fiscal policy are important determinants of aggregate demand.

MORE PROBLEMS AND APPLICATIONS

1. Give an algebraic answer to each of the following questions. Then explain in words the economics that underlies your answer.
   a. How does the sensitivity of investment to the interest rate affect the slope of the aggregate demand curve?
   b. How does the sensitivity of money demand to the interest rate affect the slope of the aggregate demand curve?
   c. How does the marginal propensity to consume affect the response of aggregate demand to changes in government purchases?