Problem set 8
More about reserve demand

Suppose the Fed does *not* pay interest on reserves. There is no reserve requirement. The Fed charges an interest rate for emergency loans
r_p to cover overdrafts. This interest rate is equal to *2*. That is, $r_p = 2$. All banks in the country are identical. Each bank has *$100* to divide between its reserve account and overnight lending. At 5 pm each bank will choose how much to leave in its reserve account.Between 5 and 6 pm. the Fed will clear payments between banks, adding a net sum *P* to each bank's reserve account. *P* can be a postive or negative number. That leaves *R+P* in the bank's reserve account at 6 pm. A bank will have overdrawn its reserve account if the balance after clearing, at 6 pm, falls below zero..A bank that overdraws its reserve account must take an emergency loan from the Fed to cover the overdraft, to bring its reserve account up to a zero balance. From a bank's point of view, *P* is a random variable, uniformly distributed between a minimum value (the smallest possible net payment into the bank's reserve account) of *-2*, and a maximum value (the largest possible payment into the bank's reserve account) of *+2*. The market overnight rate is denoted *r*.

1) Using the information given above, write an expression that gives the probability that a bank will run an overdraft in its reserve account, for any given value of *R.*, assuming *r* is greater than zero but less than 2.

Prob. $R+P<0$
$P<-R$

$$F\{-R\} = \frac{-R-(-2)}{2-(-2)} = \frac{-R+2}{4} = \frac{1}{2} - \frac{1}{4}R$$

2) *Assuming* a bank runs an overdraft in its reserve account, what is the expected value of the amount that the bank will have to borrow from the Fed, for any given value of *R*?

$$E[0-(R+P)/P<-R] = -R - E[P/P<-R]$$
$$= -R - \tfrac{1}{2}(-R+(-2)) = -R + \tfrac{1}{2}R + \tfrac{1}{2}2$$
$$= -\tfrac{1}{2}R + 1 = 1 - \tfrac{1}{2}R$$

3) Using your answers to a) and b), write an expression that gives, for any value of *R*, the expected value of the bank's profit. Remember $r_p = 2$!

$$E[\pi] = r(100-R) - (\tfrac{1}{2} - \tfrac{1}{4}R)2(1 - \tfrac{1}{2}R)$$
$$= 100r - rR - (1 - \tfrac{1}{2}R)(1 - \tfrac{1}{2}R)$$
$$= 100r - rR - (1 - \tfrac{1}{2}R - \tfrac{1}{2}R + \tfrac{1}{4}R^2)$$
$$= 100r - rR - (1 - R + \tfrac{1}{4}R^2)$$
$$= 100r - rR - 1 + R - \tfrac{1}{4}R^2$$

4) Using your answer to 3) and calculus and algebra, find the reserve balance R^D that a bank would choose to leave in its reserve account at 5 pm, as a function of r.

$$0 = \frac{\partial E[\pi]}{\partial R} = -r + 1 - \frac{1}{4} 2 R^*$$

$$0 = -r + 1 - \frac{1}{2} R^*$$

$$\frac{1}{2} R^* = -r + 1$$

$$R^* = 2 - 2r$$

5) Suppose the target overnight rate is 1/2. What is the reserve supply per bank that will cause the market overnight rate to hit the target?

$$R^S = 2 - 2\frac{1}{2} = 2 - 1 = 1$$

6) On the graph below, draw a bank's reserve demand curve, and the reserve supply per bank that will cause the market overnight rate to hit the target. Be precise; notice the numbers on the axes.