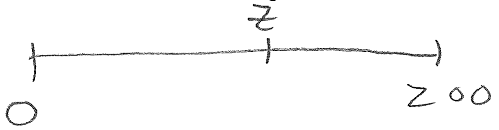
Problem set ~~9~~ 11

An analogue to the reserve demand model

Suppose you are planning a party. Let *N* denote the number of people who will come to the party. You are not sure what *N* will be - you are not sure how many people will come to the party - but you have a probability distribution for *N*. This distribution is *uniform*. The largest possible value of *N* (the most people who will possibly come) is 200. The smallest possible value of *N* (fewest people who will possibly come) is zero.

1) Let *Z* denote a number between zero and 200. What is the probability that the number of people who come to the party *N* is less than or equal to *Z*?

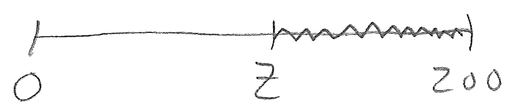


$$F\{Z\} = \frac{Z-0}{200-0} = \frac{Z}{200}$$

2) What is the probability that *N* turns out to be *more* than *Z*?

$$1 - F\{Z\} = 1 - \frac{Z}{200}$$

3) *Assuming* more than *Z* people come to the party, what is the expected value of *N* (that is, on the condition that *N*>*Z*)? (This is the same as the expected value of *N* assuming *Z* or more people come to the party.)



$$E[N/N>Z] = \tfrac{1}{2}(Z+200) = \tfrac{1}{2}Z + 100$$

4) Each guest will want to drink exactly one case of beer. (You yourself will drink no beer.) You can buy beer the day before the party at a store where beer is cheap. You don't want to buy too much beer because your mother is coming to visit the morning after the party and you don't want her to know you keep beer in your apartment. Let *B* denote the number of cases you buy the day before the party. Assuming *B* is between zero and 200, what is the probability you will run out of beer at the party? Run out of beer if $N>B$

Probability $N>B = 1 - F\{B\} = 1 - \frac{B}{200}$

Check: if $B\uparrow$, ... ok!

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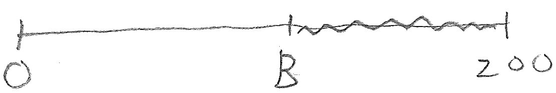
5) If you run out of beer at the party, you will have to run to an expensive beer store next to your apartment building and buy the cases you lack. Given *B* (between zero and 200), what is the expected value of the number of cases you will have to buy from the expensive beer store *assuming* you run out of beer at the party (that is, *on the condition* that you run out of beer)? If $N > B$, buy $N - B$ cases.

$$E[N-B \mid N>B] = -B + E[N \mid N>B] = -B + \tfrac{1}{2}(B+200)$$

$$= -B + \tfrac{1}{2}B + 100$$



$$= 100 - \tfrac{1}{2}B$$ Check: if... ok!

6) Suppose that the price of a case of beer at the cheap beer store is *P*. The price of a case at the expensive store next to your aparment building is $\hat{P}$. ($\hat{P}$ is greater than *P*.) Write a mathematical expression that gives the expected value of the total cost of beer that you buy, that is the total cost of beer from the cheap store plus the beer you have to buy from the expensive store, given *B*. (B still denotes the number of cases you buy at the cheap store, between zero and 200).

$$E[TC] = PB + \left(1 - \frac{B}{200}\right)\hat{P}\left(100 - \tfrac{1}{2}B\right)$$

$$= PB + \hat{P}\left(100 - \tfrac{1}{2}B - \tfrac{1}{2}B + \frac{1}{400}B^2\right)$$

$$= PB + \hat{P}\left(100 - B + \frac{1}{400}B^2\right)$$

$$= PB + \hat{P}100 - \hat{P}B + \hat{P}\frac{1}{400}B^2$$

7) When you are at the cheap beer store, you will buy the amount of beer that minimizes the expected value of the total cost of beer as you defined it in part 6). Using your answer to 6), figure out the number of cases you will buy at the cheap beer store (that is the optimal value of *B*).

$$\frac{\partial E[TC]}{\partial B} = P - \hat{P} + \hat{P}\frac{1}{400}2B = P - \hat{P} + \hat{P}\frac{1}{200}B$$

$$0 = P - \hat{P} + \hat{P}\frac{1}{200}B^*$$ Solve for B^*

$$-P + \hat{P} = \hat{P}\frac{1}{200}B^*$$

$$-\frac{P}{\hat{P}} + 1 = \frac{1}{200}B^*$$

$$B^* = 200\left(-\frac{P}{\hat{P}} + 1\right) = 200 - 200\frac{P}{\hat{P}}$$

Check: if $(P/\hat{P})\uparrow$, does $B^*\downarrow$? Yes!

What if $P = \hat{P}$? Equation says $B^* = 0$. Makes sense!