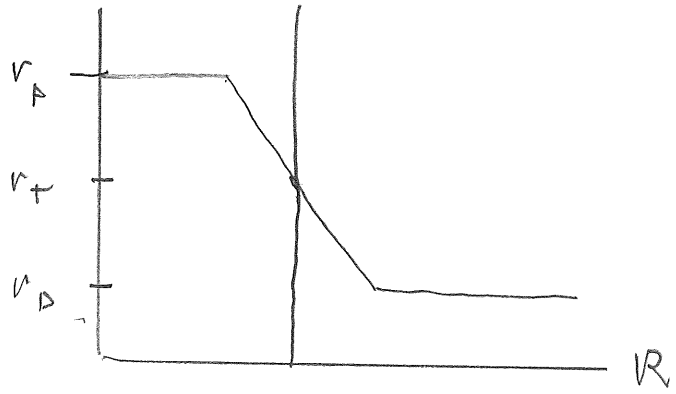
Problem set ~~4~~ 9
Reserve demand

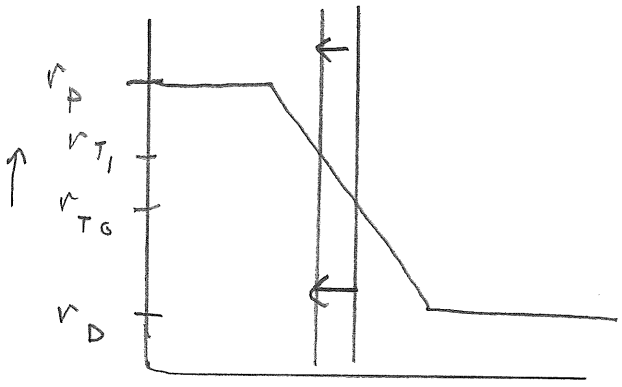
1) Consider the demand for reserves and determination of the market overnight interest rate in an economy where the central bank pays an interest rate r_D on reserve balances and charges an interest rate r_P for emergency loans to cover overdrafts. r_D is *lower* than the central bank's target overnight rate r_T. r_P is *higher* than the central bank's target overnight rate r_T.

a) Draw a graph that shows reserve demand and the reserve supply that will cause the market overnight rate r to hit the central bank's target.



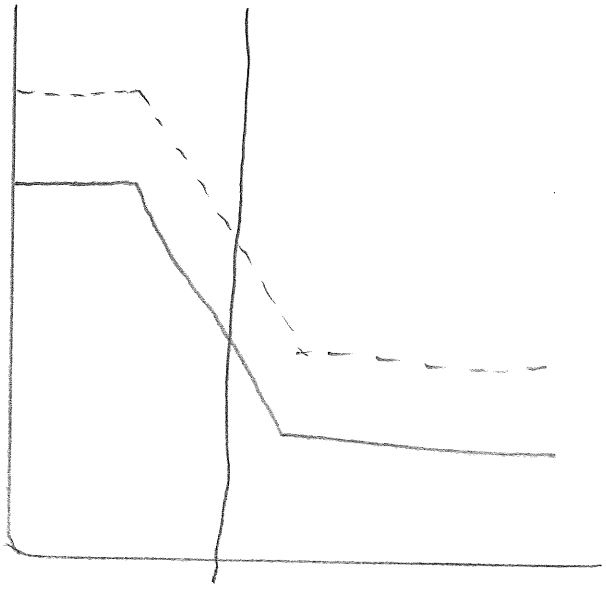
b) Suppose the central bank's policy commitee raises the target overnight rate r_T while making *no* change to r_D and r_P. (It raises r_T only a little, so that it is still between r_D and r_P.)

Draw a graph that describes this event, and what is likely to happen to reserve supply.



c) Now suppose that the central bank always adjusts r_D and r_P when it changes r_T : r_P is always equal to r_T *plus* one percent; r_D is always equal to r_T *minus* one percent.

Draw a graph that describes this event, and what is likely to happen to reserve supply.



Problem set 9

2) Consider a bank that has total funds F to divide between its reserve account at the central bank and overnight lending. The bank receives an interest rate r on overnight lending. If the bank puts a sum R in its reserve account, it has $(F-R)$ left to lend out overnight, giving earnings of $(F-R)r$.

The central bank does *not* pay interest on reserves. After the end of the day, the central bank clears payments between banks, adding a net sum P to the bank's reserve account, where P can be a negative number. That leaves $R+P$ in the bank's reserve account. From the bank's point of view, P is a random variable, uniformly distributed between a minimum value (the smallest possible net payment into the bank's reserve account) of -10, and a maximum value (the largest possible payment into the bank's reserve account) of $+10$. (Note $\bar{P}=10$, $-\bar{P}=-10$)

The reserve requirement is 5. If the balance in the bank's reserve account falls below 5 after clearing, the bank must take an emergency loan from the central bank to cover the shortfall. The central bank charges an interest rate r_P for emergency loans to cover overdrafts.

a) What is the smallest quantity of reserves that the bank will choose to hold if the market overnight rate r is equal to zero?

Make sure you can't have a shortfall: 15

(Note this is $K+\bar{P}$)

b) What is the largest quantity of reserves that the bank will choose to hold if the market interest rate r is as high as the central bank's emergency lending rate r_P?

Make sure you can't have excess: -5

(Note this is $K-\bar{P}$)

c) Given a value of R somewhere between the values in a) and b), what is the probability that a bank will run an ~~overdraft~~ shortfall in its reserve account? Check: a higher value of R should make this probability *smaller*.

Prob $R+P<K$, Prob $P<-R+K = F\{-R+K\} = \frac{-R+5-(-10)}{10-(-10)} = \frac{15}{20} - \frac{R}{20} = \frac{3}{4} - \frac{R}{20}$

d) Assuming a bank runs an ~~overdraft~~ shortfall in its reserve account, what is the expected value of the amount that the bank will have to borrow from the central bank?

IF shortfall, must borrow $K-(R+P)$

What is $E[K-(R+P)/P<-R+K]$?

$= K-R-E[P/P<-R+K]$

$= 5-R-\frac{1}{2}(-R+5+(-10))$

$= 5-R-\frac{1}{2}(-R-5) = 5-R+\frac{1}{2}R+\frac{1}{2}5$

$= 7\frac{1}{2}-\frac{1}{2}R$

Problem set 10
More about reserve demand

Suppose the Fed does *not* pay interest on reserves. There is no reserve requirement. The Fed charges an interest rate for emergency loans
r_p to cover overdrafts. This interest rate is equal to *2*. That is, $r_p = 2$. All banks in the country are identical. Each bank has *$100* to divide between its reserve account and overnight lending. At 5 pm each bank will choose how much to leave in its reserve account.Between 5 and 6 pm. the Fed will clear payments between banks, adding a net sum *P* to each bank's reserve account. *P* can be a postive or negative number. That leaves *R+P* in the bank's reserve account at 6 pm. A bank will have overdrawn its reserve account if the balance after clearing, at 6 pm, falls below zero..A bank that overdraws its reserve account must take an emergency loan from the Fed to cover the overdraft, to bring its reserve account up to a zero balance. From a bank's point of view, *P* is a random variable, uniformly distributed between a minimum value (the smallest possible net payment into the bank's reserve account) of *-2*, and a maximum value (the largest possible payment into the bank's reserve account) of *+2*. The market overnight rate is denoted *r*.

1) Using the information given above, write an expression that gives the probability that a bank will run an overdraft in its reserve account, for any given value of *R.*, assuming *r* is greater than zero but less than 2.

Prob. $R+P<0$

$P<-R$

$$F\{-R\} = \frac{-R-(-2)}{2-(-2)} = \frac{-R+2}{4} = \frac{1}{2} - \frac{1}{4}R$$

2) *Assuming* a bank runs an overdraft in its reserve account, what is the expected value of the amount that the bank will have to borrow from the Fed, for any given value of *R*?

$$E[0-(R+P)/P<-R] = -R - E[P/P<-R]$$
$$= -R - \tfrac{1}{2}(-R+(-2)) = -R + \tfrac{1}{2}R + \tfrac{1}{2}2$$
$$= -\tfrac{1}{2}R + 1 = 1 - \tfrac{1}{2}R$$

3) Using your answers to a) and b), write an expression that gives, for any value of *R*, the expected value of the bank's profit. Remember $r_p = 2$!

$$E[\pi] = r(100-R) - (\tfrac{1}{2} - \tfrac{1}{4}R)2(1-\tfrac{1}{2}R)$$
$$= 100r - rR - (1-\tfrac{1}{2}R)(1-\tfrac{1}{2}R)$$
$$= 100r - rR - (1 - \tfrac{1}{2}R - \tfrac{1}{2}R + \tfrac{1}{4}R^2)$$
$$= 100r - rR - (1 - R + \tfrac{1}{4}R^2)$$
$$= 100r - rR - 1 + R - \tfrac{1}{4}R^2$$

Problem set 10

4) Using your answer to 3) and calculus and algebra, find the reserve balance R^D that a bank would choose to leave in its reserve account at 5 pm, as a function of r.

$$0 = \frac{\partial E[\pi]}{\partial R} = -r + 1 - \frac{1}{4} 2 R^*$$

$$0 = -r + 1 - \frac{1}{2} R^*$$

$$\frac{1}{2} R^* = -r + 1$$

$$R^* = 2 - 2r$$

5) Suppose the target overnight rate is 1/2. What is the reserve supply per bank that will cause the market overnight rate to hit the target?

$$R^S = 2 - 2\,\frac{1}{2} = 2 - 1 = 1$$

6) On the graph below, draw a bank's reserve demand curve, and the reserve supply per bank that will cause the market overnight rate to hit the target. Be precise; notice the numbers on the axes.