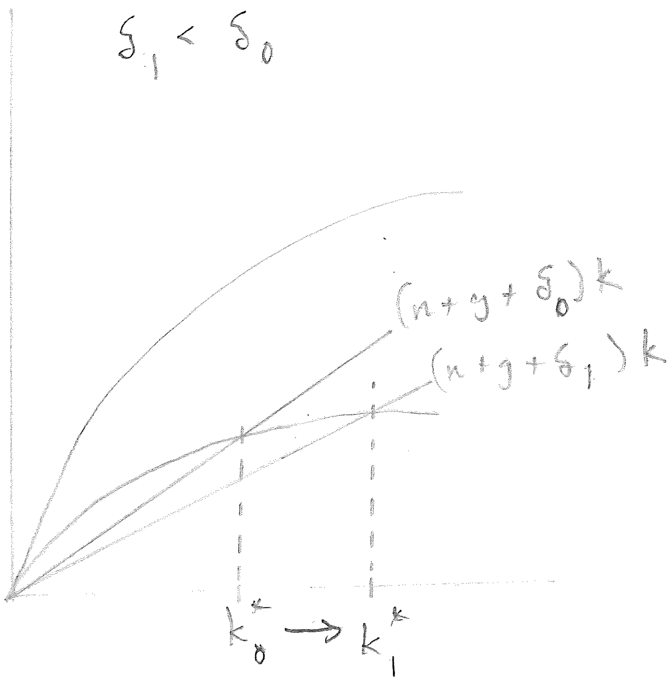
Solow Model

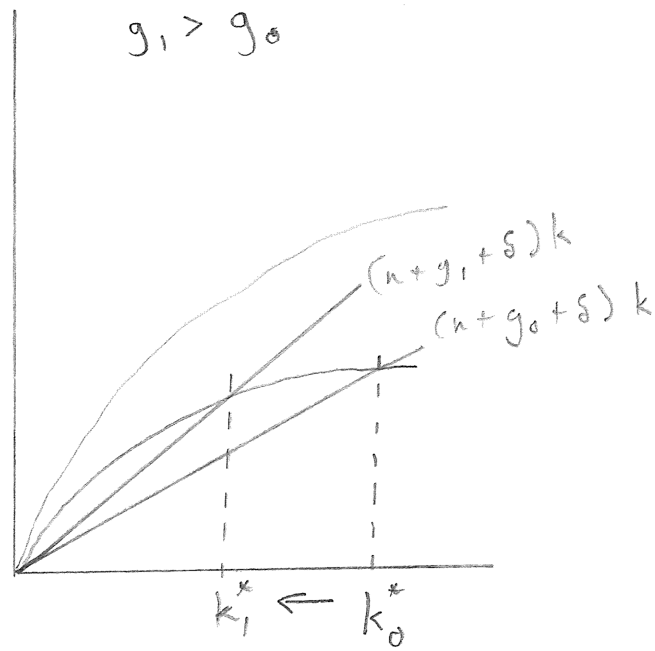
Answers to Problems

1.3

a) δ Falls



b) g rises



c) $y = f(k) = k^\alpha = \left(\frac{K}{AL}\right)^\alpha$ and $\alpha \uparrow$

If $\alpha\uparrow$, y is higher at any k

$f'(k) = \alpha k^{\alpha-1}$

If $\alpha\uparrow$, MPK (slope of y line) greater at any k

Example:

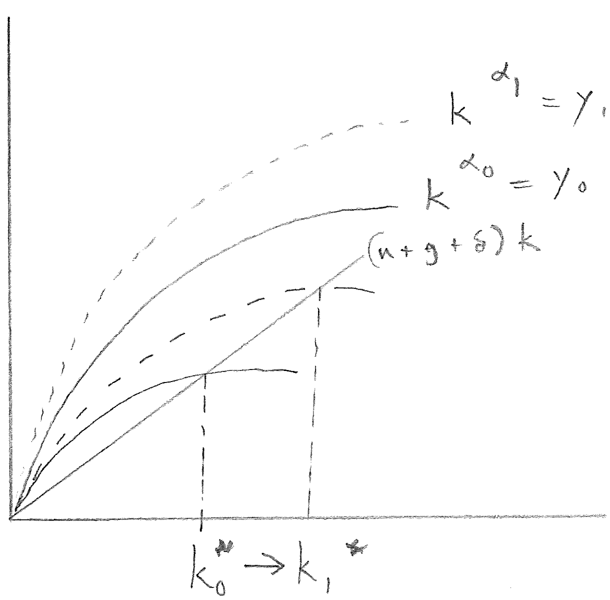
Low $\alpha = 1/3$

$f'(k) = \frac{1}{3}\frac{1}{k^{2/3}}$

High $\alpha = 1/2$

$f'(k) = \frac{1}{2}\frac{1}{k^{1/2}}$

$\alpha_1 > \alpha_0$



Solow Model

Answers to Problems (cont.)

1.3

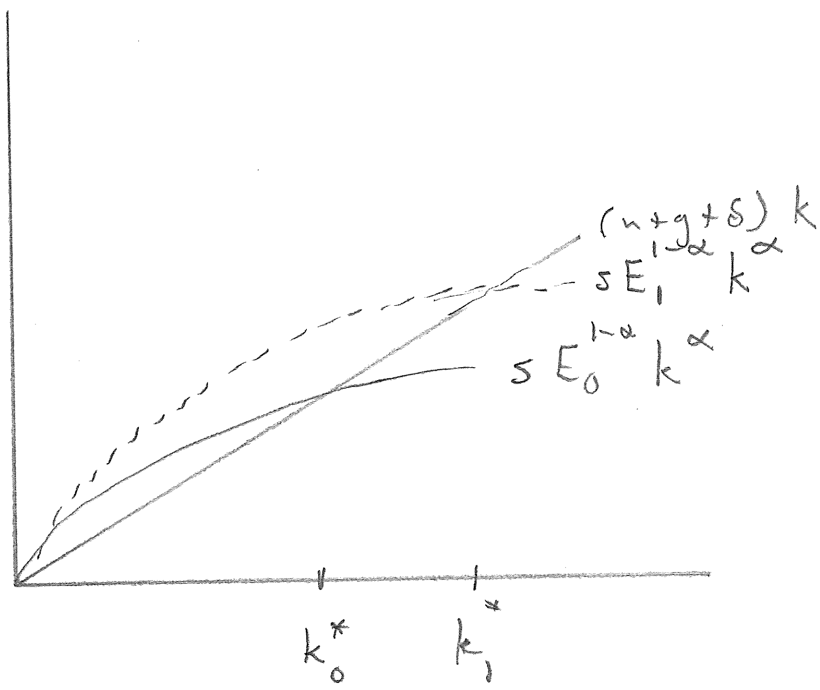
d) Effort ↑

Effort

My interpretation: $Y = K^{\alpha}(AEL)^{1-\alpha}$

means $y = K^{\alpha}(AEL)^{1-\alpha}(AL)^{-1} = E^{1-\alpha}k^{\alpha}$

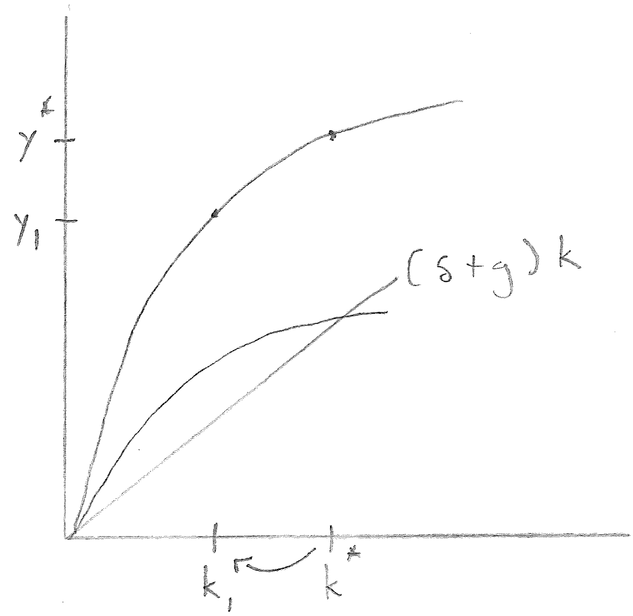
The event is that E increases from low value E_0 to $E_1 > E_0$

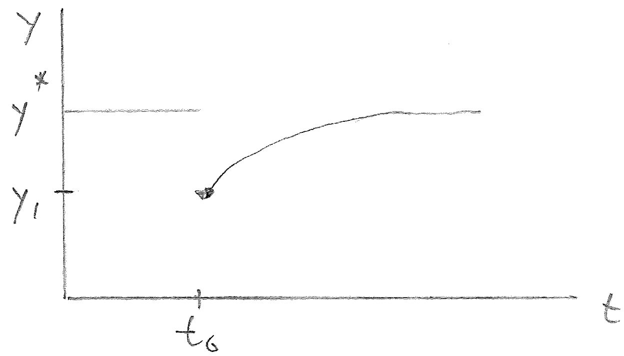


Solow Model

Answers to Problems

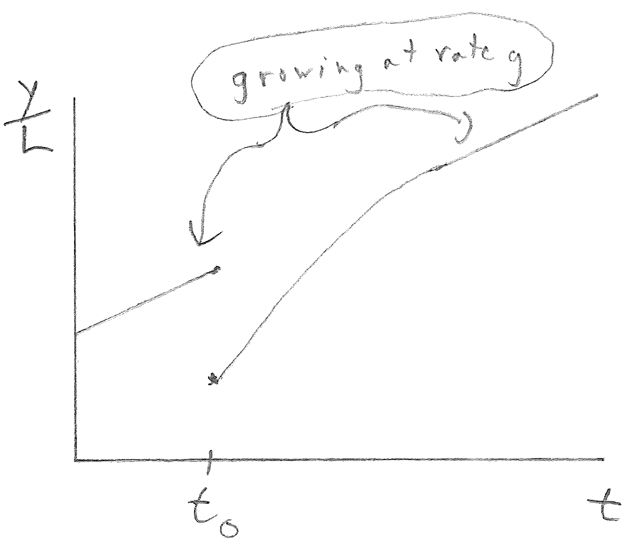
1.4) One-time jump in L





a)&b) y falls from y^* to y_1, then rises back to y^*

c) Same



Solow Model

Answers to Problems

1.5) $Y = K^{\alpha}(AL)^{1-\alpha}$

In "intensive form,"

$$y = \frac{Y}{AL} = \frac{K^{\alpha}(AL)^{1-\alpha}}{AL} = K^{\alpha}(AL)^{-\alpha} = (K/AL)^{\alpha}$$

$$f(k) = y = k^{\alpha}$$

$$f'(k) = \alpha k^{\alpha - 1}$$

a) In LRE,

$$sf(k^*) = (n+g+\delta)k^*$$

Here $s k^{\alpha} = (n+g+\delta)k^*$

$$k^{1-\alpha} = \frac{s}{n+g+\delta}$$

$$k^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}}$$

$$y^* = f(k^*)$$

Here $y^* = k^{*\alpha} = \left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}}$

$$c^* = (1-s)f(k^*)$$

Here $c^* = (1-s)\left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}}$

Solow Model

Answers to Problems

1.5) Cont.

b) Golden Rule

$$f'(k^{GR}) = n+g+\delta$$

Here $\alpha k^{GR^{\alpha-1}} = n+g+\delta$

$$k^{GR} = \left(\frac{n+g+\delta}{\alpha}\right)^{-\frac{1}{1-\alpha}} = \left(\frac{\alpha}{n+g+\delta}\right)^{\frac{1}{1-\alpha}}$$

c) s^{GR}

In any LRE,

$$sf(k) = (n+g+\delta)k$$

Here $sk^{\alpha} = (n+g+\delta)k$

$$s = (n+g+\delta)k^{1-\alpha}$$

At GR, $k = \left(\frac{\alpha}{n+g+\delta}\right)^{\frac{1}{1-\alpha}}$

$$s^{GR} = (n+g+\delta)\left(\left(\frac{\alpha}{n+g+\delta}\right)^{\frac{1}{1-\alpha}}\right)^{1-\alpha} = (n+g+\delta)\left(\frac{\alpha}{n+g+\delta}\right)$$

$$= \alpha$$

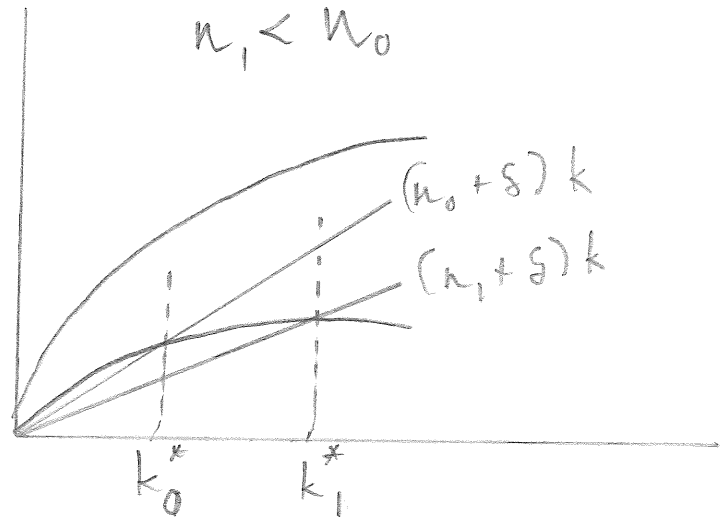
Saving rate = capital's income share

assuming $r = MPK$

Solow Model

Answers to Problems

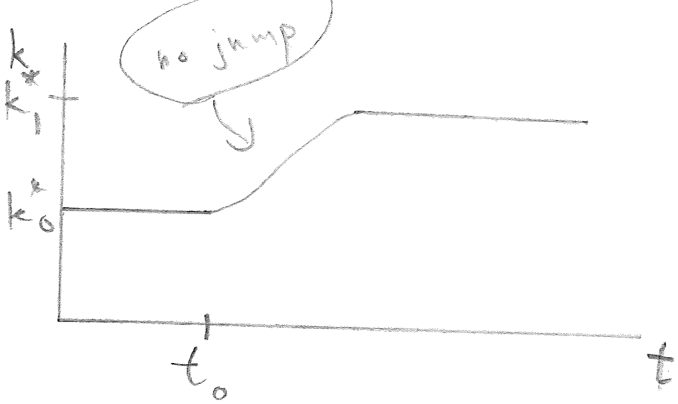
1.6) $g = 0$, n Falls



$k_1^* > k_0^*$

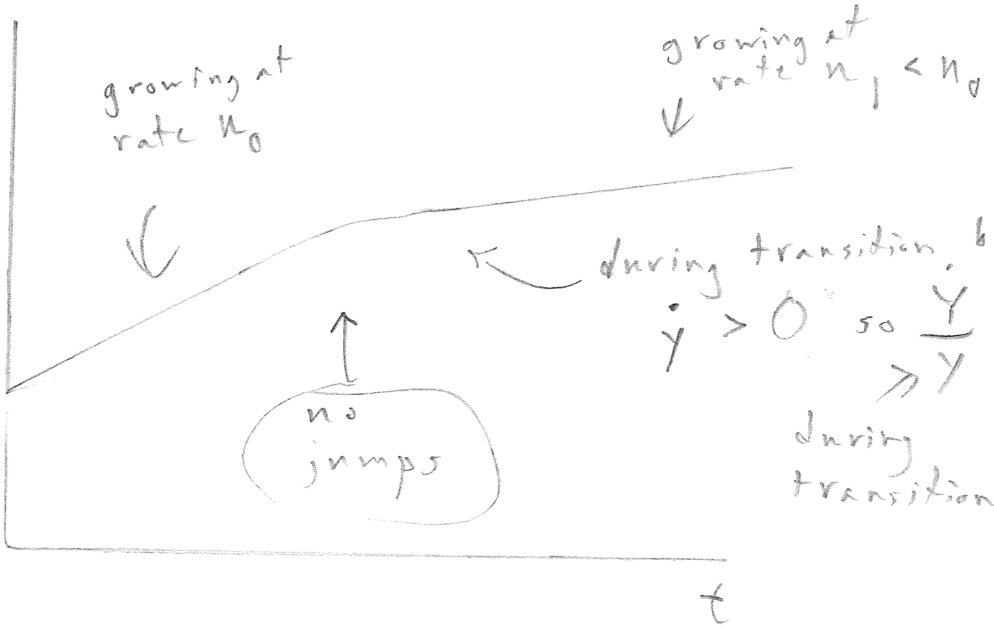
$y_1^* > y_0^*$

$c_1^* > c_0^*$



Output & consumption per worker follow similar paths

What about Y? Recall $\frac{\dot{Y}}{Y} = n+g$ here n



during transition, between y_0^* & y_1^*,

$\dot{y} > 0$ so $\underbrace{\frac{\dot{Y}}{Y}}_{\text{during transition}} > \frac{\dot{Y}_1^*}{Y_1^*} = n_1$

SOLOW MODEL

1.7) What is $\frac{\partial y^*/y}{\partial n/n} = \frac{\partial y}{\partial n}\frac{n}{y}$?

What is $\% \Delta y$ if n falls from 2% to 1%, given
$\alpha_K = 1/3$, $g = 2\%$, $\delta = 3\%$?

Answer: follow (1, 22) on

$$\frac{\partial y}{\partial n} = f'(k)\frac{\partial k}{\partial n}$$

At k^*, $sf(k^*) = (n+g+\delta)k^*$ ⟶ gives $s = \frac{(n+g+\delta)k}{f(k)}$

$$\frac{\partial(sf(k^*))}{\partial n} = \frac{\partial((n+g+\delta)k^*)}{\partial n}$$

$$sf'(k)\frac{\partial k}{\partial n} = (n+g+\delta)\frac{\partial k}{\partial n} + k$$

Solve for $\frac{\partial k}{\partial n}$:

$$\frac{\partial k}{\partial n} = \frac{k^*}{sf'(k) - (n+g+\delta)}$$ ⟵ put in for s

$$\frac{\partial y^*}{\partial n} = f'(k)\frac{\partial k}{\partial n} = \frac{f'(k)\,k^*}{\frac{(n+g+\delta)f'(k)k}{f(k)} - (n+g+\delta)}$$

$f(k)$ ⟵ y

$$\frac{\partial y^*}{\partial n}\frac{n}{y} = \frac{f'(k)k\,n}{(n+g+\delta)f'(k) - (n+g+\delta)y}$$

$$= -\frac{n}{n+g+\delta}\,\frac{f'(k)k}{y - f'(k)}$$

Divide top & bottom by y

$$= -\frac{n}{n+g+\delta}\,\frac{f'(k)k}{y} \Big/ 1 - \frac{f'(k)k}{y}$$

SOLOW MODEL

1.7) cont.

We got

$$\frac{\partial y}{\partial n}\frac{n}{y} = \frac{\%\Delta y}{\%\Delta n} = -\frac{n}{n+g+\delta}\,\frac{\overbrace{f'(k)k/y}^{\alpha}}{1-f'(k)k/y}$$

$$= -\frac{n}{n+0.02+0.03}\,\frac{1/3}{1-1/3}$$

What is $\%\Delta y$ if n falls $\underbrace{\text{from } .02 \text{ to } .01}_{\text{a } 50\% \text{ decrease}}$?

At $n = 0.02$,

$$= -.14$$

hence y^* changes by about

$$-.14(50\%) = 7\%.$$

Alternatives: calculate it at $n = 0.01$,

$$n = \frac{0.02+0.01}{2}$$