ANSWER TO 2.1

$$Y = ALf(k) \quad k = K/AL$$

Cost of a unit of labor is wA (not w)

" " " " capital is r

Firm's problem: minimize total cost $(= wAL + rK)$

subject to $\underbrace{Y = ALf(K/AL)}_{\text{constraint}}$

Note this covers any assumption about structure of product market.

Set up Lagrangian:

$$\mathcal{L} = wAL + rK + \lambda[Y - ALf(\underbrace{K/AL}_{= K(AL)^{-1}})]$$

F.O.C.'s:

$$\frac{\partial \mathcal{L}}{\partial K} = r - \lambda\left[AL^*f'(K^*/AL^*)\frac{1}{AL^*}\right] = 0$$

$$\Rightarrow (1) \quad r = \lambda f'(\overbrace{K^*/AL^*}^{k})$$

$$\frac{\partial \mathcal{L}}{\partial (AL)} = w - \lambda\left[f(K^*/AL^*) + AL^*f'(K^*/AL^*)K^*(-1)(AL^*)^{-2}\right] = 0$$

$$\Rightarrow (2) \quad w = \lambda\left[f(\overbrace{K^*/AL^*}^{k^*}) - (\overbrace{K^*/AL^*}^{k^*})f'(\overbrace{K^*/AL^*}^{k^*})\right]$$

Divide (1) by (2), λ's disappear & you have:

$$(3) \quad \frac{r}{w} = \frac{f'(k^*)}{f(k^*) - kf'(k^*)}$$

a) (3) defines k^*. Output level Y doesn't enter into it.

b) Output of a firm i with $K/AL = k^*$ is $Y_i = \overbrace{A}^{\text{same for all firms}}\underbrace{L_i}_{\text{can vary across firms}}\overbrace{f(k^*)}^{\text{same for all firms}}$

Total output of all N firms:

$$\sum_{i=1}^{N} Y_i = \sum_{i=1}^{N} AL_i f(k^*) = Af(k^*)\sum_{i=1}^{N} L_i$$

same as output for one firm employing $\sum_{i=1}^{N} L_i$ people & associated capital stock

ANSWER TO 2.2

$$U = \frac{C_1^{1-\theta}}{1-\theta} + \frac{1}{1+\rho}\frac{C_2^{1-\theta}}{1-\theta}$$

$$W = P_1 C_1 + P_2 C_2$$

a) What is optimal C_1, C_2?

b) What is $\dfrac{-\partial \ln (C_1/C_2)}{\partial \ln (P_1/P_2)}$ ⟸ elasticity of substitution

multiply by (-1) to turn negative number into positive ⟸ convention for elasticity

Note: in getting answer to a), it will be handy to define $\frac{C_1}{C_2}\left(\frac{P_1}{P_2}\right)$ in order to answer b).

"as a function of"

Two ways to do it:

— Use budget constraint to get $C_1(C_2)$ or vice-versa, take one f.o.c. with respect to C_1,

— Define Lagrangian.

ANSWER TO 2.2 (cont.)

First way:

$$C_2 = \frac{1}{P_2} W - \frac{P_1}{P_2} C_1$$

$$U_t = \text{.....} + \frac{1}{1+\rho}\frac{1}{1-\theta}\left(\frac{1}{P_2} W - \frac{P_1}{P_2} C_1\right)^{1-\theta}$$

$$\frac{\partial U_t}{\partial C_1} = 0 = C_1^{-\theta} + \frac{1}{1+\rho}\underbrace{\left(\frac{1}{P_2} W - \frac{P_1}{P_2} C_1\right)}_{C_2}^{-\theta}\left(-\frac{P_1}{P_2}\right)$$

$$0 = C_1^{-\theta} + \frac{1}{1+\rho} C_2^{-\theta}\left(-\frac{P_1}{P_2}\right)$$

$$C_1^{-\theta} = \frac{1}{1+\rho}\frac{P_1}{P_2} C_2^{-\theta}$$

Raise both sides to $(-\frac{1}{\theta})$ power

$$C_1 = \left(\frac{1}{1+\rho}\right)^{-\frac{1}{\theta}}\left(\frac{P_1}{P_2}\right)^{-\frac{1}{\theta}} C_2$$

$$C_1 = (1+\rho)^{\frac{1}{\theta}}(P_1/P_2)^{-\frac{1}{\theta}} C_2 = (1+\rho)^{\frac{1}{\theta}}(P_2/P_1)^{\frac{1}{\theta}} C_2$$

Note:

$$\frac{C_1}{C_2} = (1+\rho)^{\frac{1}{\theta}}(P_1/P_2)^{-\frac{1}{\theta}}$$

$$\ln(C_1/C_2) = \frac{1}{\theta}\ln(1+\rho) - \frac{1}{\theta}\ln(P_1/P_2)$$

we'll use this later.

ANSWER TO 2.2

First way (cont.)

Substitute $C_1(C_2)$ into $C_2 = \frac{1}{P_2} w - \frac{P_1}{P_2} C_1$

$$C_2 = \frac{W}{P_2} - \frac{P_1}{P_2}(1+\rho)^{1/\theta}(P_1/P_2)^{-1/\theta} C_2$$

$$= \frac{W}{P_2} - (1+\rho)^{1/\theta}(P_1/P_2)^{1-1/\theta} C_2$$

$$= \frac{W}{P_2} - (1+\rho)^{1/\theta}(P_1/P_2)^{\frac{\theta-1}{\theta}} C_2$$

$$= \frac{W}{P_2} \underbrace{- (1+\rho)^{1/\theta}(P_2/P_1)^{\frac{1-\theta}{\theta}} C_2}_{\text{add this to both sides}}$$

$$\left(1+(1+\rho)^{1/\theta}(P_2/P_1)^{\frac{1-\theta}{\theta}}\right) C_2 = W/P_2$$

$$C_2 = \frac{1}{1+(1+\rho)^{1/\theta}(P_2/P_1)^{\frac{1-\theta}{\theta}}} \frac{W}{P_2}$$

substitute this into equation we got earlier

$$C_1 = (1+\rho)^{1/\theta}(P_2/P_1)^{1/\theta} C_2$$

and get......

ANSWER TO 2.2

First way (cont.)

$$C_1 = \frac{(1+\rho)^{1/\theta}\,(P_2/P_1)^{1/\theta}}{1+(1+\rho)^{1/\theta}\,(P_2/P_1)^{\frac{1-\theta}{\theta}}}\;\frac{W}{P_2}$$

Now for b). We already got

$$\ln(C_1/C_2) = \frac{1}{\theta}\ln(1+\rho) - \frac{1}{\theta}\ln(P_1/P_2)$$

$$\frac{\partial \ln(C_1/C_2)}{\partial \ln(P_1/P_2)} = -\frac{1}{\theta}$$

so (−1) times $= \frac{1}{\theta}$

ANSWER TO 2.2

Second way, using Lagrangian:

$$\mathcal{L} = \frac{c_1^{1-\theta}}{1-\theta} + \frac{1}{1+\rho}\frac{c_2^{1-\theta}}{1-\theta} + \lambda\left[w - P_1 c_1 - P_2 c_2\right]$$

$$\frac{\partial \mathcal{L}}{\partial c_1} = 0 = c_1^{-\theta} - \lambda P_1$$

$$\frac{\partial \mathcal{L}}{\partial c_2} = 0 = \frac{1}{1+\rho} c_2^{-\theta} - \lambda P_2$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 = w - P_1 c_1 - P_2 c_2$$

From $\frac{\partial \mathcal{L}}{\partial c_1} = 0$, $c_1^{-\theta} = \lambda P_1$, $c_1^{-\theta} P_1^{-1} = \lambda$

From $\frac{\partial \mathcal{L}}{\partial c_2} = 0$, $\frac{1}{1+\rho} c_2^{-\theta} = \lambda P_2$, $\frac{1}{1+\rho} c_2^{-\theta} P_2^{-1} = \lambda$

hence $c_1^{-\theta} P_1^{-1} = \frac{1}{1+\rho} c_2^{-\theta} P_2^{-1}$

$$\frac{c_1^{-\theta}}{c_2^{-\theta}} = \frac{1}{1+\rho}\frac{P_1}{P_2}$$

$$\left(\frac{c_1}{c_2}\right)^{-\theta} = (1+\rho)^{-1}\frac{P_1}{P_2}$$

Raise both sides to $\left(-\frac{1}{\theta}\right)$ power, ...

ANSWER TO 2.2

Second way (cont.)

$$\frac{C_1}{C_2} = (1+\rho)^{\frac{1}{\theta}} (P_1/P_2)^{-\frac{1}{\theta}}$$

$$C_1 = (1+\rho)^{\frac{1}{\theta}} (P_1/P_2)^{-\frac{1}{\theta}} C_2$$

From $\frac{\partial \mathcal{L}}{\partial \lambda} = 0 = w - P_1 C_1 - P_2 C_2$

$$C_2 = \frac{1}{P_2} w - \frac{P_1}{P_2} C_1$$

substitute

this into here

proceed as in first way.