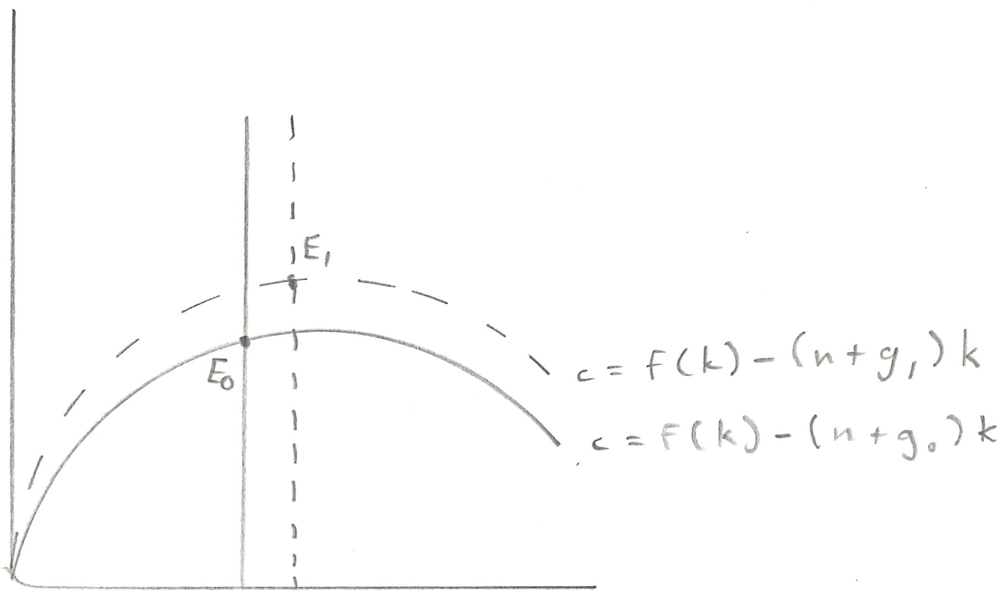
ANSWER TO 2.6

Fall in g

a) & b)

$g_1 < g_0$



k_0^E k_1^E ← $f'(k_1^E) = \rho + n + \theta g_1$

$f'(k_0^E) = \rho + n + \theta g_0$

c) Does c rise or fall when g changes?

Don't know: depends where new saddle path is relative to E_0.

ANSWER TO 2.6 (cont.)

d) Effect on $(r-g)$ at time of change

At the time of the change r is unaffected (because there's no immediate change in k) so $(r-g)$ rises.

e) Effect on $(r-g)$ in LRSS

In LRSS $r = f'(k) = \rho + n + \theta g$,

so

$$(r_1 - g_1) - (r_0 - g_0) = (\rho + n + \theta g_1 - g_1) - (\rho + n + \theta g_0 - g_0)$$

$$= (\theta - 1)(g_1 - g_0)$$

which is positive if $\theta > 1$,
negative if $\theta < 1$,

So "not possible to tell."

ANSWER TO 2.6 (cont.)

f) Effect on savings rate in LRSS

$$s = \frac{\underbrace{f(k) - c}_{\text{In LRSS, this equals } (n+g)k}}{f(k)}$$

$$\Rightarrow s^* = \frac{(n+g)k^*}{f(k^*)}$$

$$\frac{\partial s^*}{\partial g} = \frac{f(k^*)\left[(n+g)\frac{\partial k^*}{\partial g} + k^*\right] - (n+g)k^* f'(k^*)\frac{\partial k^*}{\partial g}}{(f(k^*))^2}$$

simplifies to:

$$\frac{\partial s^*}{\partial g} = \frac{(n+g)\left[f'(k^*) - k^* f'(k^*)\right]\frac{\partial k^*}{\partial g} + f(k^*)k^*}{(f(k^*))^2}$$

Recall $f'(k^E) = \rho + n + \theta g$

$f''(k^E)\frac{\partial k^E}{\partial g} = \theta$ where $f''(k) < 0$

$$\Rightarrow \frac{\partial k^E}{\partial g} = \frac{\theta}{f''(k^E)}$$

Substitute into above

$$\frac{\partial s^E}{\partial g} = \frac{\overbrace{(n+g)\left[f(k^E) - k^E f'(k^E)\right]\theta}^{+} + \overbrace{f(k^E)k^* f''(k^E)}^{(-)}}{(f(k^E))^2 f''(k^E)}$$

could be (+) or (−)

ANSWER TO 2.6 (cont.)

g) Cobb-Douglas production function:

$$Y = K^{\alpha}(AL)^{1-\alpha}$$

$$\frac{Y}{AL} = K^{\alpha}(AL)^{1-\alpha}(AL)^{-1} = \left(\frac{K}{AL}\right)^{\alpha}$$

$$f(k) = y = k^{\alpha}$$

$$f'(k) = \alpha k^{\alpha-1}$$

$$f''(k) = \alpha(\alpha-1)k^{\alpha-2}$$

Substitute into expression for $\frac{\partial s^E}{\partial g}$ gives

$$\frac{\partial s^E}{\partial g} = \frac{(n+g)\left[k^{\alpha} - k\alpha k^{\alpha-1}\right]\theta + k^{\alpha} k \alpha(\alpha-1)k^{\alpha-2}}{(k^{\alpha})^2 \alpha(\alpha-1)k^{\alpha-2}}$$

$$= \frac{(n+g)\theta k^{\alpha} - (n+g)\theta\alpha k^{\alpha} + k^{\alpha}\alpha(\alpha-1)k^{\alpha-1}}{-(1-\alpha)k^{\alpha}k^{\alpha-1}k^{\alpha-1}\alpha\alpha/\alpha}$$

$$= \alpha\frac{(n+g)\theta(1-\alpha)k^{\alpha} - (1-\alpha)k^{\alpha}\alpha k^{\alpha-1}}{-(1-\alpha)k^{\alpha}(\alpha k^{\alpha-1})(\alpha k^{\alpha-1})}$$

Divide out $(1-\alpha)$ and k^{α}

$$= -\alpha\frac{(n+g)\theta - \alpha k^{\alpha-1}}{(\alpha k^{\alpha-1})^2}$$

Use hint $f'(k^E) = \rho + n + \theta g$ $\quad \alpha k^{\alpha-1}$

$$= -\alpha\frac{(n+g)\theta - (\rho+n+\theta g)}{(\rho+n+\theta g)^2} = -\alpha\frac{(\theta-1)n - \rho}{(\rho+n+\theta g)^2}$$

ANSWER TO 2.7)

a) Rise in θ (households <u>less</u> willing to substitute c_{t+1} for c_t)

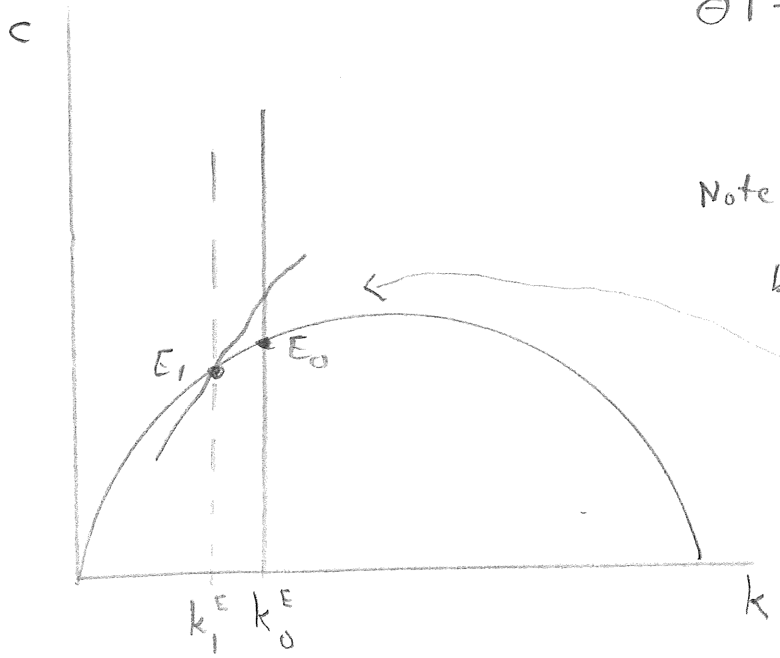
$$f'(k^E) = \rho + n + \theta g$$

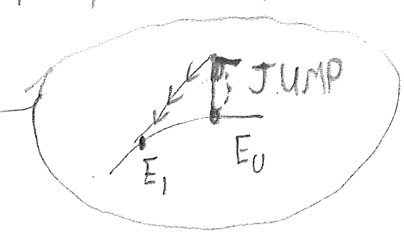
$$\theta \uparrow \rightarrow f'(k^E)\uparrow \rightarrow k^E \downarrow \text{ so } k_1^E < k_0^E$$

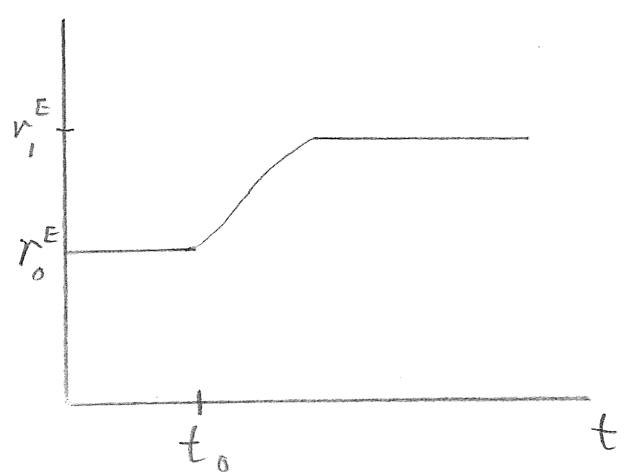
$$r_0 = f'(k_0^E) > r_1 = f'(k_1^E)$$

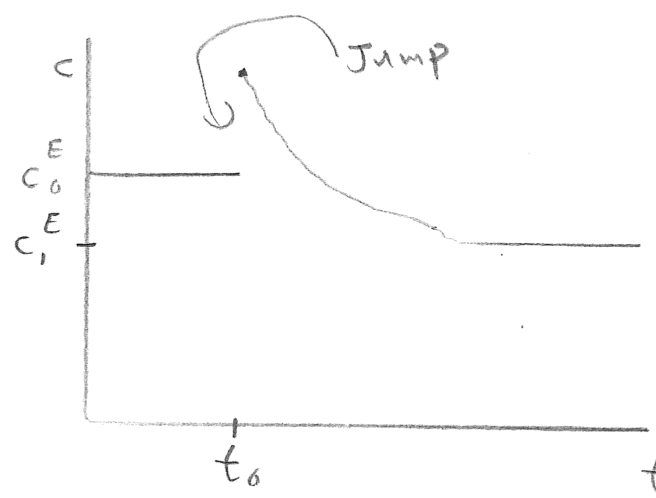
Note $c_1^E < c_0^E$,

but c jumps up at t_0









ANSWER TO 2.7) (cont.)

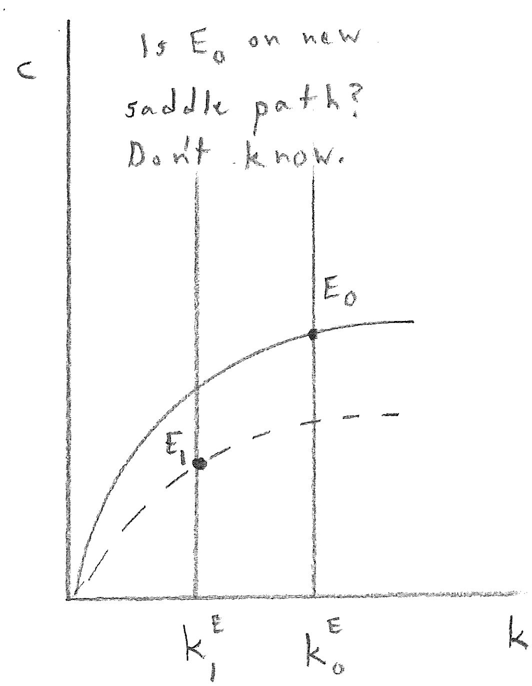
b) "Downward shift in production function"

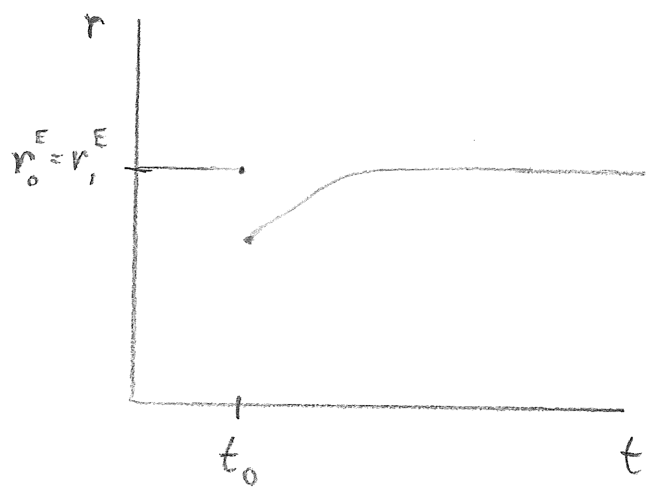
What does this mean? For any k, $f(k)$ & $f'(k)$ smaller
(like $Y = ZF(K, AL)$ & Z falls)

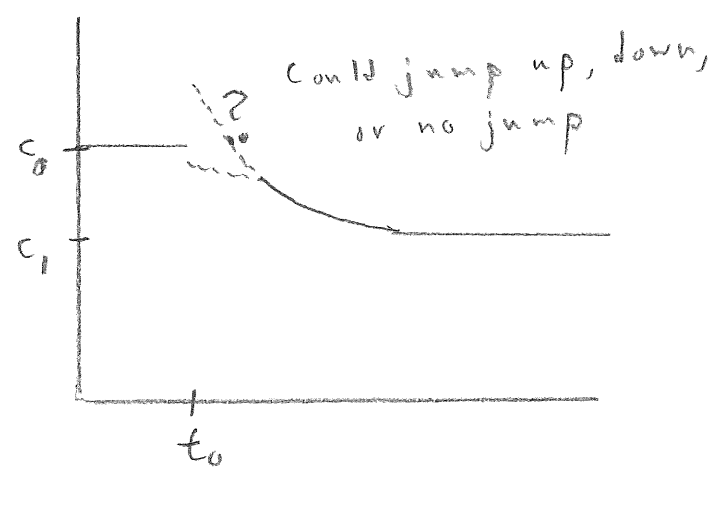
Euler equation stuff: $f'(k^E) = \rho + n + \theta g$

If $f'(k)$ lower for any k, then $f'(k_0^E)$ is now less than $\rho + n + \theta g$,
so k^E must fall: $k_1^E < k_0^E$. But $f'(k_0^E) = f'(k_1^E)$!

Solow stuff: $c^* = f(k^*) - (n+g)k$
so c^* lower for any k







<u>ANSWER TO 2,7) (cont.)</u>

<u>c) Introduce depreciation δ</u>

Solow stuff: $c^* = f(k^*) - (n+g+\delta)k^*$ so effect of δ is to <u>lower</u> c^* for any k^*

Euler equation: Net return to capital is now $f'(k) - \delta$

so for $\frac{\dot{c}}{c} = g$ we have $g = \frac{1}{\theta}\left[f'(k^E) - \delta - \rho - n\right]$

instead of $g = \frac{1}{\theta}\left[f'(k^E) - \rho - n\right]$

$$\Rightarrow f'(k^E) = \rho + n + \delta + \theta g$$

Effect of δ is to <u>increase</u> $f'(k^E)$ hence <u>decrease</u> k^E

Does c jump up or down at t_0? <u>Don't know.</u>

?where is new saddle path relative to E_0?