KEYNESIAN MACRO IS/LM Model

LM Curve

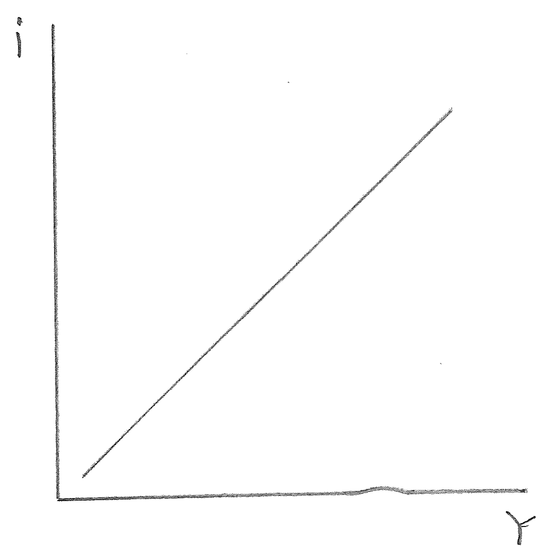
$\left(\frac{M}{P}\right)^D = L(\overset{-}{i}, \overset{+}{Y})$ e.g. $\left(\frac{M}{P}\right)^D = Ye^{-bi}$ "semilog"

"Money demand shocks"

$$\left(\frac{M}{P}\right)^D = L(\overset{-}{i}, \overset{+}{Y}) + \varepsilon$$

Say M^s fixed in a period, and take P as given

$\frac{M^s}{P} = L(\overset{-}{i}, \overset{+}{Y})$ gives LM curve



$M^s \uparrow \rightarrow$ shifts it out
$P \uparrow \rightarrow$ shifts it back
$\varepsilon \uparrow \rightarrow$ shifts it back

Example: from semilog

$$m^s - p = y - bi + \varepsilon$$

$$i = \frac{1}{b}(-m^s + p + y + \varepsilon)$$

More generally....

Keynesian macro IS/LM

LM curve (cont.)

Why does LM slope up?

Upward slope means that $\partial i / \partial Y > 0$ for given $\frac{M}{P}$

How to show this is true:

- Differentiate L, then hold $\frac{M}{P}$ fixed.

$$\partial\left(\frac{M^D}{P}\right) = \frac{\partial L(i,Y)}{\partial i}\partial i + \frac{\partial L(i,Y)}{\partial Y}\partial Y = L_i\,\partial i + L_y\,\partial y$$

If $\partial\left(\frac{M^D}{P}\right) = 0$ (hold M/P fixed)

$$0 = L_i\,\partial i + L_y\,\partial Y$$

$$\frac{\partial i}{\partial Y} = -\frac{\overset{+}{L_Y}}{\underset{-}{L_i}} > 0$$

$\boxed{\text{OK}}$

- "Implicit function theorem."

If $F(X_1, X_2, X_3 \ldots) = 0$, $\frac{\partial X_1}{\partial X_2} = -\frac{F_{X_2}}{F_{X_1}}$

hence for $L(i,Y) = \frac{M}{P}$ where $\frac{M}{P}$ fixed,

$L(i,Y) - \frac{M}{P} = 0$, $\frac{\partial i}{\partial Y} = -\frac{L_Y}{L_i}$

Note: LM steeper ($\partial i / \partial Y$ bigger) when

L_Y big (money demand sensitive to income)

L_i small (money demand insensitive to i)

IS-LM

LM curve (cont.)

How does $\Delta(\frac{M}{P})$ shift curve?

Holding i fixed, what's $\partial Y / \partial(\frac{M}{P})$?

$$\partial\left(\frac{M}{P}\right) = L_i \, \partial i + L_Y \, \partial Y$$

$$= 0 + L_Y \, \partial Y$$

$$\frac{\partial Y}{\partial M/P} = \frac{1}{L_Y} > 0$$ hence $\frac{M}{P}\uparrow$ shifts LM out (or up)

Note: $P\uparrow$ equivalent to $M\downarrow$

Holding M fixed, changes in P shift LM curve.

$P\uparrow \rightarrow \frac{M}{P}\downarrow \rightarrow$ LM shifts in.

Money demand shocks

$$\frac{M^D}{P} = L(i, Y) + \varepsilon$$ ← anything other than i, Y

$$\frac{M}{P} = L(i, Y) + \varepsilon$$

$$\frac{M}{P} - \varepsilon = L(i, Y)$$

Note: $\varepsilon\uparrow$ ← something increases $\frac{M^D}{P}$ given i, Y

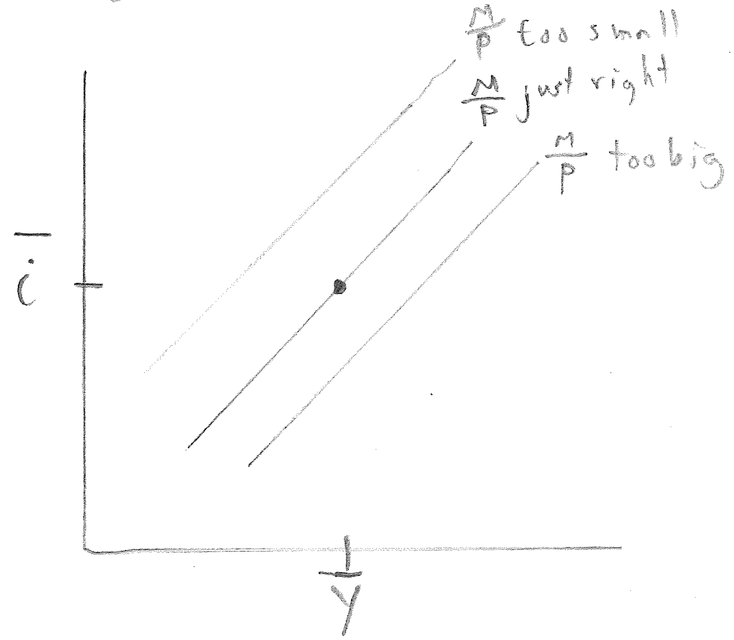
equivalent to $\frac{M}{P}\downarrow$

hence LM shifts back.

IS/LM

LM curve & natural rates $\bar{r}$, $\bar{Y}$

$\bar{i} = \bar{r} + \pi^e$ Nominal interest rate matching $\bar{r}$



For any $\bar{r}$, $\bar{Y}$, and π^e

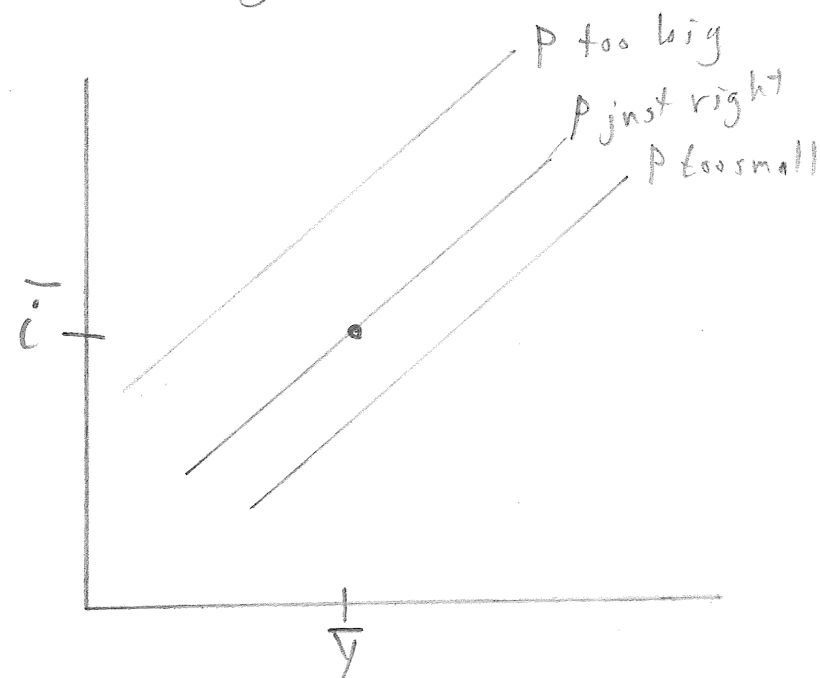
a value $\overline{\left(\frac{M}{P}\right)}$

for which

$$\overline{\left(\frac{M}{P}\right)} = L(\bar{i}, \bar{Y})$$

$\bar{r} + \pi^e$

Taking M as given, $\overline{\left(\frac{M}{P}\right)}$ implies $\bar{P}$



for which

$$\frac{M}{P} = L(\bar{i}, \bar{Y})$$

required price level

Note: $\bar{P}$ moves proportionately with M

to make $\frac{M}{P} = \overline{\left(\frac{M}{P}\right)}$ real balances consistent with $\bar{Y}$, $\bar{r}$, π^e

IS Curve

Keynesian Cross

A specific version of IS curve, not needed for most purposes.

E Expenditure or "planned expenditure"

must always equal Y (so I've ignored E so far)

$$E = C + I + G = E(\overset{+}{Y}, \overset{(-)}{r}, \overset{+}{G}, \overset{(-)}{T})$$

r: $i - \pi^e$

where $0 < E_Y < 1$

Stories for this:

1) current income affects C

$\frac{\partial C}{\partial Y} > 0$ ⇐ "Marginal propensity to consume"

2) Current sales affect I "Accelerator"

- current sales affect expectations of future product demand
- some firms are liquidity-constrained, must finance investment out of earnings

$$Y = E(Y, r, G, T)$$

affects relation between Y and r, G, T

Example: what is $\partial Y / \partial i$? $\boxed{\text{inverse of slope of IS curve}}$

From above,

$$\frac{\partial Y}{\partial i} = E_Y \frac{\partial Y}{\partial i} + E_r$$

Solve for $\frac{\partial Y}{\partial i}$:

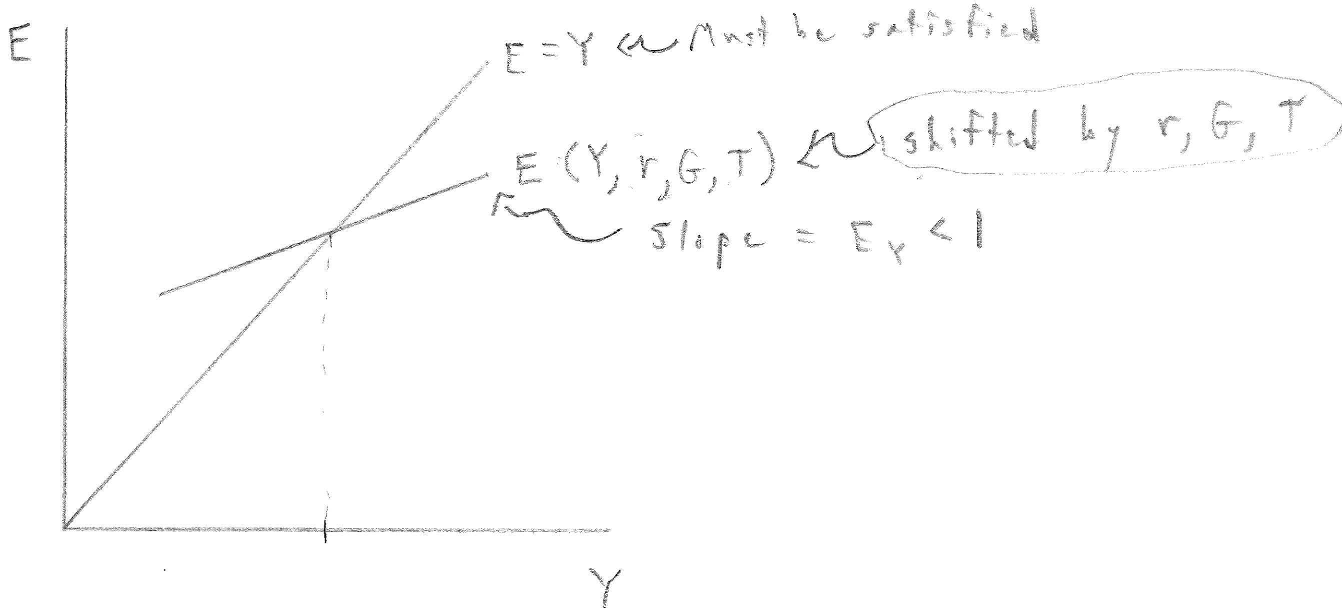
$$\frac{\partial Y}{\partial i} = \frac{E_r}{1 - E_Y} < 0$$

Note: $\frac{1}{1-E_Y}$ is "the spending multiplier"

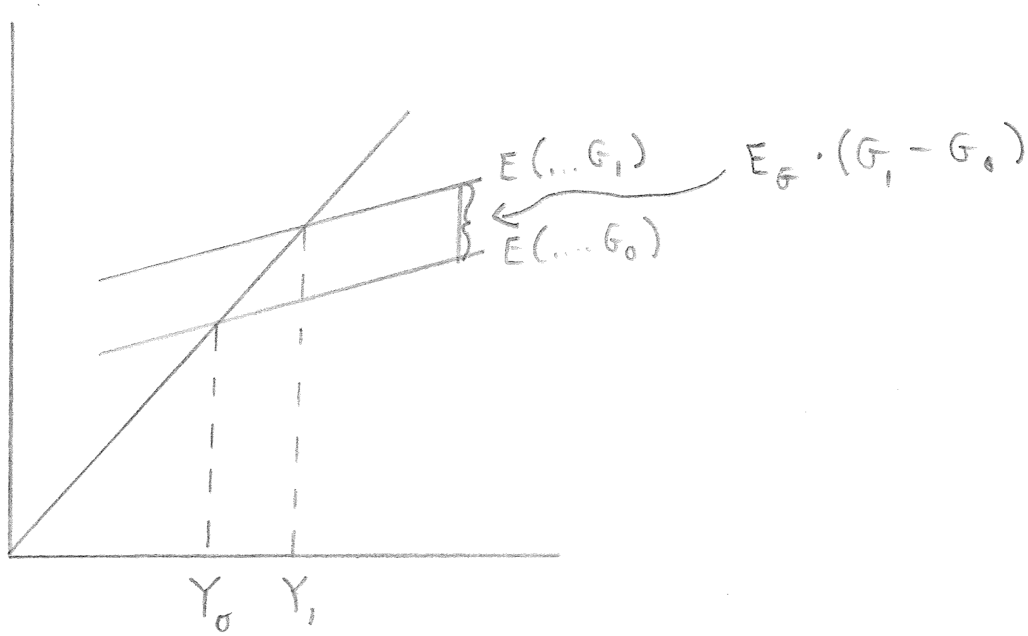
IS-Curve

Keynesian Cross (cont.)

Graph



A change in G $\quad G_1 > G_0$



$$\underbrace{Y_1 - Y_0}_{} = E_G(G_1 - G_0)\underbrace{\frac{1}{1-E_Y}}_{\text{spending multiplier}} > E_G(G_1 - G_0)$$

IS curve (cont.)

Equation of IS curve

$$Y = E(Y, r, G, T, \ldots)$$

Solve out for Y to get expression corresponding to IS curve

$$Y(r, G, T, \ldots)$$

Inverse of this is function plotted as IS curve

$$r(Y, \ldots)$$

horizontal axis

vertical axis

Note from $Y = E(Y, r, G, T \ldots)$

$$\frac{\partial Y}{\partial r} = E_Y \frac{\partial Y}{\partial r} + E_r \Rightarrow \partial Y / \partial r = \frac{E_r}{(1 - E_Y)}$$

slope of IS

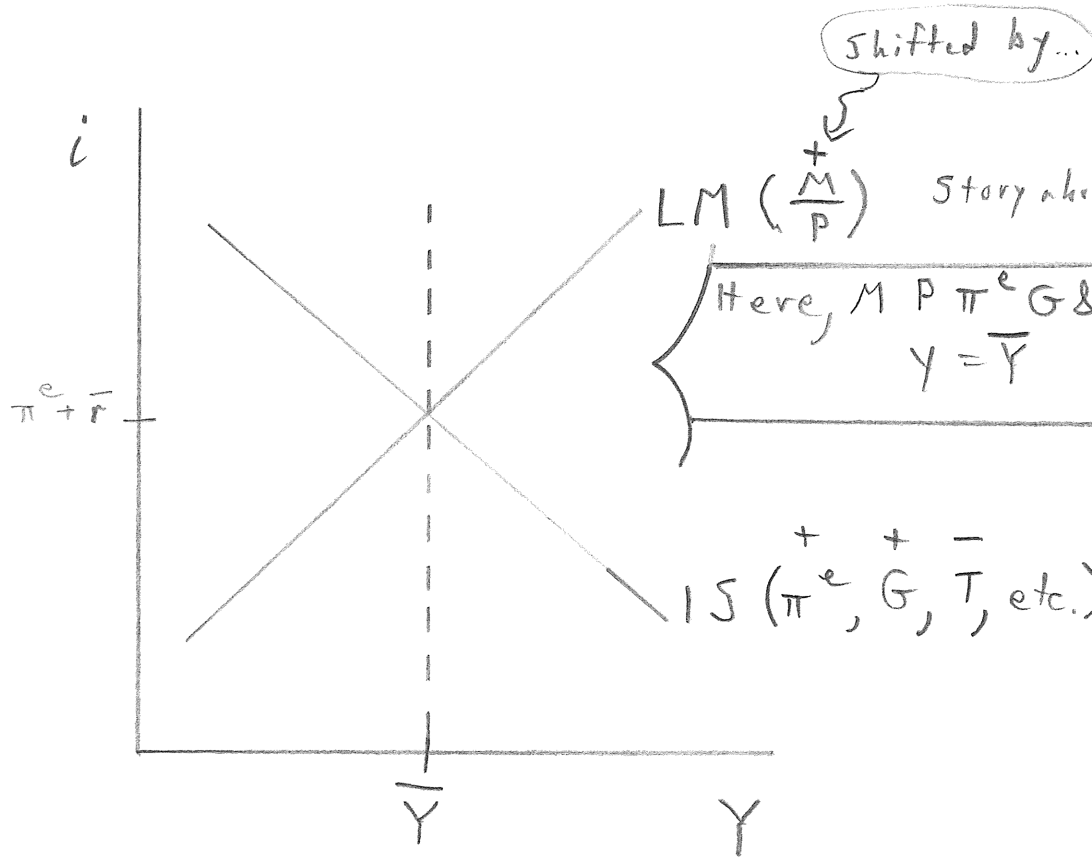
$$\frac{\partial r}{\partial Y} = \frac{(1 - E_Y)}{E_r}$$

From $Y(r, G, T \ldots)$

$$\frac{\partial Y}{\partial r} = Y_r$$

IS-LM

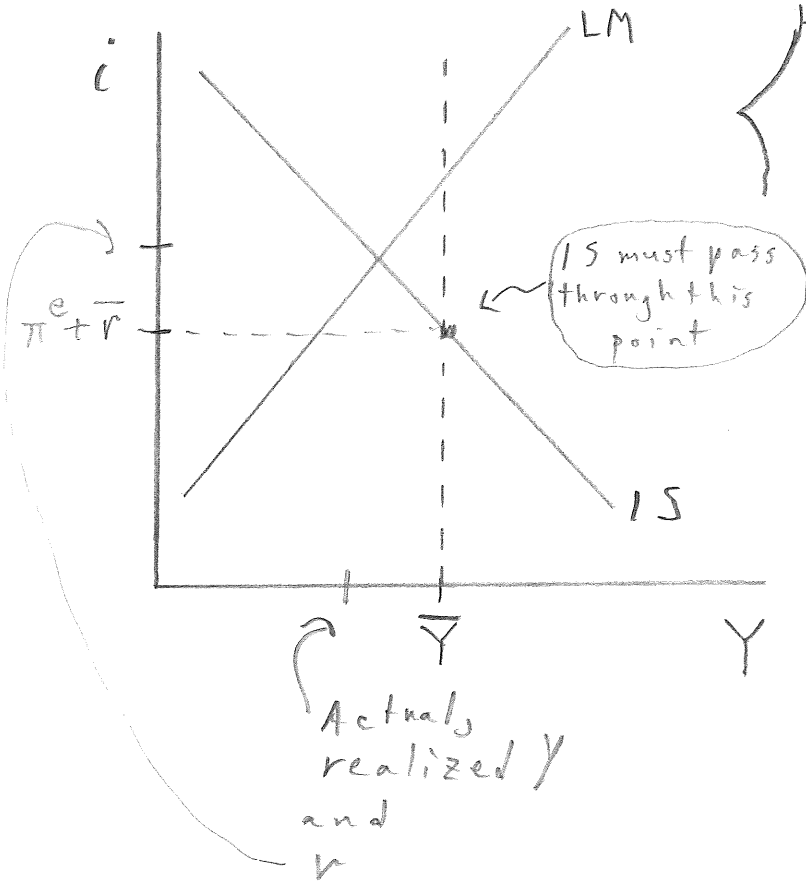
IS-LM Graph



Story about "money supply & demand"

Here, M P π^e G & T are such that $Y = \bar{Y}$

Story about real interest rate & spending, including $\bar{r}$



Here, M P π^e G & T are such that $Y < \bar{Y}$

Mechanically, Y can be increased to equal $\bar{Y}$ by any of the following:

$\downarrow P$
$\uparrow M$ } equivalent

$\pi^e \uparrow$ shifts IS up

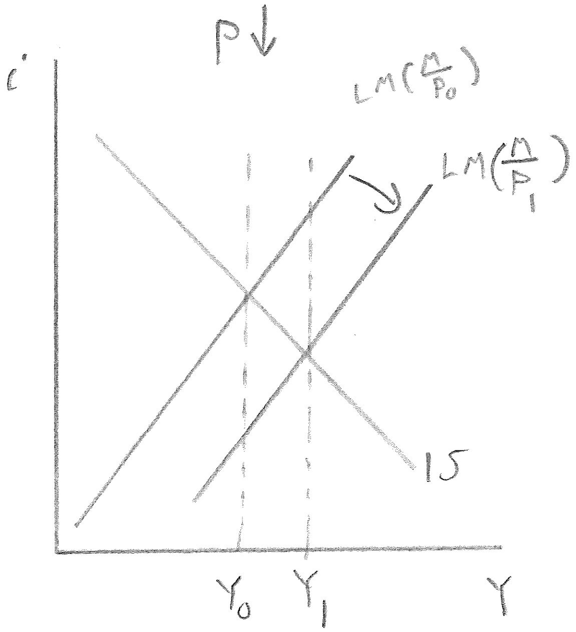
IS-LM (closed economy)

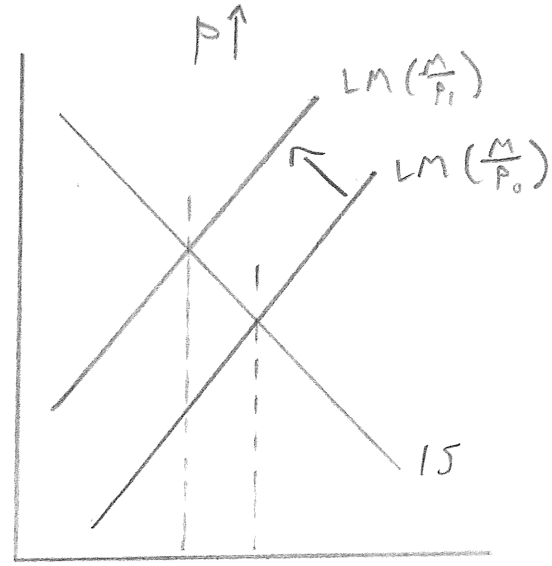
Aggregate Demand Curve

Hold M fixed, vary P: what must happen to Y?

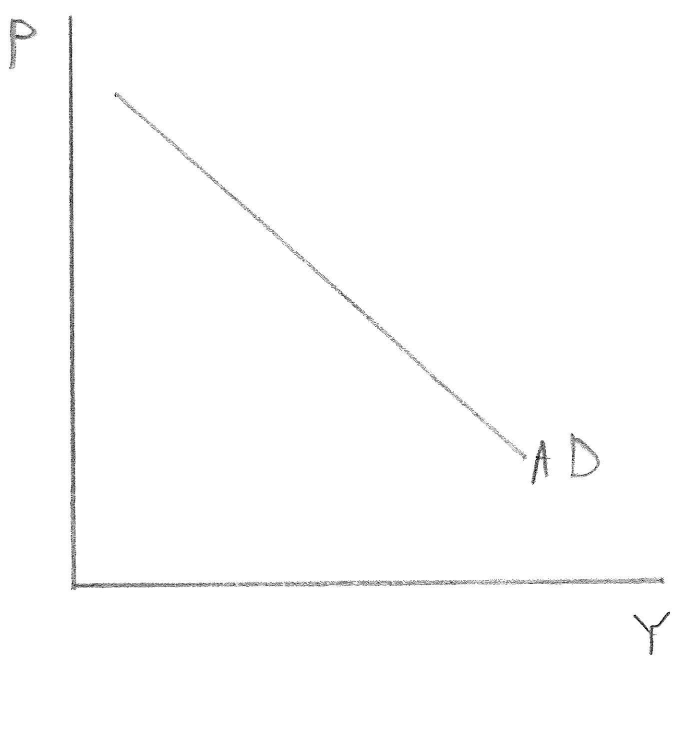
For fixed M, $P\downarrow \rightarrow \frac{M}{P}\uparrow$

LM shifts out $\rightarrow Y\uparrow$





Aggregate Demand Curve



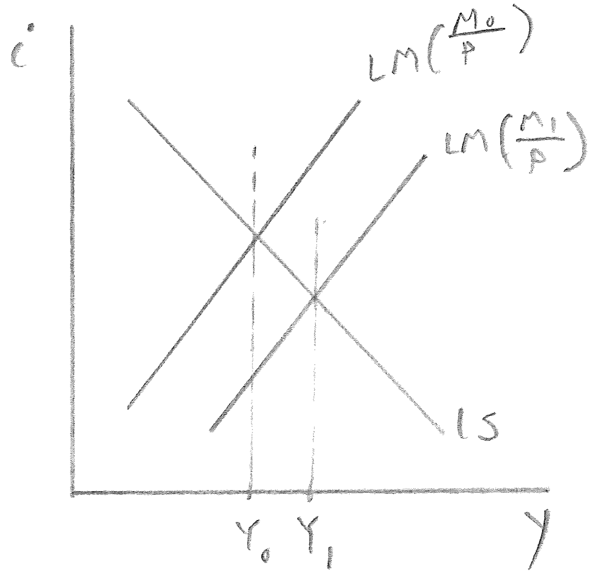
Given $M, G, T, \& \pi^e$, plot out P vs. Y

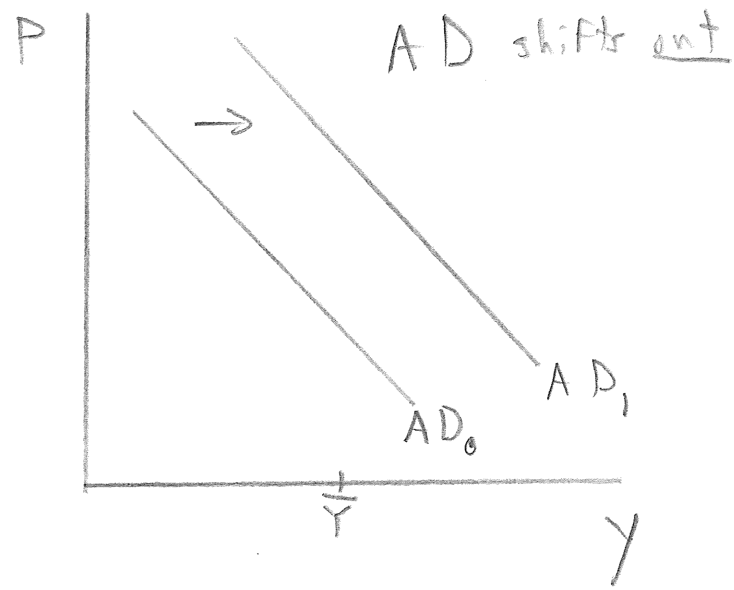
IS-LM (closed economy)

Aggregate Demand Curve (cont.)

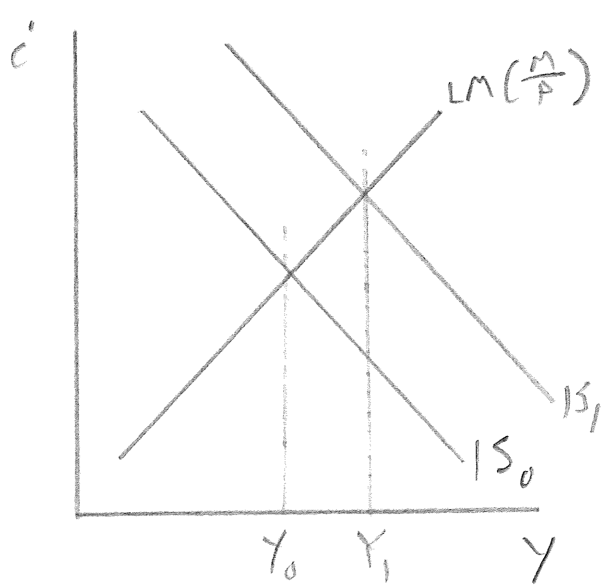
Changes in M, G, T, π^e shift AD curve

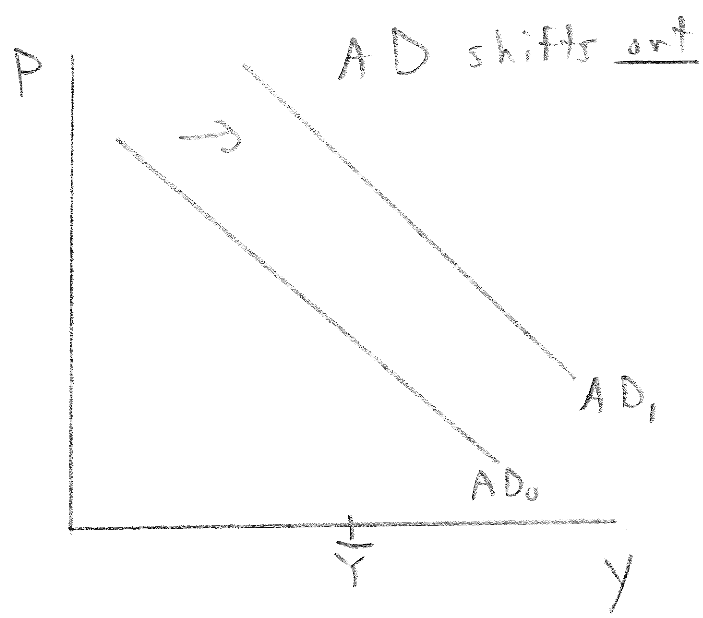
M↑ : For any P, Y↑





G↑, T↓: For any P, Y↑





IS-LM (closed economy)

Aggregate Demand curve (cont.)

Derive slope of AD curve using $E(Y, r, G, T)$

holding M etc. fixed

$$Y = E(Y, r, G, T)$$

$$\frac{\partial Y}{\partial P} = E_Y \frac{\partial Y}{\partial P} + E_r \frac{\partial i}{\partial P}$$

$$(1 - E_Y)\frac{\partial Y}{\partial P} = E_r \frac{\partial i}{\partial P}$$

$$\frac{\partial Y}{\partial P} = \frac{E_r}{(1-E_Y)} \frac{\partial i}{\partial P}$$

$$\frac{M}{P} = L(i, Y)$$

$$-\frac{M}{P^2} = L_i \frac{\partial i}{\partial P} + L_Y \frac{\partial Y}{\partial P}$$

$$\frac{\partial i}{\partial P} = -\frac{1}{L_i}\frac{M}{P^2} - \frac{L_Y}{L_i}\frac{\partial Y}{\partial P}$$

$$\frac{\partial Y}{\partial P} = \frac{E_r}{(1-E_Y)}\left(-\frac{1}{L_i}\right)\left(\frac{M}{P^2} + L_Y \frac{\partial Y}{\partial P}\right)$$

rearrange to get

$$\frac{\partial Y}{\partial P} = -\frac{\frac{M}{P^2}}{\frac{(1-E_Y)L_i}{E_r} + L_Y} < 0$$

This is *inverse* of function plotted as AD curve

IS-LM (closed economy)

Review: What the math of IS-LM does

By itself, IS-LM

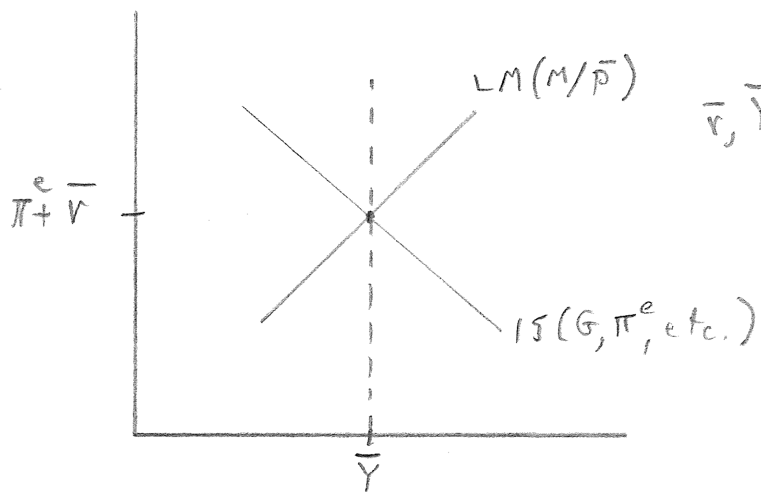
describes relations between M, P, π^e, i, Y, G etc. that must hold if

① $Y(\overset{-}{r}, \overset{+}{G}, \ldots)$

② $\frac{M}{P} = L(\overset{-}{i}, \overset{+}{Y})$

What's endogenous/exogenous? Depends on assumptions

• Assuming $Y = \bar{Y}$ (output must equal natural rate)

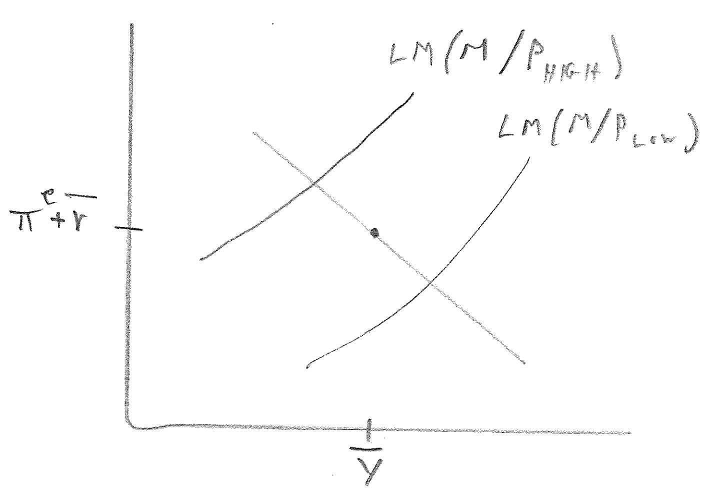


Exogenous

$\bar{r}, \bar{Y}, M, \pi^e, G$ etc. $\longrightarrow \bar{P}$

price level that would prevail in RBC model

• Assuming $P = P_0$ (fixed price level



Exogenous

P, M, π^e, G, etc. $\longrightarrow Y, i, r$

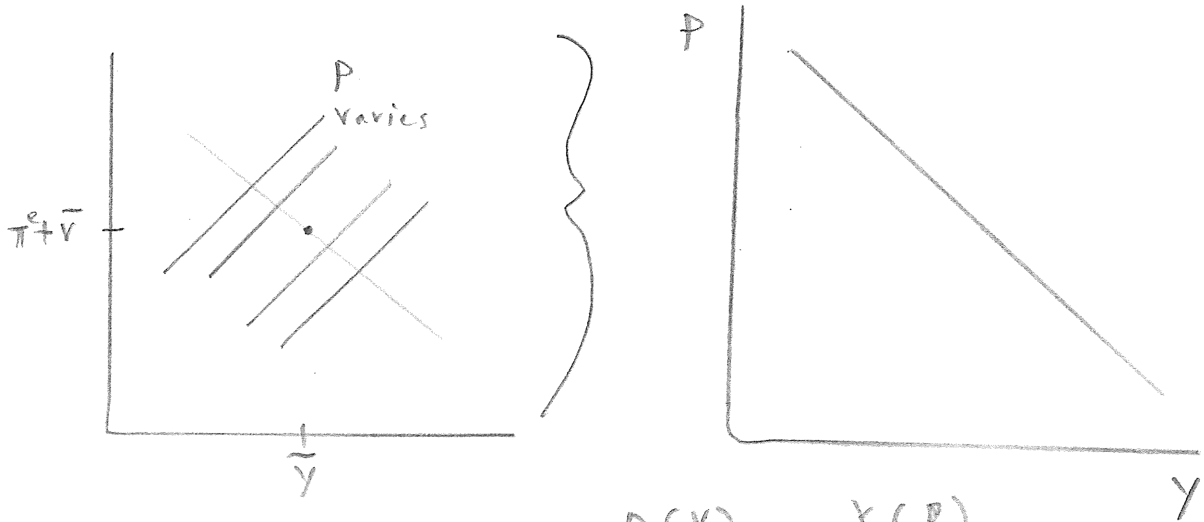
$i - \pi^e$

need not equal $\bar{Y}$

IS-LM (closed economy)

Review: math of IS-LM (cont.)

- Assuming M fixed, P & Y can vary: AD curve



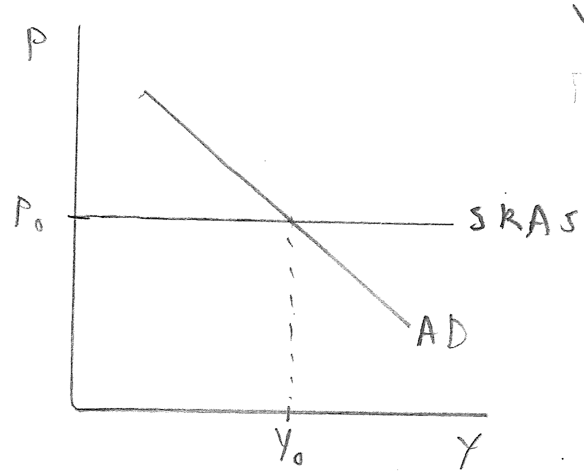
M, π^e, G etc, $\longrightarrow$ $P(Y)$ or $Y(P)$

To determine P & Y, need "aggregate supply" curve

"Short run": fixed P

Y_0 can be $\neq \bar{Y}$

Variables affecting AD determine Y



"Long run": P adjusts to $\bar{P}$ where $Y = \bar{Y}$

Variables affecting AD determine $P = \bar{P}$