Problem set on production functions

Consider the aggregate production function $Y = \left(K^{\rho} + L^{\rho}\right)^{(1/\rho)}$ *where* $\rho \leq 1$.

1) Suppose you add a factor A to this production function to take account of progress in knowledge, technology etc.

a) Write down an equation that depicts "Harrod-neutral" progress.

b) Write down an equation that depicts "capital-augmenting" progress.

c) Write down an equation that depicts "Hick-neutral" progress.

d) Are the three types of progress equivalent in this production function, as they are in the Cobb-Douglas production function?

2) Does this production function have constant returns to scale? Prove your answer is true.

3) What is the marginal product of capital and marginal product of labor for this production function?

4) What is the elasticity of substitution between capital and labor?

5) Let S denote labor's share of income. Derive an equation that gives S as a function of the capital-labor ratio (K / L).

6) Using your answer to 5), what is $\partial S / \partial (K / L)$? Is $\partial S / \partial (K / L)$ positive or negative?

7) Suppose $\rho = 1/2$.
a) Using your answer to 4), what is the elasticity of substitution?
b) Using your answers to 5) and 6), what happens to labor's share of income as (K / L) grows - increase, decrease or no effect?

8) Suppose $\rho = -1/2$.
a) Using your answer to 4), what is the elasticity of substitution?
b) Using your answers to 5) and 6), what happens to labor's share of income as (K / L) grows - increase, decrease or no effect?