Romer 2nd edition (2001)

4.15. The derivation of the log-linearized equation of motion for capital. Consider the equation of motion for capital, $K_{t+1} = K_t + K_t^\alpha (A_t L_t)^{1-\alpha} - C_t - G_t - \delta K_t$.

(*a*) (*i*) Show that $\partial \ln K_{t+1}/\partial \ln K_t$ (holding A_t, L_t, C_t, and G_t fixed) is $(1 + r_{t+1})(K_t/K_{t+1})$.

(*ii*) Show that this implies that $\partial \ln K_{t+1}/\partial \ln K_t$ evaluated at the balanced growth path is $(1 + r^*)/e^{n+g}$.[44]

(*b*) Show that

$$\tilde{K}_{t+1} \simeq \lambda_1 \tilde{K}_t + \lambda_2(\tilde{A}_t + \tilde{L}_t) + \lambda_3 \tilde{G}_t + (1 - \lambda_1 - \lambda_2 - \lambda_3)\tilde{C}_t,$$

where $\lambda_1 \equiv (1 + r^*)/e^{n+g}$, $\lambda_2 \equiv (1 - \alpha)(r^* + \delta)/(\alpha e^{n+g})$, and $\lambda_3 = -(r^* + \delta)(G/Y)^*/(\alpha e^{n+g})$; and where $(G/Y)^*$ denotes the ratio of G to Y on the balanced growth path without shocks. (Hints: Since the production function is Cobb-Douglas, $Y^* = (r^* + \delta)K^*/\alpha$. On the balanced growth path, $K_{t+1} = e^{n+g}K_t$, which implies that $C^* = Y^* - G^* - \delta K^* - (e^{n+g} - 1)K^*$.)

(*c*) Use the result in (*b*) and equations (4.43)–(4.44) to derive (4.52), where $b_{KK} = \lambda_1 + \lambda_2 a_{LK} + (1 - \lambda_1 - \lambda_2 - \lambda_3)a_{CK}$, $b_{KA} = \lambda_2(1 + a_{LA}) + (1 - \lambda_1 - \lambda_2 - \lambda_3)a_{CA}$, and $b_{KG} = \lambda_2 a_{LG} + \lambda_3 + (1 - \lambda_1 - \lambda_2 - \lambda_3)a_{CG}$.

4.16. A Monte Carlo experiment and the source of bias in OLS estimates of trend reversion. Suppose output growth is described simply by $\Delta \ln y_t = \varepsilon_t$, where the ε's are independent, mean-zero disturbances. Normalize the initial value of $\ln y$, denoted by $\ln y_0$, to 0. This problem asks you to consider what occurs in this situation if one estimates equation (4.56), $\Delta \ln y_t = \alpha' + b \ln y_{t-1} + \varepsilon_t$, by ordinary least squares.

(*a*) Suppose the sample size is 3, and suppose each ε is equal to 1 with probability $\frac{1}{2}$ and −1 with probability $\frac{1}{2}$. For each of the eight possible realizations of $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ ((1, 1, 1), (1, 1, −1), and so on), what is the OLS estimate of b? What is the average of the estimates? Explain intuitively why the estimates differ systematically from the true value of $b = 0$.

(*b*) Suppose the sample size is 200, and suppose each ε is normally distributed with a mean of 0 and a variance of 1. Using a random-number generator on a computer, generate 200 such ε's; then generate $\ln y$'s using $\Delta \ln y_t = \varepsilon_t$ and $\ln y_0 = 0$; then estimate (4.56) by OLS; finally, record the estimate of b. Repeat this process 500 times. What is the average estimate of b? What fraction of the estimated b's is negative?

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[44] One could express r^* in terms of the discount rate ρ. Campbell (1994) argues, however, that it is easier to discuss the model's implications in terms of r^* than ρ.

Chapter 5
TRADITIONAL KEYNESIAN THEORIES OF FLUCTUATIONS

This chapter and the next develop models of fluctuations based on the assumption that there are barriers to the instantaneous adjustment of nominal prices and wages. As we will see, sluggish nominal adjustment causes changes in the aggregate demand for goods at a given level of prices to affect the amount that firms produce. As a result, it causes purely monetary disturbances (which affect only demand) to change employment and output. In addition, many real shocks, including changes in government purchases, investment demand, and technology, affect aggregate demand at a given price level; thus sluggish price adjustment creates a channel other than the intertemporal-substitution and capital-accumulation mechanisms of basic real-business-cycle models through which these shocks affect employment and output.

This chapter takes nominal stickiness as given. It has two main goals. The first is to investigate aggregate demand. We will examine the determinants of aggregate demand at a given price level and the effects of changes in the price level. The second is to consider alternative assumptions about the form of nominal rigidity. We will investigate different assumptions' implications for firms' willingness to change output in response to changes in aggregate demand and for the behavior of real wages, markups, and inflation. Chapter 6 then turns to the questions of why nominal prices and wages might not adjust immediately to disturbances.

The models of this chapter are based on traditional Keynesian models. Thus both their substance and their modeling strategy are at the other extreme from the pure real-business-cycle models of Chapter 4. The models in this chapter often directly specify relationships among aggregate variables. The relationships are often static, and the models' implications for the behavior of some variables (such as the capital stock) are sometimes omitted from the analysis. In addition, rather than specifying stochastic processes for the exogenous variables, the analysis focuses on the effects of one-time changes. And the models are so stylized that any effort to see how well they match overall features of the economy is of little value.

The remainder of the chapter consists of six sections. Sections 5.1 and 5.2 develop the aggregate demand side of the standard Keynesian model. These sections take as given that nominal prices and wages are not completely flexible, and that firms change their output in response to changes in demand. Section 5.1 assumes a closed economy, and Section 5.2 considers the open-economy case.

Sections 5.3 and 5.4 consider aggregate supply. Section 5.3 shows how different combinations of wage rigidity, price rigidity, and non-Walrasian features of the labor and goods markets yield different implications about the effect of shifts in aggregate demand on output, unemployment, the real wage, and the markup. Section 5.4 discusses short-run and long-run output-inflation tradeoffs.

Finally, Sections 5.5 and 5.6 discuss some empirical evidence about the real effects of monetary changes and the cyclical behavior of the real wage.

5.1 Review of the Textbook Keynesian Model of Aggregate Demand

The textbook Keynesian model is traditionally summarized by two curves in output-price or output-inflation space, an aggregate demand (*AD*) curve and an aggregate supply (*AS*) curve. The *AD* curve slopes down and the *AS* curve slopes up. These curves are shown in Figure 5.1.

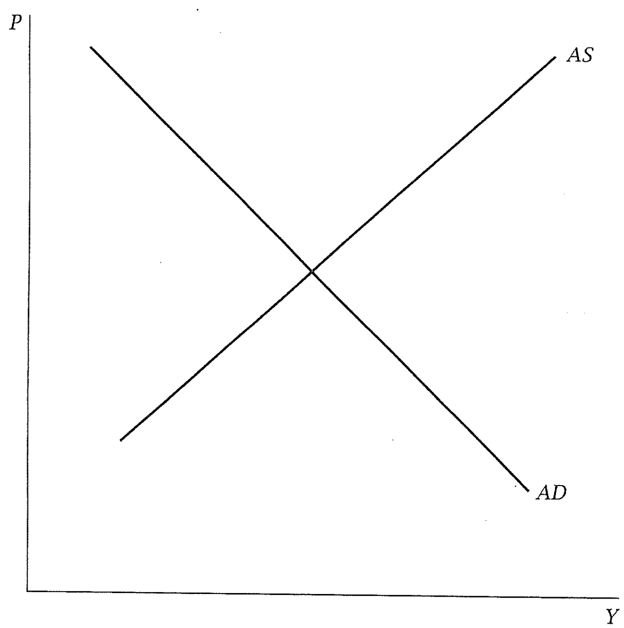

FIGURE 5.1 The *AS-AD* diagram

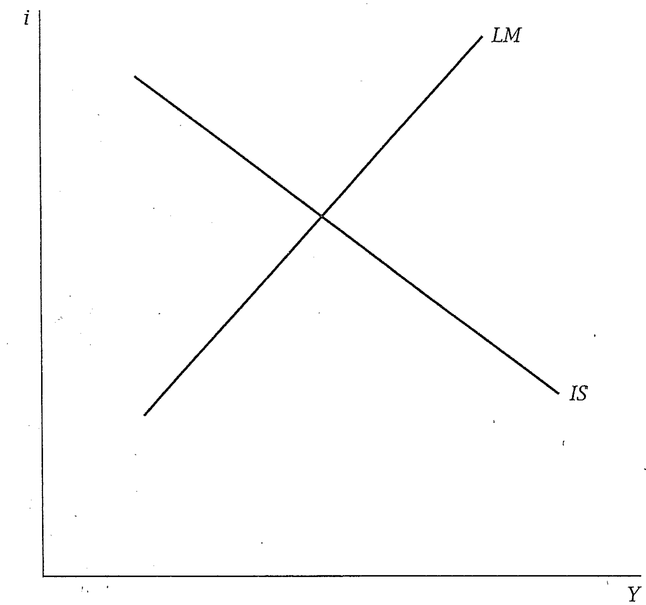

FIGURE 5.2 The *IS-LM* diagram

The fact that the aggregate supply curve is upward-sloping rather than vertical is the critical feature of the model. If the *AS* curve is vertical, changes on the demand side of the economy affect only prices. But if it is merely upward-sloping, changes in aggregate demand affect both prices and output.

The *AD* curve summarizes the demand side of the economy. It is derived from two familiar curves in output-interest rate space, the *IS* and *LM* curves. These are shown in Figure 5.2. The curves are drawn for a given price level; as we will see shortly, considering different values of the price level allows us to use the *IS* and *LM* curves to derive the *AD* curve. Although there are innumerable variations and extensions of the *IS-LM* model, here we consider a standard version.

The *IS* Curve

The *IS* curve shows the combinations of output and the interest rate such that planned and actual expenditures on output are equal.[1] Planned real expenditure depends positively on real income, negatively on the real interest rate, positively on government purchases of goods and services, and

[1] The *IS* curve is often described as showing equilibrium in the goods market. But since supply is ignored, this is not an accurate description.

negatively on taxes:

$$E = E(Y, i - \pi^e, G, T), \quad 0 < E_Y < 1,\ E_{i-\pi^e} < 0,\ E_G > 0,\ E_T < 0. \tag{5.1}$$

Here E is planned real expenditure, Y is real output, i is the nominal interest rate, π^e is expected inflation, G is real government purchases, and T is real taxes. E_Y, $E_{i-\pi^e}$, and so on denote the partial derivatives of $E(\bullet)$. G, T, and π^e are all taken as given.[2] The negative effect of the real interest rate on planned expenditure operates through firms' investment decisions and through consumers' purchases, particularly of durable goods. Planned expenditure is assumed to increase less than one-for-one with income; that is, $0 < E_Y < 1$.

In textbook treatments, E is often expressed in terms of its component parts, and strong assumptions are made about how the determinants of planned expenditure enter. A standard formulation is

$$E = C(Y - T) + I(i - \pi^e) + G, \tag{5.2}$$

where $C(\bullet)$ is consumption and $I(\bullet)$ is investment. The restrictions imposed in this specification may be highly unrealistic. For example, there is considerable evidence that the real interest rate affects consumption, and almost overwhelming evidence that income influences investment. To give another example, there is little basis for assuming that income and taxes have equal and opposite effects on spending. Since the general formulation in (5.1) is only slightly more difficult, we will use it in what follows.

If one treats goods that a firm produces and then holds as inventories as purchased by the firm, then all output is purchased by someone. Thus actual expenditure equals the economy's output, Y. In equilibrium, planned and actual expenditures must be equal. If planned expenditure falls short of actual expenditure, for example, firms are accumulating unwanted inventories; they will respond by cutting their production. Thus equilibrium requires

$$E = Y. \tag{5.3}$$

Substituting (5.3) into (5.1) yields

$$Y = E(Y, i - \pi^e, G, T). \tag{5.4}$$

Figure 5.3, the *Keynesian cross*, depicts equations (5.1) and (5.3) in (Y, E) space for a given level of the interest rate. Equation (5.3) is just the 45-degree line. Since planned expenditure increases less than one-for-one with Y, the set of points satisfying (5.1) is less steep than the 45-degree line. The point where the planned expenditure curve crosses the 45-degree line (Point A)

2 Properly speaking, expected inflation should be determined within the model rather than taken as given, since the path of the price level will be determined within the model. Taking π^e as given here simplifies the discussion without altering the model's main implications, however.

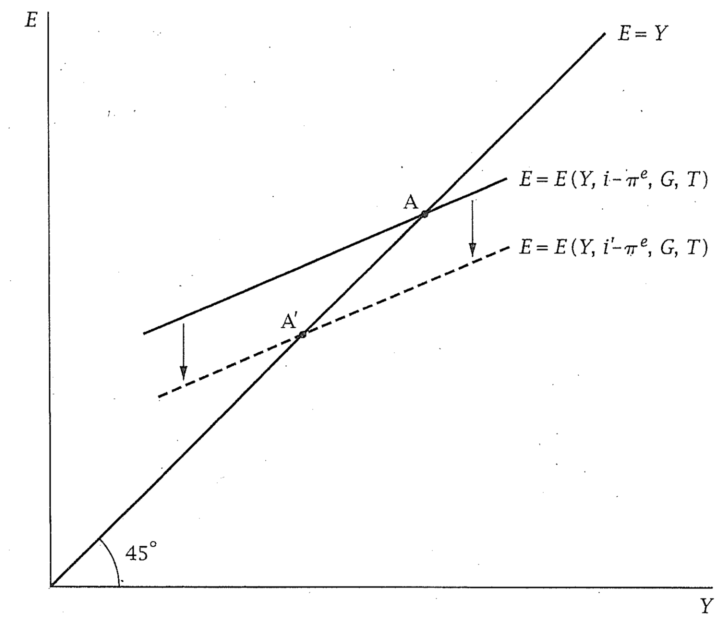

FIGURE 5.3 The Keynesian cross

shows the unique level of income where actual and planned expenditures are equal for the given interest rate.[3]

An increase in the interest rate shifts the planned expenditure line down (since $E(\bullet)$ is decreasing in $i - \pi^e$), and thus reduces the level of income at which actual and planned expenditures are equal; in terms of the diagram, an increase in the interest rate from i to i' shifts the intersection of the two lines from Point A to Point A′. Thus the IS curve slopes down.

Differentiating both sides of (5.4) with respect to i yields

$$\left.\frac{dY}{di}\right|_{IS} = E_Y \left(\left.\frac{dY}{di}\right|_{IS}\right) + E_{i-\pi^e}, \tag{5.5}$$

or

$$\left.\frac{dY}{di}\right|_{IS} = \frac{E_{i-\pi^e}}{1 - E_Y}, \tag{5.6}$$

where $\left.\frac{dY}{di}\right|_{IS}$ denotes dY/di along the IS curve. Since this is an expression for dY/di (rather than di/dY), it implies that the IS curve is flatter when either $E_{i-\pi^e}$ or E_Y is larger. Intuitively, the larger the effect of the interest

3 The Keynesian cross is sometimes described as a theory of income determination. But this is correct only if the interest rate can be treated as fixed, which is often inappropriate.

ate on planned expenditure, the larger the downward shift of the planned xpenditure line, and thus the larger the fall in output. Similarly, the steeper he planned expenditure line, the more output must fall in response to a iven downward shift of the planned expenditure line to reach a point where lanned and actual expenditures are again in balance, and thus the larger the all in output. This last effect is the famous *multiplier:* because E depends n Y, the fall in Y needed to restore the equality of E and Y is larger than he amount that E falls at a given Y.

The *LM* Curve

The *LM* curve shows the combinations of output and the interest rate that ead to equilibrium in the money market for a given price level. It is simplest o think of money as high-powered money—currency and reserves—issued y the government. Since high-powered money pays no nominal interest, he opportunity cost of holding it is the nominal interest rate. The demand or real money balances is therefore a decreasing function of the nominal nterest rate. In addition, since the volume of transactions is greater when utput is higher, the demand for real balances is increasing in output. The ominal money supply is set by the government. Putting all this together, he condition for the supply and demand of real balances to be equal at a iven price level is

$$\frac{M}{P} = L(i, Y), \qquad L_i < 0, \qquad L_Y > 0, \tag{5.7}$$

vhere M is the quantity of money and P is the price level.

Since $L(\bullet)$ is decreasing in i and increasing in Y, the set of combinations f i and Y that satisfy (5.7) is upward-sloping. Formally, differentiating both ides of (5.7) with respect to Y and rearranging yields

$$\left.\frac{di}{dY}\right|_{LM} = -\frac{L_Y}{L_i} > 0. \tag{5.8}$$

Thus increases in the income elasticity of money demand and decreases in he interest elasticity (in absolute value) make the *LM* curve steeper.[4]

Implicitly, the *IS-LM* model treats all assets other than money as perect substitutes. The market for these other assets is then suppressed by Valras's law. Specifically, total wealth in the economy equals the total value f all assets, and the total value of any individual's asset holdings must equal is or her total wealth. Thus if the market for every asset but one clears,

[4] This presentation makes the standard assumption that M is exogenous. Taylor (1998) nd D. Romer (2000) have recently proposed replacing this assumption with an assumption hat the central bank adjusts M to make the real interest rate an increasing function of nflation, and perhaps of output as well. They argue that this alternative better describes hat central banks do and leads to a model that is easier to analyze. It is too soon to now whether this approach will become the standard textbook formulation. Since this pproach and the usual one have similar implications for our purposes, we maintain the sual assumption of an exogenous money supply.

the market for the remaining asset must clear as well. In the *IS-LM* model there are only two assets (money and everything else), and so only one asset-market equilibrium condition is needed. Many important extensions of the *IS-LM* model investigate the consequences of relaxing the assumption that all assets other than money are perfect substitutes.[5]

The *AD* Curve

The intersection of the *IS* and *LM* curves shows the values of i and Y such that the money market clears and actual and planned expenditures are equal for given levels of M, P, π^e, G, and T. To see how the *IS* and *LM* curves imply the existence of a downward-sloping relationship between P and Y, consider the effects of assuming a higher value of P. Since the price level does not enter the planned expenditure function, $E(\bullet)$, the *IS* curve is unaffected. The rise in the price level reduces the supply of real money balances, however. Thus a higher interest rate is needed to clear the money market for a given level of income, and so the *LM* curve shifts up. As a result, i rises and Y falls. This is shown in Figure 5.4. Thus the level of output at the

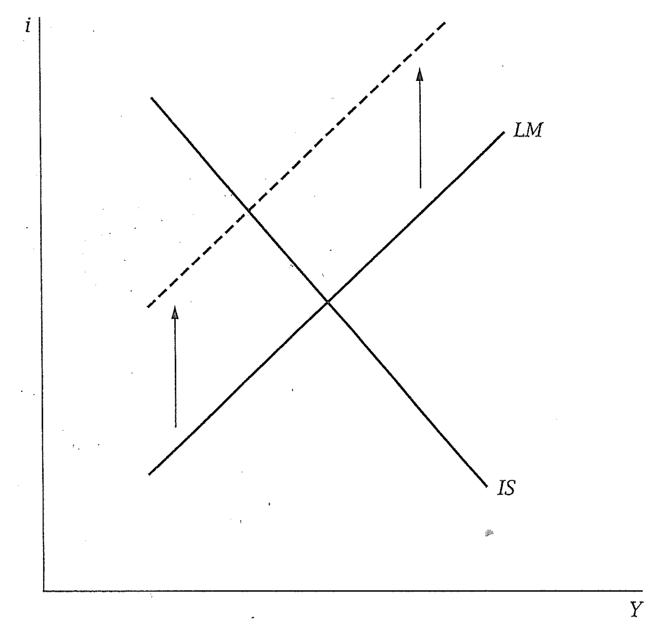

FIGURE 5.4 The effects of an increase in the price level

[5] Two classic references are Tobin and Brainard (1963) and Tobin (1969). A large recent literature relaxes the assumption that assets held by banks, particularly their loans, are perfect substitutes for other interest-bearing assets. See Bernanke and Blinder (1988) and Kashyap and Stein (1994).

intersection of the IS and LM curves is a decreasing function of the price level. This is what is shown by the aggregate demand curve.

To find the slope of the AD curve, differentiate (5.4) and (5.7) with respect to P. This yields two equations in two unknowns:

$$\left.\frac{dY}{dP}\right|_{AD} = E_Y \left.\frac{dY}{dP}\right|_{AD} + E_{i-\pi^e} \left.\frac{di}{dP}\right|_{AD}, \tag{5.9}$$

$$-\frac{M}{P^2} = L_i \left.\frac{di}{dP}\right|_{AD} + L_Y \left.\frac{dY}{dP}\right|_{AD}. \tag{5.10}$$

These can be solved to obtain

$$\left.\frac{dY}{dP}\right|_{AD} = \frac{-M/P^2}{[(1-E_Y)L_i/E_{i-\pi^e}] + L_Y}. \tag{5.11}$$

This expression is unambiguously negative, and it shows the determinants of the slope of the aggregate demand curve.

Example: The Effects of an Increase in Government Purchases

The IS and LM curves provide a simple model of aggregate demand that can be used to analyze many issues. Suppose, for example, that government purchases rise. The increase in G raises planned expenditure for a given level of output and the interest rate. The planned expenditure line in Figure 5.3 therefore shifts up, and so the level of Y such that actual and planned expenditures are equal is higher for a given level of the interest rate. Thus the IS curve shifts to the right; this is shown in Panel (a) of Figure 5.5. The shift in the IS curve raises Y (and i) for a given price level, and thus moves the AD curve outward; this is shown in Panel (b) of the figure.[6]

The impact of this change in aggregate demand on output and the price level depends on the aggregate supply curve. If it is vertical, only the price level increases. If it is horizontal, only output increases. And if it is upward-sloping but not vertical, both output and the price level increase.

Thus, incomplete adjustment of nominal prices introduces a new channel through which shocks affect output. For some reason, which we have not yet specified, nominal prices do not adjust fully in the short run. As a result, any change in the demand for goods at a given price level affects output. In contrast, the intertemporal-substitution and wealth effects that drive employment fluctuations in real-business-cycle models would correspond to effects of government purchases on the aggregate supply curve—that is, they would affect not the quantity of output that households and firms want to buy at a given price level, but the quantity that firms want to produce at a given price level.

[6] The IS-LM diagram is drawn for a given value of P. Thus the amount that output increases in the IS-LM diagram is the same as the amount that the aggregate demand curve shifts to the right at the value of P assumed in the IS-LM diagram.

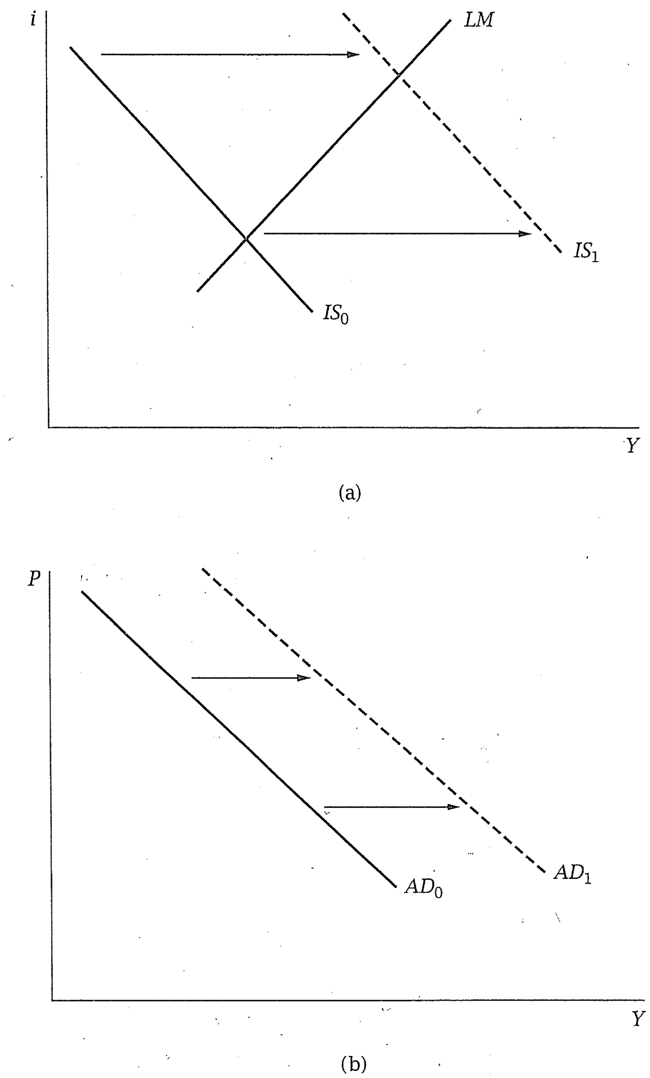

FIGURE 5.5 The effects of an increase in government purchases

5.2 The Open Economy

In most practical applications, the exchange rate and international trade are important to short-run fluctuations. This section therefore extends the IS-LM model to the case of an open economy.[7]

[7] See Obstfeld and Rogoff (1996) for the state-of-the-art treatment of open-economy macroeconomics.