MONETARY POLICY

Interest-rate policy with uncertainty

"IS shocks"

$$Y_t = -\beta(r - \bar{r}_t)$$ varies over time

$\hat{r}_t$ Central Bank's guess at $\bar{r}_t$

$$\hat{r}_t = \bar{r}_t + \varepsilon_t$$ error in Central Bank's guess

$E[\varepsilon] = 0$ it's a rational expectation. σ_ε^2 variance in error

$$Y_t = -\beta(r_t - \hat{r}_t + \varepsilon_t)$$

$$= -\beta(r_t - \hat{r}_t) + \beta\varepsilon_t$$ equivalent to "shock" to IS curve

$$\pi_t = {}_{t-1}\pi_t^e + \alpha Y_t = {}_{t-1}\pi_t^e + \alpha\beta(r_t - \hat{r}_t) + \alpha\beta\varepsilon_t$$

$$L = \tfrac{1}{2}E[y^2] + \tfrac{1}{2}aE[(\pi - \pi^*)^2]$$

$$= \tfrac{1}{2}(E[y])^2 + \tfrac{1}{2}a(E[\pi - \pi^*])^2 + \tfrac{1}{2}Var(y) + \tfrac{1}{2}a\,Var(\pi)$$

$$= \tfrac{1}{2}\underbrace{\beta^2(r_t - \hat{r}_t)^2}_{(E[y])^2} + \tfrac{1}{2}a\underbrace{\left({}_{t-1}\pi^e - \pi^* - \alpha\beta(r_t - \hat{r}_t)\right)^2}_{(E[\pi - \pi^*])^2} + \tfrac{1}{2}\beta^2(1 + a\alpha^2)\sigma_\varepsilon^2$$

Calculate $\frac{\partial L}{\partial r}$, set $\frac{\partial L}{\partial r} = 0$ and solve for r^*.

MONETARY POLICY
Interest-rate policy with...
IS shocks (cont.)

$$\frac{\partial L}{\partial r} = \beta^2 (r_t - \hat{r}_t) + a\left({}_{t-1}\pi^e - \pi^* - \alpha\beta(r_t - \hat{r}_t)\right)(-\alpha\beta)$$

Setting $\frac{\partial L}{\partial r} = 0$ gives

$$r_t^* - \hat{r}_t = \frac{a\alpha}{\beta(1+a\alpha^2)}\left({}_{t-1}\pi_t^e - \pi^*\right)$$

same as without uncertainty!
"Certainty equivalence"

$$y_t = -\frac{a\alpha}{1+a\alpha^2}\left({}_{t-1}\pi_t^e - \pi^*\right) + \beta\varepsilon_t$$

$$\pi_t = \pi^* + \frac{1}{1+a\alpha^2}\left({}_{t-1}\pi_t^e - \pi^*\right) + \alpha\beta\varepsilon_t.$$

What is value of L at r^*?

$$L = \tfrac{1}{2}\left(E[y]\right)^2 + a\tfrac{1}{2}\left(E[\pi - \pi^*]\right)^2 + \tfrac{1}{2}Var(y) + \tfrac{1}{2}a\,Var(\pi - \pi^*)$$

$$= \tfrac{1}{2}\left(\frac{a\alpha}{1+a\alpha^2}\right)^2 \left({}_{t-1}\pi_t^e - \pi^*\right)^2 + \tfrac{1}{2}a\left(\frac{1}{1+a\alpha^2}\right)^2\left({}_{t-1}\pi_t^e - \pi^*\right)^2 + \tfrac{1}{2}\beta^2\sigma_\varepsilon^2 + \tfrac{1}{2}a\alpha^2\beta^2\sigma_\varepsilon^2$$

$$= \tfrac{1}{2}\frac{a}{1+a\alpha^2}\left({}_{t-1}\pi_t^e - \pi^*\right)^2 + \tfrac{1}{2}\beta^2(1+a\alpha^2)\sigma_\varepsilon^2$$

Note: bigger $\left({}_{t-1}\pi_t^e - \pi^*\right)$ gives bigger loss.

Lesson: announce inflation target...

MONETARY POLICY

Interest rate policy with...

IS shocks

In rational expectations equilibrium

Assume ${}_{t-1}\pi^e_t = E[\pi_t]$

$${}_{t-1}\pi^e_t = E\left[\pi^* + \frac{1}{1+a\alpha^2}\left({}_{t-1}\pi^e_t - \pi^*\right) + \alpha\beta\varepsilon_t\right]$$

where $E[\varepsilon_t] = 0$

$${}_{t-1}\pi^e_t = \pi^* + \frac{1}{1+a\alpha^2}\left({}_{t-1}\pi^e_t - \pi^*\right)$$

$$\Rightarrow {}_{t-1}\pi^e_t = \pi^*$$

so $r^*_t - \hat{r}_t = 0$

$$y_t = 0 + \beta\varepsilon_t$$

$$\pi_t = \pi^* + \alpha\beta\varepsilon_t$$

$$i_t = \hat{r}_t + \pi^e = \hat{r}_t + \pi^*$$

$$L = \tfrac{1}{2}\beta^2(1+a\alpha^2)\sigma^2_\varepsilon$$

Note: ε_t must be uncorrelated with $\hat{r}_t$,

so y_t and π_t are uncorrelated with i_t!

It looks like monetary policy has no effect on π or y!

MONETARY POLICY

Interest rate policy with…
IS shocks
In rational expectations equilibrium
"Inflation forecast targeting"

For ${}_{t-1}\pi^e_t = \pi^*$,

$$E[\pi_t] = \pi^* - \alpha\beta(r_t - \hat{r}_t) + \underbrace{E[\alpha\beta\varepsilon_t]}_{\text{zero}}$$

r^*_t is value of r_t for which $E[\pi_t] = \pi^*$

$i^*_t = r^*_t + \pi^*$ is value of i_t for which $E[\pi_t] = \pi^*$

Optimal policy means setting i_t so that central banks forecast of π is equal to π^*, forecast of y is equal to 0.

What's relation between i_t and $y_{t-1}, \pi_{t-1}, i_{t-1}$?

$$i_t = \hat{r}_t + \pi^*$$

bank's guess at $\bar{r}_t$

so it depends on relation between $\hat{r}_t$ and y_{t-1}, etc.

We'll get back to this