RBC Thy Problems

5.8) $E[U] = E\left[\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t u(C_t)\right]$

where $u(C_t) = C_t - \theta C_t^2$ hence $u'(C_t) = 1 - 2\theta C_t$

$Y_t = A K_t + e_t$ means $\frac{\partial Y}{\partial K} = A$ ← ("interest rate")

$K_{t+1} = K_t + Y_t - C_t$ hence $\partial K_{t+1}/\partial C_t = -1$, $C_t = K_{t+1} + K_t + Y_t$

$e_t = \phi e_{t-1} + \varepsilon_t$ where $E[\varepsilon] = 0$ hence $E[e_t] = \phi e_{t-1}$

and $A = \rho$

("productivity shock")

a) Find Euler equation

Variables that define current state: K_t, e_t

$$V(K_t, e_t) = \underset{C_t}{\text{Max}}\left\{u_c(C_t) + \frac{1}{1+\rho} E_t\left[V(K_{t+1}, e_{t+1})\right]\right\}$$

$$\frac{\partial V}{\partial C_t} = 0 = 1 - 2\theta C_t + \frac{1}{1+\rho} E_t\left[V_K(\quad)\underbrace{\frac{\partial K_{t+1}}{\partial C_t}}_{-1}\right]$$

What's V_K? Envelope theorem, Benveniste-Scheinkman

$$V_K(K_t, \varepsilon_t) = \frac{\partial u}{\partial C_t}\frac{\partial C_t}{\partial K_t} = (1 - 2\theta C_t)\frac{\partial C_t}{\partial K_t}$$

and $\frac{\partial C_t}{\partial K_t} = 1 + \frac{\partial Y_t}{\partial K_t} = 1 + A = 1 + \rho$

so $V_K = (1 - 2\theta C_t)(1+\rho)$

$$E_t[V_K(\ _{+1}\ )] = E[(1 - 2\theta C_{t+1})(1+\rho)]$$
$$= (1+\rho)(1 - 2\theta C_{t+1}^e)$$

5,8) a) (cont.)

$$0 = 1 - 2\theta C_t + \frac{1}{1+\rho}(1+\rho)(1 - 2\theta C^e_{t+1})$$

$$C_t = C^e_{t+1}$$ "Consumption is a random walk"

b) Conjecture $C_t = \alpha + \beta K_t + \gamma e_t$

then $K_{t+1} = K_t + \underbrace{Y_t}_{AK_t + e_t} - C_t = (1+\rho-\beta)K_t + (1-\gamma)e_t - \alpha$

c) What values of α, β, γ could work (satisfy Euler equation)?

$$C^e_{t+1} = E_t[\alpha + \beta K_{t+1} + \gamma e_{t+1}] = \alpha + \beta E[K_{t+1}] + \gamma \overbrace{E[e_{t+1}]}^{\phi e_{t-1}}$$

$$= \alpha + \beta((1+\rho-\beta)K_t + (1-\gamma)e_t - \alpha) + \gamma\phi e_{t-1}$$

$$= (1-\beta)\alpha + (1+\rho-\beta)\beta K_t + (\beta + \gamma(\phi - \beta))e_t$$

and $C^e_{t+1} = C_t = \alpha + \beta K_t + \gamma e_t$

so question is, what values of α, β, γ satisfy

$$\overbrace{\alpha + \beta K_t + \gamma e_t}^{C_t} = \ldots\ldots$$

means
$$\alpha = (1-\beta)\alpha$$
$$\beta = (1+\rho-\beta)\beta$$
$$\gamma = \beta + \gamma(\phi - \beta)$$

5.8)

c) (cont.)

$$\alpha = (1-\beta)\alpha \qquad \beta = (1+\rho-\beta)\beta \qquad \gamma = \beta + \gamma(\phi - \beta)$$

One solution:

$$1 = (1-\beta)$$

hence $\beta = 0 \Rightarrow 0 = (1+\rho-0)0 \Rightarrow \gamma = 0 + \gamma(\phi - 0)$

$$\gamma = \gamma\phi$$

$$\gamma = 0$$

and α can be anything.

Another solution:

$$\alpha = 0$$

$$\beta = (1+\rho-\beta)\beta$$

$$1 = 1+\rho-\beta$$

$$\beta = \rho \; (=A) \Rightarrow \gamma = \rho + \gamma(\phi - \rho)$$

$$= \rho + \gamma\phi - \gamma\rho$$

$$\gamma + \gamma\rho - \gamma\phi = \rho$$

$$(1+\rho-\phi)\gamma = \rho$$

$$\gamma = \frac{\rho}{1+\rho-\phi}$$

substituting these into conjectured equation for C_t gives

$$C_t = \rho K_t + \left(\frac{\rho}{1+\rho-\phi}\right) e_t$$

and

$$K_{t+1} = (1+\rho-\rho) K_t + \left(1 - \frac{\rho}{1+\rho-\phi}\right) e_t$$

$$= K_t + \frac{1-\phi}{1-\phi+\rho} e_t$$

With these two equations, you can trace out path of C & K over time in response to ε_t

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5.8) cont.

d) Effects of one-time ε shock on paths of Y, K, C

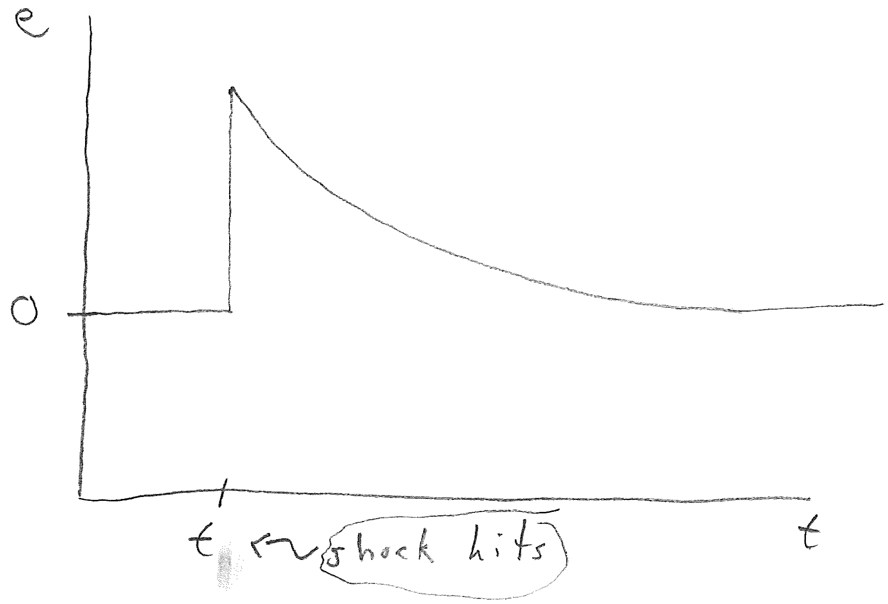
$$e = \phi e_{t-1} + \varepsilon_t$$

$$C_t = \rho K_t + \left(\frac{\rho}{1+\rho-\phi}\right) e_t$$

$$K_{t+1} = K_t + \frac{1-\phi}{1-\phi+\rho} e_t$$

$$Y_t = A K_t + e_t$$

path for e:



Note: e is zero before & eventually after.

with zero e,

$$K_{t+1} = K_t \quad \text{hence}$$

before shock $\quad K_t = K_{t-1} = K_{t-2} = K_{t-3} \cdots$

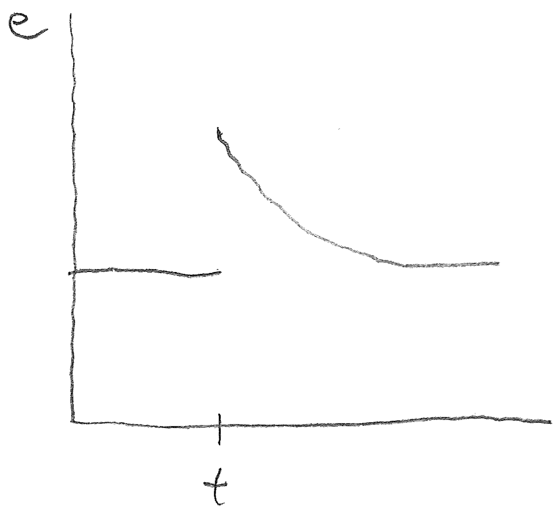
eventually, after shock $\quad K_{t+j} = K_{t+j+1} = K_{t+j+2} \cdots$

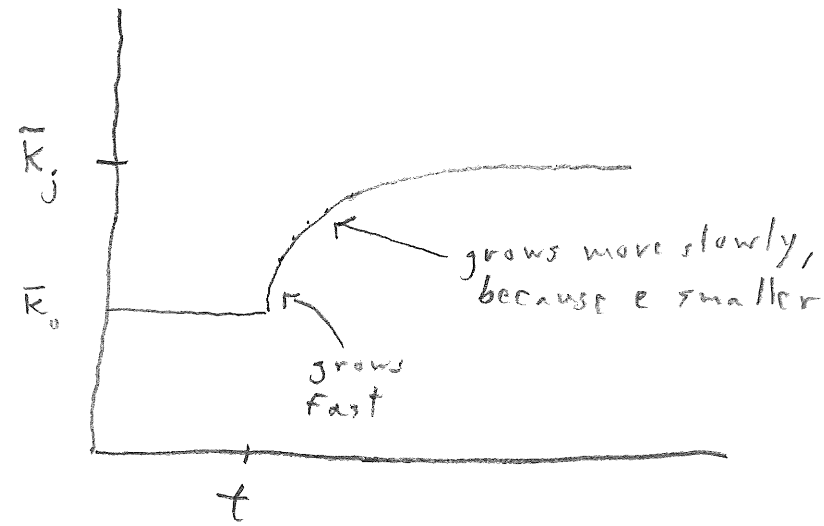
RBC thy

5.8) cont.

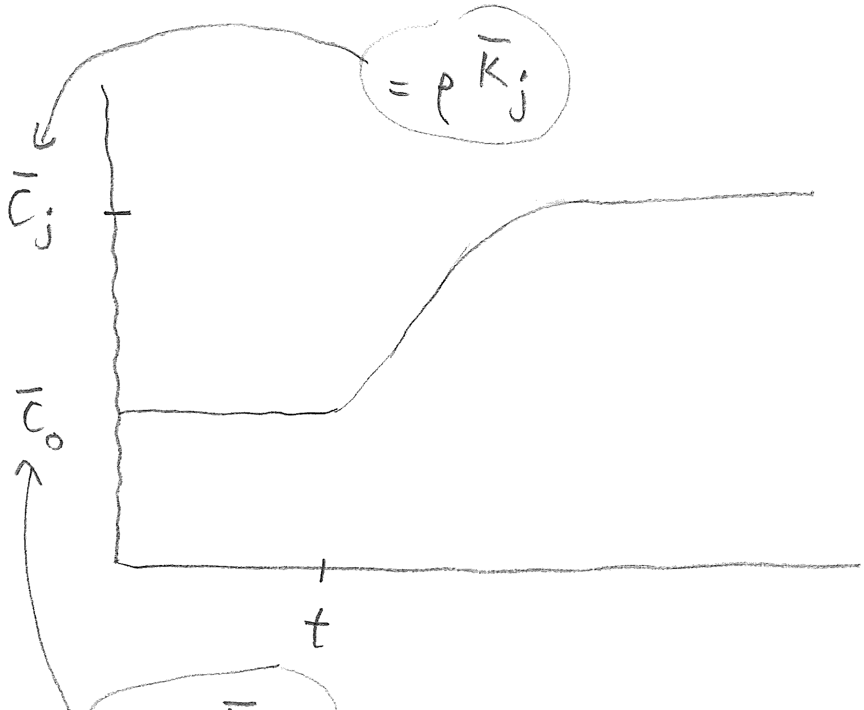
In the meantime, before e has settled down, K grows

$$K_{t+1} = K_t + \frac{1-\phi}{1-\phi+\rho}\, e_t$$



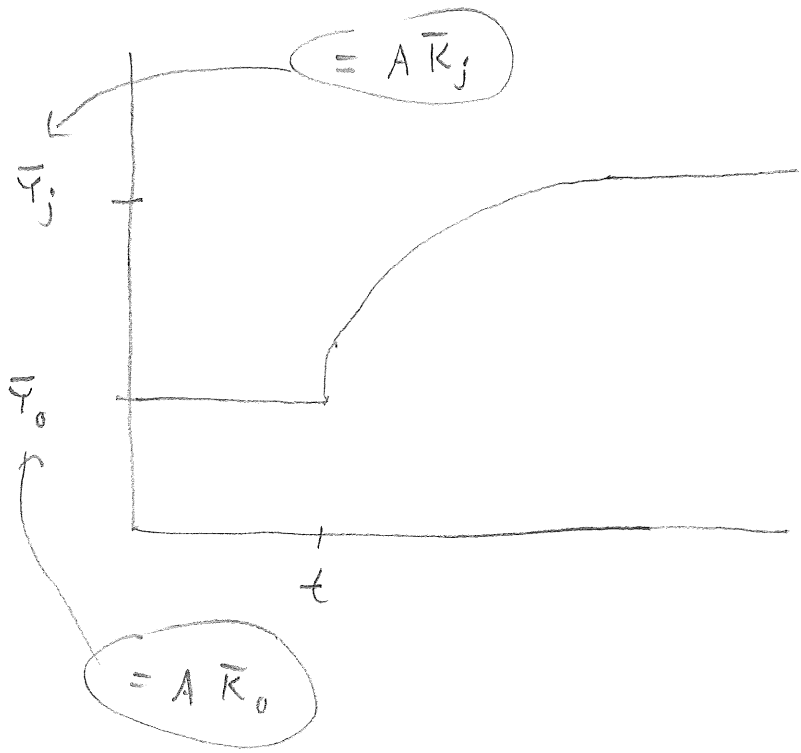


$$C_t = \rho K_t + (\quad) e_t$$



5. 8) cont.

Finally, $Y_t = A K_t + e_t$



RBC Thy

5.10) For LRSS without shocks,

a) Get expressions defining Y^*, k^*, c^*, w^*, l^*, r^*

recall C/AL vs. c in utility function

b) Assuming $\alpha = \frac{1}{3}$, $g = 0.5\% = 0.005$, $n = 0.25\% = 0.0025$, $\delta = 2.5\% = 0.025$, $(G^*/Y) = 0.2$, ← share of G in Y

$r^* = 1.5\% = 0.015$, $l^* = \frac{1}{3}$,

what is $\frac{C}{Y}$, $\frac{I}{Y}$, $\frac{K}{\text{Annual } Y} = \frac{K}{4 \cdot Y}$ ← quarterly Y

Note: in class I derived LRSS values for $G = 0$.

a) From $\frac{1}{c_t} = e^{-\rho} \frac{1}{c_{t+1}} (1 + r^*)$ and $\frac{c_{t+1}}{c_t} = e^{g}$,

we get $(1 + r^*) = e^{g+\rho}$

$$r^* = e^{g+\rho} - 1$$

From $y^* = k^{*\alpha}$ we get $r^* = f'(k) - \delta = \alpha k^{*-(1-\alpha)} - \delta$

solve for k^*,

$$k^* = \left(\frac{\alpha}{e^{g+\rho} + \delta - 1}\right)^{\frac{1}{1-\alpha}} = \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{1}{1-\alpha}}$$

$$y^* = \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{\alpha}{1-\alpha}}$$

5.10) (cont.)

Recall $w = MPL$ where $Y = K^{\alpha}(AL)^{1-\alpha}$

hence $w = K^{\alpha}(1-\alpha)(AL)^{-\alpha} A = A(1-\alpha)\left(\frac{K}{AL}\right)^{\alpha} = A(1-\alpha)k^{\alpha}$

$w^* = A(1-\alpha)k^{*\alpha} \leftarrow$ (defined above

What about $\frac{C}{L}$ and $\frac{C}{AL}$?

$c^* = \left(\frac{C}{AL}\right)^* = f(k^*) - (n+g+\delta)k^* - G^* \leftarrow (G/AL)^*$

We're not told G^*; we're told $\frac{G^*}{y} \leftarrow \frac{Govt./AL}{Y/AL} = \frac{Govt.}{Y} = 0.2$

so put it in terms of G/Y:

$$c^* = y^* - y\frac{G^*}{Y} - (n+g+\delta)k^*$$

$$= \left(1 - \frac{G^*}{Y}\right)y^* - (n+g+\delta)k^*$$

$$= \left(1 - G^*/Y\right)k^{*\alpha} - (n+g+\delta)k^*$$

and

$$\frac{C^*}{L} = \frac{C}{AL}\cdot A = Ac^* = A\left(\left(1 - \frac{G^*}{Y}\right)k^{*\alpha} - (n+g+\delta)k^*\right)$$

Finally, what's l^*?

From $U'_{1-l}(1-l) = wU'_c(C/L)$, here $b\frac{1}{1-l} = w\frac{1}{C/L}$,

we have $l^* = 1 - b\frac{(C/L)^*}{w^*}$

$$= 1 - b\frac{((1 - G/Y^*)k^{*\alpha} - (n+g+\delta)k^*}{(1-\alpha)k^{*\alpha}}$$

5.10) (cont.)

b) Put in parameter values

$$k^* = \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{1}{1-\alpha}} = \left(\frac{1/3}{0.015 + 0.025}\right)^{\frac{1}{1-1/3}} \approx 24.1$$

$$y^* = k^{\alpha} = 24.1^{1/3} \approx 2.9$$

$$c^* = \frac{C}{AL} = (1 - G^*)\, y^* - (n + g + \delta)\, k^*$$

$$= (1 - 0.2)\, 2.9 - (0.0025 + 0.0050 + .025)\, 24.1$$

$$\approx 1.53$$

What's C/Y, $\frac{I}{Y}$, $\frac{K}{4Y}$?

$$\frac{C}{Y} = \frac{C/AL}{Y/AL} = \frac{1.53}{2.9} \approx 0.53 \text{ or } 53\%$$

$$\frac{I}{Y} = 1 - \frac{G}{Y} - \frac{C}{Y} = 1 - 0.2 - 0.53 = .27 \text{ or } 27\%$$

$$\frac{K}{4Y} = \frac{1}{4}\,\frac{K/AL}{Y/AL} = \frac{1}{4}\,\frac{24.1}{2.9} \approx 2.07$$