Economics 501, Macroeconomics, Christopher Hanes Fall 2007
Midterm examination. Look over the entire examination before you begin.

1) Consider a closed economy where the central bank's policy committee does not follow an interest-rate rule. The price level and expected inflation are fixed.The equations describing the economy are:

$$Y = E(r, G, T) \quad \text{where } E_r < 0 \;,\; E_G > 0 \;,\; E_T < 0$$

$$\frac{M}{P} = L(i, Y) \quad \text{where } L_i < 0 \;,\; L_Y > 0$$

(3) a) Assume the central bank fixes the money supply at a value M^S. Derive the response of output to a change in government expenditure $\partial Y/\partial G$.

$Y = E(r, G, T)$

$$\frac{\partial Y}{\partial G} = E_r \frac{\partial i}{\partial G} + E_G$$

$M^S/P = L(i, Y)$

$$\frac{\partial (M^S/P)}{\partial G} = 0 = L_i \frac{\partial i}{\partial G} + L_Y \frac{\partial Y}{\partial G}$$

$$L_i \frac{\partial i}{\partial G} = -L_Y \frac{\partial Y}{\partial G}$$

$$\frac{\partial i}{\partial G} = -\frac{L_Y}{L_i} \frac{\partial Y}{\partial G}$$

$$\frac{\partial Y}{\partial G} = E_r \left(-\frac{L_Y}{L_i} \frac{\partial Y}{\partial G} \right) + E_G$$

$$\frac{\partial Y}{\partial G} + E_r \frac{L_Y}{L_i} \frac{\partial Y}{\partial G} = E_G$$

$$\left(1 + E_r \frac{L_Y}{L_i} \right) \frac{\partial Y}{\partial G} = E_G$$

$$\frac{\partial Y}{\partial G} = \frac{E_G}{1 + E_r \frac{L_Y}{L_i}} = \frac{1}{1 + E_r \frac{L_Y}{L_i}} E_G$$

this is + but less than one

(3) b) Now assume the central bank follows a "money supply rule." The central bank sets the money supply as a negative function of the interest rate (the central bank reduces the money supply when the interest rate is higher). This behavior can be described by an expression:

$M^S(i)$ *where* $M_i^S < 0$

Derive the response of output to a change in government expenditure $\partial Y/\partial G$.

$$Y = E(r, G, T)$$

$$\frac{\partial Y}{\partial G} = E_r \frac{\partial i}{\partial G} + E_G$$

$$M^s(i)/P = L(i, Y)$$

$$\frac{1}{P} M_i^s \frac{\partial i}{\partial G} = L_i \frac{\partial i}{\partial G} + L_Y \frac{\partial Y}{\partial G}$$

$$L_i \frac{\partial i}{\partial G} - \frac{M_i^s}{P} \frac{\partial i}{\partial G} = -L_Y \frac{\partial Y}{\partial G}$$

$$\left(L_i - \frac{M_i^s}{P}\right) \frac{\partial i}{\partial G} = -L_Y \frac{\partial Y}{\partial G}$$

$$\frac{\partial i}{\partial G} = -\frac{L_Y}{L_i - \frac{M_i^s}{P}} \frac{\partial Y}{\partial G}$$

$$\frac{\partial Y}{\partial G} = E_r \left(-\frac{L_Y}{L_i - \frac{M_i^s}{P}} \frac{\partial Y}{\partial G}\right) + E_G$$

$$\left(1 + E_r \frac{L_Y}{L_i - \frac{M_i^s}{P}}\right) \frac{\partial Y}{\partial G} = E_G$$

$$\frac{\partial Y}{\partial G} = \frac{1}{1 + E_r \frac{L_Y}{L_i - \frac{M_i^s}{P}}} E_G$$

this is +, less than one.

2) Consider an economy that can be described by the open-economy Keynesian model with imperfect capital mobility and a floating exchange rate. The central bank does not follow an interest-rate rule: instead the central bank policy committee chooses values for the money supply. The expressions describing the economy are:

$$Y = Z(Y,r,G,T) + NX(\varepsilon) \quad \text{where } E_Y > 0 \text{ , } E_r < 0 \text{ , } E_G > 0 \text{ , } E_T < 0 \text{ , } NX_\varepsilon > 0$$

$$\frac{M}{P} = L(i,Y) \quad \text{where } L_i < 0 \text{ , } L_Y > 0$$

$$NX(\varepsilon) + CF\left(r - r^* - \frac{\dot{\varepsilon}^e}{\varepsilon}\right) = 0 \text{ where } CF_{r-r^*+\dot{\varepsilon}^*/\varepsilon} > 0$$

Assume "static exchange-rate expectations" $\frac{\dot{\varepsilon}^e}{\varepsilon} = 0$ (means $\frac{\partial r}{\partial i} = 1$)

and expected future inflation is always equal to zero $\pi^e = 0$.

(3) a) Derive the slope of the IS** curve $\partial i/\partial Y$.

$Y = Z(Y, r, G, T) + NX(\varepsilon)$

$\frac{\partial Y}{\partial i} = E_Y \frac{\partial Y}{\partial i} + E_r + NX_\varepsilon \frac{\partial \varepsilon}{\partial i}$

$(1 - E_Y)\frac{\partial Y}{\partial i} = E_r + NX_\varepsilon \frac{\partial \varepsilon}{\partial i}$

$NX(\varepsilon) + CF(r - r^*) = 0$

$NX_\varepsilon \frac{\partial \varepsilon}{\partial i} + CF_r = 0$

$NX_\varepsilon \frac{\partial \varepsilon}{\partial i} = -CF_r$

$\frac{\partial Y}{\partial i} = \frac{1}{1-E_Y}\left(E_r + NX_\varepsilon \frac{\partial \varepsilon}{\partial i}\right)$

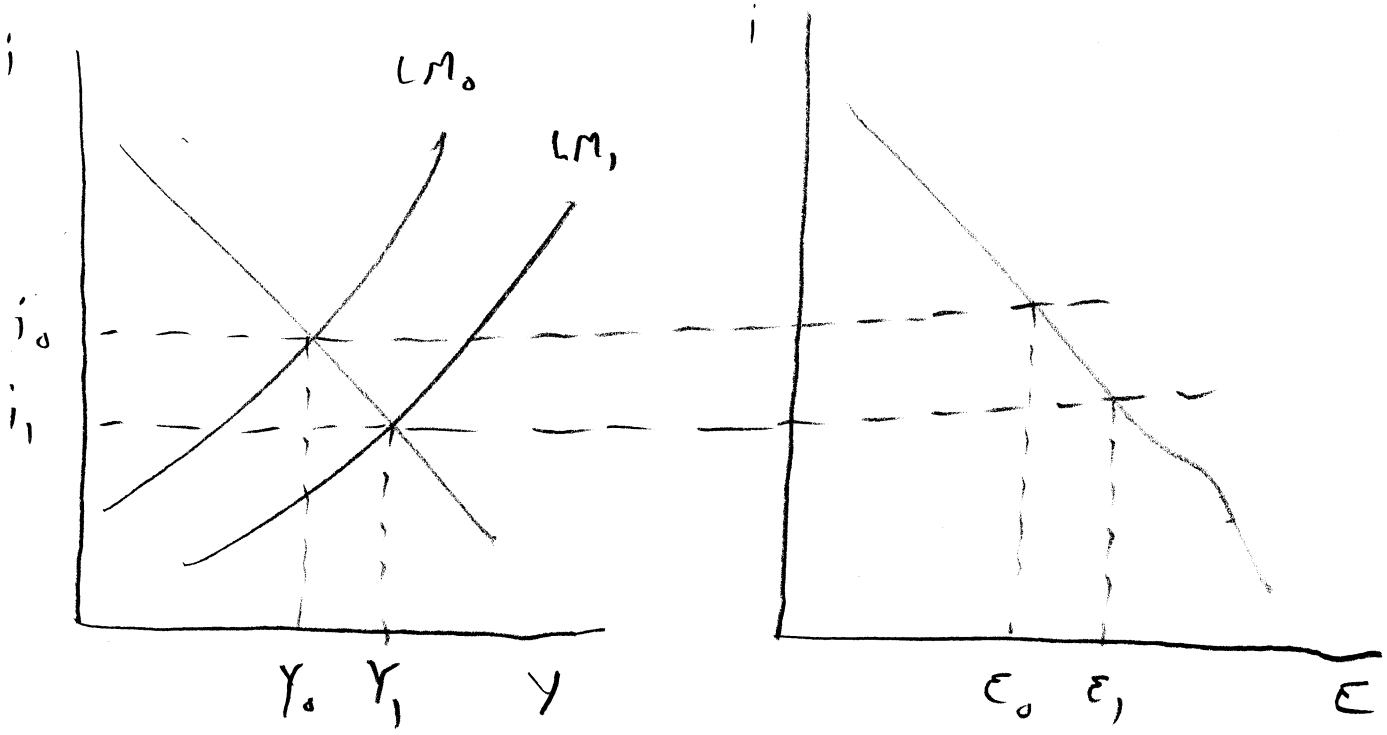
$\frac{\partial Y}{\partial i} = \frac{1}{1-E_Y}\left(E_r - CF_r\right) = \frac{E_r - CF_r}{1 - E_Y}$

$\frac{\partial i}{\partial Y} = \frac{1 - E_Y}{E_r - CF_r}$ or $\frac{1 - Z_Y}{Z_r - CF_r}$

b) Suppose the central bank wants to increase the domestic money supply.

(3) i) What happens to output Y, the interest rate i, and the real exchange rate ε, if the central bank increases the money supply by buying domestic assets (such as bonds issued by the country's government)? Illustrate your answer with a graph or graphs.

Buying domestic assets (bonds) with domestic assets (money) has no effect on RG. So this is just outward shift in LM.



Y↑, i↓, ε↑ (depreciation)

this is from

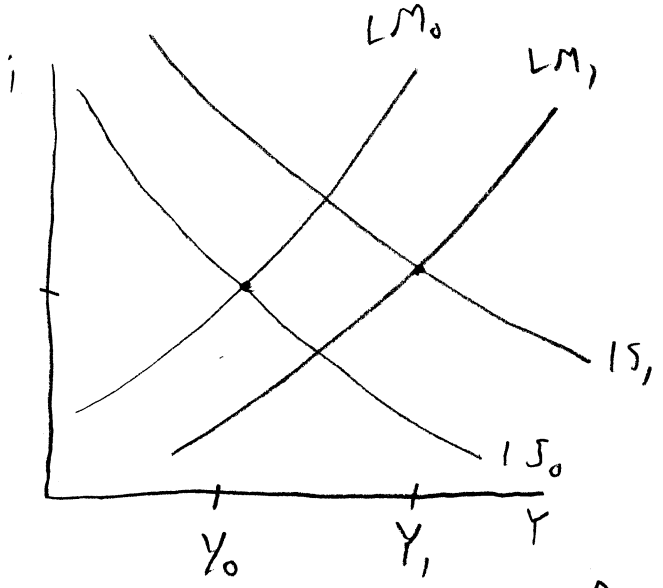
$CF(i - i^*) = -NX(\varepsilon)$

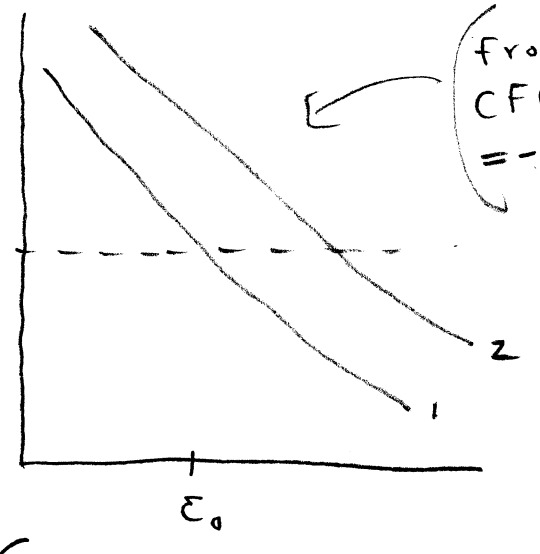
(3) ii) What happens to output Y, the interest rate i, and the real exchange rate ε, if the central bank increases the money supply by buying foreign assets (such as bonds issued by a foreign government)? Illustrate your answer with a graph or graphs.

Buying foreign assets means $RG > 0$.

$CF(r - r^*) + NX(\varepsilon) - RG = 0$

At given r, $\varepsilon\uparrow$ (to make $NX\uparrow$) and IS^{**} shifts out, other curve shifts out.





$Y\uparrow$, i ambiguous, $\varepsilon\uparrow$ ← How do I know? $RG\uparrow$ by itself causes $\varepsilon\uparrow$; $M^S\uparrow$ by itself causes $\varepsilon\uparrow$; so $RG\uparrow$ and $M^S\uparrow$ cause $\varepsilon\uparrow$

(3) iii) Is the effect of the central bank's action on Y bigger in *i)*, bigger in *ii)*, or the same magnitude in both cases? Is the effect of the central bank's action on i bigger in *i)*, bigger in *ii)*, or the same magnitude in both cases?

Effect on Y bigger in ii)

Effect on i smaller in ii)

3) Consider a small open economy subject to perfect international capital mobility, which can be described by the Mundell-Fleming model. The exchange rate is floating. Assume that expected inflation π^e, the price level P, and the foreign real interest rate r^* (and foreign output Y^*) all remain fixed. Using a graph or graphs, describe what happens to *output* and the *exchange rate* if the public comes to expect that the country's currency will depreciate in the future, assuming that:

(3)

a) The central bank fixes the money supply.

$r = r^* + \frac{\dot{\varepsilon}^e}{\varepsilon}$

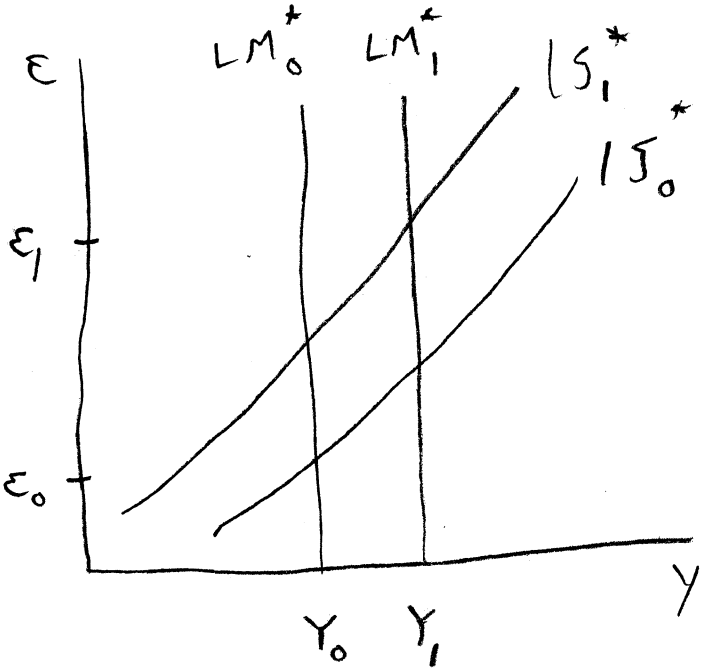
$i = r + \pi^e = r^* + \pi^e + \frac{\dot{\varepsilon}^e}{\varepsilon}$

If $\frac{\dot{\varepsilon}^e}{\varepsilon}$ goes from zero to $+$, (expected future depreciation)

then r and i increase.

From $\frac{M}{P} = L(i, Y)$, (−) (+), if i increases, Y must increase.

↑ (LM^* shifts out.)



From $Y = E(Y, r, \varepsilon, \ldots)$, since $r\uparrow$, any given ε corresponds to lower Y.

↑ (IS^* shifts back)

So $Y\uparrow$, $\varepsilon\uparrow$ (depreciation)

(3) b) The central bank follows an interest-rate rule $r(\pi, Y)$ *where* $r_\pi > 0$, $r_Y > 0$. For this case, also describe what happens to the domestic money supply.

$$r(\pi, Y) = r^* + \frac{\dot{\mathcal{E}}^e}{\mathcal{E}}$$

$$i = r(\pi, Y) + \pi^e = r^* + \pi^e + \frac{\dot{\mathcal{E}}^e}{\mathcal{E}}$$

If $\frac{\dot{\mathcal{E}}^e}{\mathcal{E}}$ goes from zero to +, then r & i increase.

From $r(\pi, Y) = r$, (+ +) if r increases Y must increase.

(we know $\pi = 0$ because I said P is fixed)

(MP^* shifts out)

From $Y = E(Y, r, \mathcal{E}, \ldots)$ since r ↑, any given $\mathcal{E}$ gives lower Y

(IS^* shifts back)

So Y ↑, $\mathcal{E}$ ↑.

What about M^S?

$$\frac{M^S}{P} = L(i, Y)$$

(M^S fixed; i went up; Y went up)

Can't say! Depends on L_i and L_Y.

4) Consider an economy where the central bank does not follow an interest-rate rule. Instead it follows a money-supply rule that creates an automatic negative relation between the current money supply and current output. Thus, the economy is described by these expressions:

AD: $y_t = m_t - p_t + \varepsilon_t$

AS/Phillips curve: $\pi_t = {}_{t-1}\pi_t^e + fy_t$

Money-supply rule: $m_t = \bar{m} - jy_t$

where y is the log output gap, $\bar{m}$ is a fixed value, and ε is a mean-zero, i.i.d. random variable with variance σ_ε^2. The public knows the distribution for ε_t, but as of time (t-1) the public cannot predict what the realized value of ε_t will be.
The economy is in rational expectations equilibrium.

(3) a) Derive expected inflation ${}_{t-1}p_t^e$.

First, get realized p_t in terms of ${}_{t-1}p_t^e$.

From AS, $p_t - p_{t-1} = {}_{t-1}p_t^e - p_{t-1} + fy_t$

$\Rightarrow p_t = {}_{t-1}p_t^e + fy_t$

From AD and money-supply rule,

$y_t = \underbrace{\bar{m} - jy_t}_{m_t} - p_t + \varepsilon_t$

$(1+j)y_t = \bar{m} - p_t + \varepsilon_t$

$y_t = \frac{1}{1+j}(\bar{m} - p_t + \varepsilon_t)$ put that in here

$p_t = {}_{t-1}p_t^e + \frac{f}{1+j}(\bar{m} - p_t + \varepsilon_t)$

$p_t = {}_{t-1}p_t^e + \frac{f}{1+j}\bar{m} - \frac{f}{1+j}p_t + \frac{f}{1+j}\varepsilon_t$

$\left(1 + \frac{f}{1+j}\right)p_t = {}_{t-1}p_t^e + \frac{f}{1+j}\bar{m} + \frac{f}{1+j}\varepsilon_t$

$$\frac{1+j+f}{1+j} p_t = \text{------}$$

$$p_t = \frac{1+j}{1+j+f} {}_{t-1}p_t^e + \frac{f}{1+j+f} \bar{m} + \frac{f}{1+j+f} \varepsilon_t$$

Here we are! Now, R.E.E.:

$$p^e = E[p] = \frac{1+j}{1+j+f} p^e + \frac{f}{1+j+f} \bar{m} + \frac{f}{1+j+f} \underbrace{E[\varepsilon_t]}_{\text{zero}}$$

$$\left(1 - \frac{1+j}{1+j+f}\right) p^e = \frac{f}{1+j+f} \bar{m}$$

$$\left(\frac{1+j+f}{1+j+F} - \frac{1+j}{1+j+f}\right) p^e = \frac{F}{1+j+f} \bar{m}$$

$$\frac{f}{1+j+f} p^e = \frac{f}{1+j+f} \bar{m}$$

so $$p^e = \bar{m}$$

(3) b) Derive the realized price level p_t in terms of the realized AD shock ε_t.

From

$$p_t = \frac{1+j}{1+j+f} p^e + \frac{f}{1+j+f} \bar{m} + \frac{f}{1+j+f} \varepsilon_t,$$

$$\Rightarrow p_t = \frac{1+j}{1+j+f} \bar{m} + \frac{f}{1+j+f} \bar{m} + \frac{f}{1+j+f} \varepsilon_t$$

$$p_t = \bar{m} + \frac{f}{1+j+f} \varepsilon_t$$

(3) c) Derive realized output y_t in terms of the realized AD shock ε_t.

From

$$y_t = \frac{1}{1+j}(\bar{m} - p_t + \varepsilon_t)$$

$$\Rightarrow y_t = \frac{1}{1+j}\left(\bar{m} - \bar{m} - \frac{f}{1+j+f}\varepsilon_t + \varepsilon_t\right)$$

$$y_t = \frac{1}{1+j}\left(\varepsilon_t - \frac{f}{1+j+f}\varepsilon_t\right)$$

$$y_t = \frac{1}{1+j}\left(\frac{1+j+f}{1+j+f}\varepsilon_t - \frac{f}{1+j+f}\varepsilon_t\right)$$

$$y_t = \frac{1}{1+j}\,\frac{1+j}{1+j+f}\,\varepsilon_t$$

$$y_t = \frac{1}{1+j+f}\,\varepsilon_t$$

(3) d) If the central bank's goal is to minimize variance in output y, is it better to set the money-supply rule parameter $j > 0$, or $j = 0$?

$$\sigma_y^2 = \left(\frac{1}{1+j+f}\right)^2 \sigma_\varepsilon^2$$

so $j > 0$ is better,

gives smaller σ_y^2,