Problem set on IS-LM

Assume

$\frac{M}{P} = L(i, Y)$ $\qquad$ $Y = E(Y, r, G, T)$ where $r = i - \pi^e$

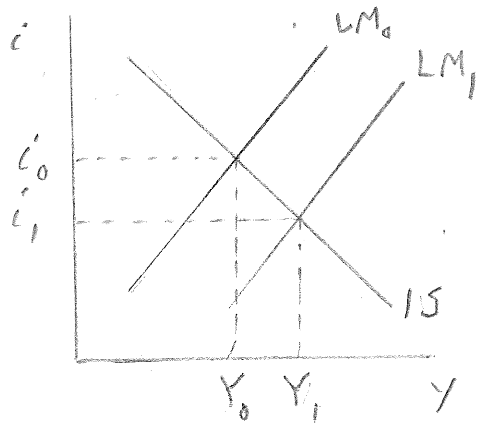
1) Derive expressions for $\frac{\partial i}{\partial M}$ and $\frac{\partial Y}{\partial M}$ in terms of L_Y, L_i, E_r, E_Y assuming π^e and P are held fixed.

2) Derive expressions for $\frac{\partial i}{\partial \pi^e}$ and $\frac{\partial Y}{\partial \pi^e}$ holding M & P fixed

Hint: use method we used to derive slope of AD curve.

ANSWERS TO PROBLEMS on IS/LM

What's $\frac{\partial i}{\partial M}$ & $\frac{\partial Y}{\partial M}$ holding P fixed?



See: $\frac{\Delta i}{\Delta M} < 0 \qquad \frac{\Delta Y}{\Delta M} > 0$

$$\frac{M}{P} = L(i, Y)$$

$$M = P\,L(i, Y)$$

Differentiate w.r.t. M

$$1 = P L_i \frac{\partial i}{\partial M} + P L_Y \frac{\partial Y}{\partial M}$$

$$\frac{\partial Y}{\partial M} = \frac{1}{P L_Y} - \frac{L_i}{L_Y}\frac{\partial i}{\partial M}$$

$$\frac{\partial Y}{\partial M} = \frac{1}{P L_Y} - \frac{L_i}{L_Y}\,\frac{1-E_Y}{E_r}\,\frac{\partial Y}{\partial M}$$

$$\left(1 + \frac{L_i}{L_Y}\,\frac{1-E_Y}{E_r}\right)\frac{\partial Y}{\partial M} = \frac{1}{P L_Y}$$

$$\frac{\partial Y}{\partial M} = \frac{1}{P\left(L_Y + L_i \frac{1-E_Y}{E_r}\right)} > 0$$

Alternatively, $Y = Y(i - \pi^e, G, T)$

$$Y = E(Y, \overbrace{i - \pi^e}^{r}, G, T)$$

Differentiate w.r.t. M

$$\frac{\partial Y}{\partial M} = E_Y \frac{\partial Y}{\partial M} + E_r \frac{\partial i}{\partial M}$$

Recall $E_Y < 1$

$$(1 - E_Y)\frac{\partial Y}{\partial M} = E_r \frac{\partial i}{\partial M}$$

$$\frac{\partial i}{\partial M} = \frac{1-E_Y}{E_r}\,\frac{\partial Y}{\partial M}$$

$$\frac{\partial i}{\partial M} = \frac{1-E_Y}{E_r}\,\frac{1}{P\left(L_Y + L_i \frac{1-E_Y}{E_r}\right)}$$

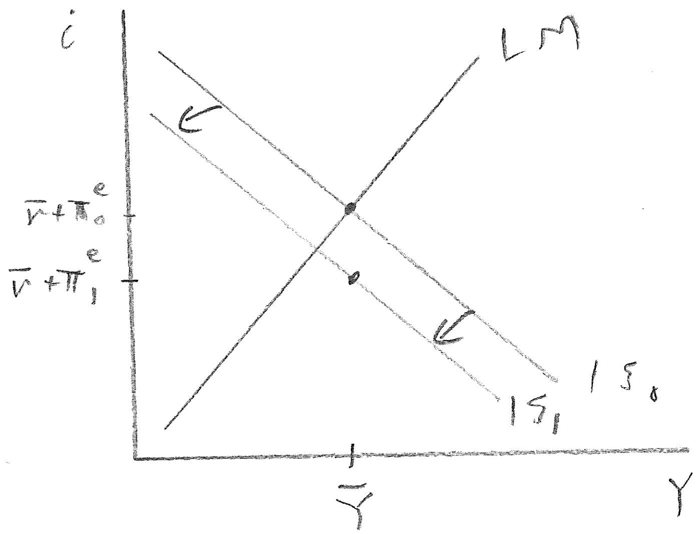
$$= \frac{1-E_Y}{P\left[L_Y E_r + L_i (1 - E_Y)\right]}$$

Divide top & bottom by $(1 - E_Y)$

$$\frac{\partial i}{\partial M} = \frac{1}{P\left(L_Y \frac{E_r}{1-E_Y} + L_i\right)} < 0$$

ANSWERS TO PROBLEMS (cont.)

What's effect of $\partial \pi^e$?



For any i, $r = i - \pi^e$ is bigger, which makes IS shift back.

Look: i falls less than π^e, so $(i-\pi^e)$ rises, and Y falls.

$$Y = E(Y, \overbrace{i-\pi^e}^{r}, G, T)$$

$$\frac{\partial Y}{\partial \pi^e} = E_Y \frac{\partial Y}{\partial \pi^e} + E_r \frac{\partial i}{\partial \pi^e} - E_r$$

$$\frac{M}{P} = L(i, Y)$$

$$0 = L_i \frac{\partial i}{\partial \pi^e} + L_Y \frac{\partial Y}{\partial \pi^e}$$

$$\Rightarrow \frac{\partial Y}{\partial \pi^e} = -\frac{L_i}{L_Y} \frac{\partial i}{\partial \pi^e}$$

gives

$$\frac{\partial i}{\partial \pi^e} = \frac{E_r L_Y}{E_r L_Y + (1-E_Y) L_i}$$

which is positive but less than one, so i falls less than π^e, r rises.

$$\frac{\partial Y}{\partial \pi^e} = E_Y \frac{\partial Y}{\partial \pi^e} + E_r \frac{\partial i}{\partial \pi^e} - E_r$$

$$\Rightarrow \frac{\partial i}{\partial \pi^e} = -\frac{L_Y}{L_i} \frac{\partial Y}{\partial \pi^e}$$

gives

$$\frac{\partial Y}{\partial \pi^e} = \frac{-\overset{(-)}{L_i}\,\overset{(-)}{E_r}}{\underset{(-)}{E_r}\,\underset{(+)}{L_Y} + \underset{(+)}{(1-E_Y)}\,\underset{(-)}{L_i}} > 0$$

so $\pi^e \downarrow \rightarrow Y \downarrow$

ANSWERS TO PROBLEMS

5.3) a) $E = C(Y-T) + I(r) + G$

i) How does $\Delta G = \Delta T$ affect position of IS?

That is, what's $\left(\frac{\partial Y}{\partial G} + \frac{\partial Y}{\partial T}\right)$ assuming $\frac{\partial G}{\partial T} = 1$ and holding i fixed?

$$Y = C(Y-T) + I(r) + G$$

$$\frac{\partial Y}{\partial G} = \underbrace{C_{Y-T}\frac{\partial Y}{\partial G} + 1}_{\partial Y/\partial G} - \underbrace{C_{Y-T}}_{\partial Y/\partial T}\underbrace{\frac{\partial T}{\partial G}}_{\text{equal to one}}$$

Solve for $\partial Y/\partial G$:

$$(1 - C_{Y-T})\frac{\partial Y}{\partial G} = 1 - C_{Y-T}$$

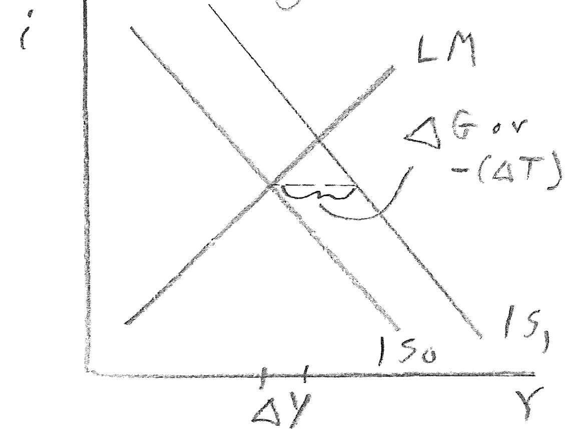
$$\frac{\partial Y}{\partial G} = 1$$

Hence, for $\Delta G = \Delta T$, IS shifts out/in a distance equal to ΔG (or $-\Delta T$) } "Balanced budget multiplier"

ii) How does $\Delta G = \Delta T$ affect AD?

That is, what's $\left(\frac{\partial Y}{\partial G} + \frac{\partial Y}{\partial T}\right)$ assuming $\frac{\partial G}{\partial T} = 1$

and holding M/P fixed, but allowing i to vary?



Look!

$\Delta Y < \Delta G$

5.3 a) (cont.)

$$\frac{\partial Y}{\partial G} = C_{Y-T}\frac{\partial Y}{\partial G} + 1 - C_{Y-T} + I_r \frac{\partial i}{\partial G}$$

new

$$\frac{M}{P} = L(i, Y)$$

$$0 = L_i \frac{\partial i}{\partial G} + L_Y \frac{\partial Y}{\partial G}$$

$$\frac{\partial i}{\partial G} = -\frac{L_Y}{L_i}\frac{\partial Y}{\partial G}$$

$$\frac{\partial Y}{\partial G} = C_{Y-T}\frac{\partial Y}{\partial G} + 1 - C_{Y-T} - I_r \frac{L_Y}{L_i}\frac{\partial Y}{\partial G}$$

$$\left(1 - C_{Y-T} + I_r \frac{L_Y}{L_i}\right)\frac{\partial Y}{\partial G} = 1 - C_{Y-T}$$

$$\frac{\partial Y}{\partial G} = \frac{1 - C_{Y-T}}{1 - C_{Y-T} + \underset{(-)}{I_r}\frac{L_Y}{\underset{(-)}{L_i}}} > 0$$

but less than one. "Crowding-out"

b) $T(Y)$ where $T_Y > 0$

i) What's $\frac{\partial i}{\partial Y}$ a slope of IS

$$Y = E(Y, r, G, T(Y))$$

$$\frac{\partial Y}{\partial i} = E_Y \frac{\partial Y}{\partial i} + E_r + E_T T_Y \frac{\partial Y}{\partial i}$$

$$\Rightarrow \frac{\partial i}{\partial Y} = \frac{1 - E_Y - \overset{(-)}{E_T}\overset{(+)}{T_Y}}{\underset{(-)}{E_r}}$$

so adding $\overbrace{(-E_T T_Y)}^{(+)}$ makes top of fraction larger, $\frac{\partial i}{\partial Y}$ greater, IS steeper

5.3) b) ii) How does T_Y affect $\frac{\partial Y}{\partial G}$ and $\frac{\partial Y}{\partial M}$ holding fixed P?

First, $\frac{\partial Y}{\partial G}$:

$$Y = E(Y, r, G, T(Y))$$

$$\frac{\partial Y}{\partial G} = E_Y \frac{\partial Y}{\partial G} + E_r \frac{\partial i}{\partial G} + E_G + E_T T_Y \frac{\partial Y}{\partial G}$$

$$\frac{M}{P} = L(i, Y)$$

$$0 = L_i \frac{\partial i}{\partial G} + L_Y \frac{\partial Y}{\partial G}$$

$$\frac{\partial i}{\partial G} = -\frac{L_Y}{L_i} \frac{\partial Y}{\partial G}$$

$$\frac{\partial Y}{\partial G} = E_Y \frac{\partial Y}{\partial G} - \frac{E_r L_Y}{L_i} \frac{\partial Y}{\partial G} + E_G + E_T T_Y \frac{\partial Y}{\partial G}$$

$$\left(1 - E_Y + \frac{E_r L_Y}{L_i} - E_T T_Y\right) \frac{\partial Y}{\partial G} = E_G$$

$$\frac{\partial Y}{\partial G} = \frac{E_G}{1 - E_Y + \frac{E_r L_Y}{L_i} - \underset{(+)}{E_T} \underset{(-)}{T_Y}} > 0$$

so introducing $\underbrace{(-E_T T_Y)}_{(+)}$ makes bottom of fraction bigger, reduces $\frac{\partial Y}{\partial G}$.

ANSWERS TO PROBLEMS (CONT.)

5. 3) b) ii) (cont.)

Now, $\frac{\partial Y}{\partial M}$:

$$Y = E(Y, r, G, T(Y))$$

$$\frac{\partial Y}{\partial M} = E_Y \frac{\partial Y}{\partial M} + E_r \frac{\partial i}{\partial M} + E_T T_Y \frac{\partial Y}{\partial M}$$

$$\frac{M}{P} = L(i, Y)$$

$$\frac{1}{P} = L_i \frac{\partial i}{\partial M} + L_Y \frac{\partial Y}{\partial M}$$

$$\frac{\partial i}{\partial M} = \frac{1}{PL_i} - \frac{L_Y}{L_i} \frac{\partial Y}{\partial M}$$

$$\frac{\partial Y}{\partial M} = E_Y \frac{\partial Y}{\partial M} + \frac{E_r}{PL_i} - \frac{E_r L_Y}{L_i} \frac{\partial Y}{\partial M} + E_T T_Y \frac{\partial Y}{\partial M}$$

$$\left(1 - E_Y + \frac{E_r L_Y}{L_i} - E_T T_Y\right) \frac{\partial Y}{\partial M} = \frac{E_r}{PL_i}$$

$$\frac{\partial Y}{\partial M} = \frac{E_r}{PL_i\left(1 - E_Y + \frac{E_r L_Y}{L_i} - \underset{(-)}{E_T}\,\underset{(+)}{T_Y}\right)} > 0$$

so introducing $\underbrace{(-E_T T_Y)}_{(+)}$ makes bottom of fraction bigger,

reduces $\frac{\partial Y}{\partial M}$

ANSWERS TO PROBLEMS (cont.)

5.4) a) "Liquidity trap"

Suppose $i \approx 0$, $L_i \approx \infty$ $\leftarrow \dfrac{\partial\left(\frac{M^D}{P}\right)}{\partial i}$

a) What is slope of AD curve?

Recall slope of AD curve:

$P\downarrow \rightarrow \frac{M}{P}\uparrow \rightarrow i\downarrow \rightarrow Y\uparrow$

$\nwarrow$ but here, huge change in $\frac{M}{P}$ has infinitesimal effect on i

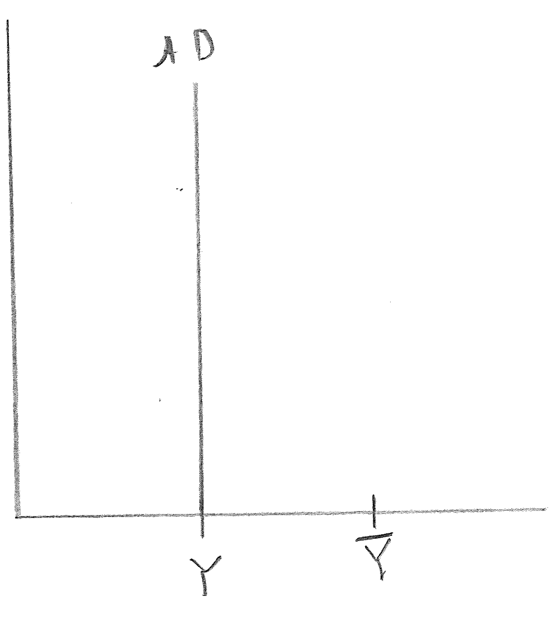
Hence AD is again vertical: decrease in P does not raise Y

Expression for AD we derived earlier:

$$\frac{\partial Y}{\partial P} = \frac{-\frac{M}{P^2}}{L_Y + \frac{L_i}{Y_r}}$$

As $L_i \rightarrow \infty$, $\frac{\partial Y}{\partial P} \rightarrow 0$

If prices are flexible (fall whenever $Y < \bar{Y}$), does AD matter for output?



Prices just keep falling, output does not rise to $\bar{Y}$.

Effect of π^e makes things worse — shifts AD back!

But $G\uparrow$ or $T\downarrow$ might work!

ANSWERS TO PROBLEMS (cont.)

5.4) b) "Pigou effect"

Real wealth affects expenditure,
$\frac{M}{P}$ is part of real wealth.

Is AD irrelevant to output?

New IS: $Y(r, M, T, \overset{+}{\frac{M}{P}})$ $\quad Y_{\frac{M}{P}} > 0$

Derive new AD curve: what is $\frac{\partial Y}{\partial P}$ holding M fixed?

$$Y = Y(r, G, T, \tfrac{M}{P})$$

$$\frac{\partial Y}{\partial P} = Y_r \frac{\partial i}{\partial P} - Y_{\frac{M}{P}} \frac{M}{P^2}$$

$$\frac{M}{P} = L(i, Y)$$

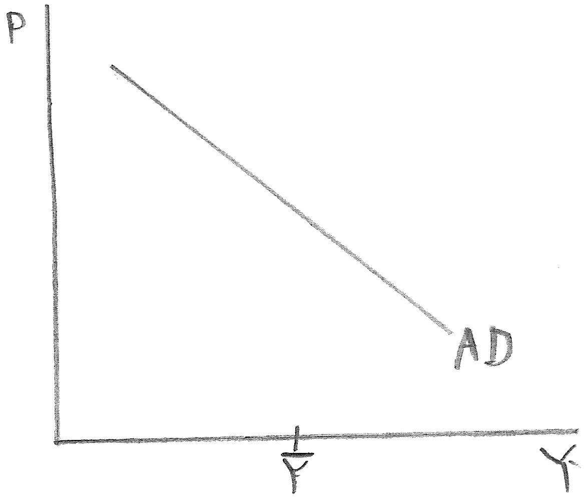
$$-\frac{M}{P^2} = L_i \frac{\partial i}{\partial P} + L_Y \frac{\partial Y}{\partial P}$$

$$\frac{\partial i}{\partial P} = -\frac{1}{L_i}\frac{M}{P^2} - \frac{L_Y}{L_i}\frac{\partial Y}{\partial P}$$

$$\frac{\partial Y}{\partial P} = \frac{-\frac{M}{P^2}}{L_Y + \frac{L_i}{Y_r}} - Y_{\frac{M}{P}} \frac{M}{P^2}$$

Note: even as $L_i \to \infty$, $\frac{\partial Y}{\partial P} < 0$

Hence AD slopes down



If $P\downarrow$ as long as $Y < \bar{Y}$,
then there's a $\bar{P}$ for which
$Y = \bar{Y}$

But what about effect of π^e?