RATIONAL EXPECTATIONS PROBLEMS

ANSWERS

1) AD: $y_t = a(m_t - p_t)$

AS: $\pi_t = {}_{t-1}\pi^e_t + f y_t$

means $p_t - p_{t-1} = {}_{t-1}p^e_t - p_{t-1} + f y_t$

$$p_t = {}_{t-1}p^e_t + f y_t$$

and $m_t = m_{t-1} + \varepsilon_t$ random variable

What is R.E.E.?

a) Derive ${}_{t-1}p^e_t$ in terms of m_{t-1} this is ${}_{t-1}m^e_t$

Reverse AS to get

$$y_t = \frac{1}{f} p_t - \frac{1}{f}\, {}_{t-1}p^e_t$$

using AD,

$$a(m_t - p_t) = \frac{1}{f} p_t - \frac{1}{f}\, {}_{t-1}p^e_t$$

realized p_t expected p_t

$$m_t - p_t = \frac{1}{af} p_t - \frac{1}{af}\, {}_{t-1}p^e_t$$

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ANSWERS

1) a) cont.

$$p_t + \frac{1}{af} p_t = \frac{1}{af} p_t^e + m_t$$

$$\left(1 + \frac{1}{af}\right) p_t =$$

$$\frac{af+1}{af} p_t =$$

$$p_t = \frac{1}{af+1} p^e + \frac{af}{af+1} m_t$$

Apply rational expectations

$$p^e = E[p_t] = E\left[\frac{1}{af+1} p^e + \frac{af}{af+1} m_t\right]$$

$$= \frac{1}{af+1} p^e + \frac{af}{af+1} E[m_t]$$ This is m_{t-1}

$$= \text{\_ \_ \_ \_ \_ \_ \_ \_} \; E[m_{t-1} + \varepsilon_t]$$

$$= \text{\_ \_ \_ \_ \_ \_ \_ \_ \_ \_} \; m_{t-1} + E[\varepsilon_t]$$ zero

$$p^e = \frac{1}{af+1} p^e + \frac{af}{af+1} m_{t-1}$$

1) a)

$$p^e - \frac{1}{af+1} p^e = \frac{af}{af+1} m_{t-1}$$

$$\left(1 - \frac{1}{af+1}\right) p^e = \frac{af}{af+1} m_{t-1}$$

$$\left(\frac{af-1}{af+1} - \frac{1}{af+1}\right) p^e = \frac{af}{af+1} m_{t-1}$$

$$\frac{af}{af+1} p^e = \frac{af}{af+1} m_{t-1}$$

$$_{t-1}p_t^e = m_{t-1} = E[m_t] = {}_{t-1}m_t^e$$

b) What is p_t interms of m_{t-1} and ε_t?

From above,

$$p_t = \frac{1}{af+1} p^e + \frac{af}{af+1} m_t$$

$$p_t = \frac{1}{af+1} m_{t-1} + \frac{af}{af+1} m_t$$

We know $m_t = m_{t-1} + \varepsilon_t$ so...

1) b) (cont.)

$$p_t = \frac{1}{af+1}\, m_{t-1} + \frac{af}{af+1}\left(m_{t-1} + \varepsilon_t\right)$$

$$p_t = m_{t-1} + \frac{af}{af+1}\, \varepsilon_t$$

$${}_{t-1}m^{e}_{t}$$

c) π_t?

$$\pi_t = p_t - p_{t-1} = m_{t-1} - p_{t-1} + \frac{af}{af+1}\, \varepsilon_t$$

d) y_t in terms of ε_t

One way:

$$y_t = a(m_t - p_t) = a\left(m_t - m_{t-1} - \frac{af}{af+1}\, \varepsilon_t\right)$$

$$y_t = a\left(m_{t-1} + \varepsilon_t - m_{t-1} - \frac{af}{af+1}\, \varepsilon_t\right)$$

$$= a\left(\varepsilon_t - \frac{af}{af+1}\, \varepsilon_t\right) = a\left(1 - \frac{af}{af+1}\right) \varepsilon_t$$

1) d) cont.

$$y_t = a\left(\frac{af+1}{af+1} - \frac{af}{af+1}\right)\varepsilon_t$$

$$= \frac{a}{af+1}\varepsilon_t$$

e) Variance of y?

$$\sigma_y^2 = \left(\frac{a}{af+1}\right)^2 \sigma_\varepsilon^2$$

2) AD: $y_t = a(m_t - p_t) + \varepsilon_t$ ← random variable

AS: $\pi_t = {}_{t-1}\pi_t^e + f y_t$

$$\Rightarrow p_t = {}_{t-1}p_t^e + f y_t$$

and $m_t = \bar{m}$ for all t

a) Derive p^e in terms of $\bar{m}$.

From AS,

$$\frac{1}{f} p_t - \frac{1}{f} p^e = y = a(\bar{m} - p_t) + \varepsilon_t$$

$$\bar{m} - p_t = \frac{1}{a}\left(\frac{1}{f} p_t - \frac{1}{f} p^e - \varepsilon_t\right)$$

$$\bar{m} - p_t = \frac{1}{af} p_t - \frac{1}{af} p^e - \frac{1}{a} \varepsilon_t$$

$$\left(1 + \frac{1}{af}\right) p_t = \bar{m} + \frac{1}{af} p^e + \frac{1}{a} \varepsilon_t$$

$$\frac{af+1}{af} p_t = \dots$$

$$p_t = \frac{af}{af+1} \bar{m} + \frac{1}{af+1} p^e + \frac{f}{af+1} \varepsilon_t$$

p_t ← realized p^e ← expected

2) a) cont.

Apply R.E.E.

$$p^e = E[p_t] = E\left[\frac{af}{af+1}\bar{m} + \frac{1}{af+1}p^e + \frac{f}{af+1}\varepsilon_t\right]$$

$$p^e = \frac{af}{af+1}\bar{m} + \frac{1}{af+1}p^e + \frac{f}{af+1}\underbrace{E[\varepsilon_t]}_{\text{zero}}$$

$$\left(1 - \frac{1}{af+1}\right)p^e = \frac{af}{af+1}\bar{m}$$

$$\left(\frac{af+1}{af+1} - \frac{1}{af+1}\right)p^e = \frac{af}{af+1}\bar{m}$$

$$\frac{af}{af+1}p^e = \frac{af}{af+1}\bar{m}$$

b) What's p_t in terms of $\bar{m}$ and ε_t?

From above,

$$p_t = \frac{af}{af+1}\bar{m} + \frac{1}{af+1}\bar{m} + \frac{f}{af+1}\varepsilon_t$$

$$p_t = \left(\frac{1+af}{af+1}\right)\bar{m} + \frac{f}{af+1}\varepsilon_t$$

$$p_t = \bar{m} + \frac{f}{af+1}\varepsilon_t$$

2) c) Derive π_t in terms of $\bar{m}$, p_{t-1} and ε_t

$$\pi_t = p_t - p_{t-1} = \bar{m} - p_{t-1} + \frac{f}{af+1}\varepsilon_t$$

d) What's y_t in terms of ε_t?

$$y_t = a(m_t - p_t) + \varepsilon_t = a\left(\bar{m} - \bar{m} - \frac{f}{af+1}\varepsilon_t\right) + \varepsilon_t$$

$$= -\frac{af}{af+1}\varepsilon_t + \varepsilon_t$$

$$= \left(1 - \frac{af}{af+1}\right)\varepsilon_t$$

$$= \left(\frac{af+1}{af+1} - \frac{af}{af+1}\right)\varepsilon_t$$

$$= \frac{1}{af+1}\varepsilon_t$$

e) σ_y^2?

$$\sigma_y^2 = \left(\frac{1}{af+1}\right)^2 \sigma_y^2$$

3) IS: $y_t = -a r_t$

$$\pi_t = {}_{t-1}\pi_t^e + f y_t$$

No random variables

$r(\pi, y)$: $r_t = b y_t + c(\pi_t - \bar{\pi})$

From IS and i.r.r.,

$$y_t = -a(b y_t + c(\pi - \bar{\pi}))$$

$$= -aby - ac\pi + ac\bar{\pi}$$

$$(1+ab)y = -ac\pi + ac\bar{\pi}$$

$$y = -\frac{ac}{1+ab}\pi + \frac{ac}{1+ab}\bar{\pi}$$

put into AS

$$\pi = \pi^e + f\left(-\frac{ac}{1+ab}\pi + \frac{ac}{1+ab}\right)\bar{\pi}$$

$$\pi = \pi^e - \frac{fac}{1+ab}\pi + \frac{fac}{1+ab}\bar{\pi}$$

$$\left(1 + \frac{fac}{1+ab}\right)\pi = \pi^e + \frac{fac}{1+ab}\bar{\pi}$$

3) cont.

$$\frac{1+ab+fac}{1+ab}\pi = \pi^e + \frac{fac}{1+ab}\overline{\pi}$$

$$\pi = \frac{1+ab}{1+ab+fac}\pi^e + \frac{fac}{1+ab+fac}\overline{\pi}$$

Apply R.E.E.

$$\pi^e = E[\pi] = \frac{1+ab}{1+ab+fac}\pi^e + \frac{fac}{1+ab+fac}\overline{\pi}$$

and $E[\pi] = \pi$ because no uncertainty

$$\pi^e - \frac{1+ab}{1+ab+fac}\pi^e = \frac{fac}{1+ab+fac}\overline{\pi}$$

$$\left(\frac{1+ab+fac}{1+ab+fac} - \frac{1+ab}{1+ab+fac}\right)\pi^e = \frac{fac}{1+ab+fac}\overline{\pi}$$

$$\frac{fac}{1+ab+fac}\pi^e = \frac{fac}{1+ab+fac}\overline{\pi}$$

$$\pi^e = \overline{\pi}$$

$$\pi = \overline{\pi}$$

and $y = 0$

4) IS $y_t = -a r_t + \varepsilon_t$ ← random variable

AS $\pi_t = {}_{t-1}\pi_t^e + f y_t$

$r(\pi, y)$ $r_t = b y_t + c(\pi_t - \bar{\pi})$

a) Solve for ${}_{t-1}\pi_t^e$

From IS & i.r.r,

$$y_t = -a(b y_t + c(\pi - \bar{\pi})) + \varepsilon_t$$

$$= -\frac{ac}{1+ab}\pi + \frac{ac}{1+ab}\bar{\pi} + \frac{1}{1+ab}\varepsilon_t$$

Put into AS:

$$\pi = \pi^e + f(\qquad\qquad)$$

$$\pi = \pi^e - \frac{fac}{1+ab}\pi + \frac{fac}{1+ab}\bar{\pi} + \frac{f}{1+ab}\varepsilon_t$$

$$\left(1 + \frac{fac}{1+ab}\right)\pi = \text{---}$$

$$\frac{1+ab+fac}{1+ab}\pi =$$

4) a) cont.

$$\pi = \frac{1+ab}{1+ab+fac}\pi^e + \frac{fac}{1+ab+fac}\bar{\pi} + \frac{f}{1+ab+fac}\varepsilon_t$$

Apply R.E.E.

$$\pi^e = E[\pi] = \frac{1+ab}{1+ab+fac}\pi^e + \frac{fac}{1+ab+fac}\bar{\pi} + \frac{f}{1+ab+fac}\underbrace{E[\varepsilon_t]}_{\text{zero}}$$

From here, same as before, so

$$\pi^e = \bar{\pi}$$

but now π_t may not equal π^e.

b) π_t in terms of ε_t and $\bar{\pi}$

From $\pi^e = \bar{\pi}$ and here

$$\pi = \frac{1+ab}{1+ab+fac}\bar{\pi} + \frac{fac}{1+ab+fac}\bar{\pi} + \frac{f}{1+ab+fac}\varepsilon_t$$

$$\pi = \bar{\pi} + \frac{f}{1+ab+fac}\varepsilon_t$$

Note!

$$E[\pi] = \bar{\pi} + \frac{f}{1+ab+fac}E[\varepsilon_t] = \bar{\pi}$$

$$\pi - \bar{\pi} = \frac{f}{1+ab+fac}\varepsilon_t$$

4) c) What's y_t?

From above,

$$y_t = -\frac{ac}{1+ab}(\pi - \bar{\pi}) + \frac{1}{1+ab}\varepsilon_t$$

$$= -\frac{ac}{1+ab}\left(\frac{f}{1+ab+fac}\varepsilon_t\right) + \frac{1}{1+ab}\varepsilon_t$$

$$y_t = -\frac{1}{1+ab}\left(\frac{fac}{1+ab+fac}\right)\varepsilon_t + \frac{1}{1+ab}\varepsilon_t$$

$$y_t = \frac{1}{1+ab}\left(1 - \frac{fac}{1+ab+fac}\right)\varepsilon_t$$

$$= \frac{1}{1+ab}\left(\frac{1+ab}{1+ab+fac}\right)\varepsilon_t$$

$$= \frac{1}{1+ab+fac}\varepsilon_t$$

Note $E[y_t] = 0$

d) σ_y^2?

$$\left(\frac{1}{1+ab+fac}\right)^2 \sigma_\varepsilon^2 = \sigma_y^2$$

Note on 3) and 4)

See that if central bank follows rule

$$r_t = b y_t + c(\pi_t - \bar{\pi})$$

then π will be centered on $\bar{\pi}$.

This might be a good policy if central bank wants $\pi = \bar{\pi}$ ← Inflation target

Rule can also be written

$$r_t = -c\bar{\pi} + b y_t + c\pi_t$$

For <u>nominal</u> interest rate,

$$i_t = r_t + \pi^e = -c\bar{\pi} + b y_t + c\pi_t + \pi^e$$

If $\pi^e \approx \pi_t$, then

$$i_t = -c\bar{\pi} + b y_t + (1+c)\pi_t$$