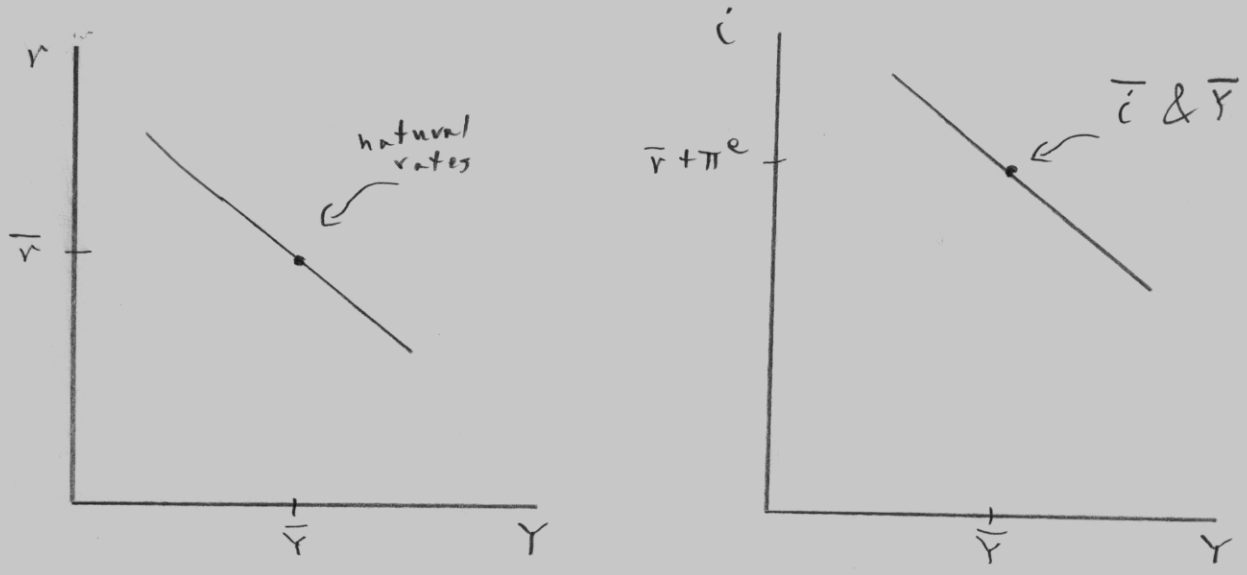
IS Curve

If $r < \bar{r}$, $Y > \bar{Y}$ if $r > \bar{r}$, $Y < \bar{Y}$



Why does IS curve slope down?

C Consumption Real output produced & purchased for use by people ("households") to make themselves happy

I Investment Real output in the form of machines, buildings, inventories of materials & outputs for use in future production.

r What it means for

Households: if I consume X less units of output today & increase my savings (purchase K or lend) I'll receive $X \cdot (1+r)$ more units of output tomorrow.

Firms: if I borrow Z more units of output today & install it as capital (or trade it to get capital), I must pay someone $Z \cdot (1+r)$ more units tomorrow.

G Government purchases of output

IS Curve

r & C

If $r_t > \bar{r}$, this is an extraordinarily good time to save:
I get an extraordinarily high return from postponing consumption.
I'll consume <u>less</u> now because that allows me to consume <u>more</u> in future.

If $r_t < \bar{r}$, this is an extraordinarily good time to party:
I give up less future consumption than I usually would.

r & I

Recall profit-maximizing firms set

$$r = MPK = F_K(K, L)$$

and $\dot{K}_t = I_t - \delta K$

If $r_t < r_{t-1}$, I want to install <u>more</u> capital,
I is bigger relative to δK

If $r_t > r_{t-1}$, I want to get rid of capital: let I fall relative to δK so that K falls.

Hence I is smaller (bigger) if r is smaller (bigger) relative to r_{t-1}.

$Y = I + C + G$

IS curve

r & Y

If $\bar{r}$ is long-run trend value of r,

then if $r_t < \bar{r}$, $r_t <$ expected future r
$<$ past r

so $C_t > \bar{C}$, $I_t > \bar{I}$, $Y > \bar{Y}$

What shifts IS curve?

1) Anything that changes $\bar{r}$, $\bar{Y}$ (r & Y from growth model)
E.g. in OLG model, changes in n, δ, g, ρ ....

2) Factors not in growth model that change $\bar{r}$, $\bar{Y}$

G Government purchases of output
$G\uparrow \rightarrow Y\uparrow$

T Taxes
One view: $T\uparrow \rightarrow Y\downarrow$ ("after-tax income")
Another view: "Ricardian equivalence"

Holding G fixed, a tax cut today means
tax hikes tomorrow with interest
We'll save all of our tax cut, earn
enough extra interest to pay future taxes.

3) π^e
for graph drawn in i | ___ space
Y

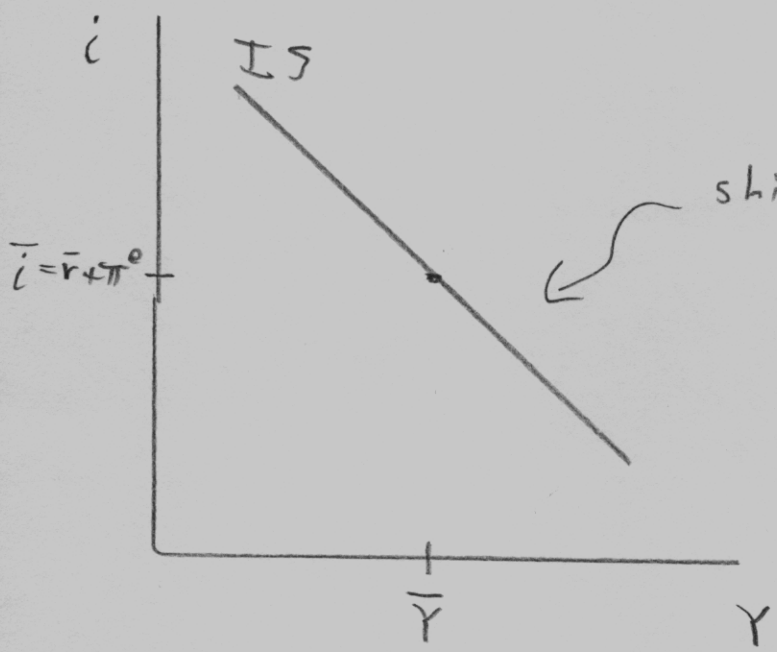
IS Curve

IS curve in general Form

$$E(Y, \bar{r}, \overset{+}{G}, \overset{-(?)}{T}, \ldots) = Y$$

Key assumption: if households & businesses want to buy more Y, more Y will be produced.

This is an assumption about "aggregate supply." We'll get back to it later



shifted by π^e, G, T, & anything else that affects $\bar{r}$ or $\bar{Y}$

IS Curve

Keynesian Cross

A specific version of IS curve, not needed for most purposes.

E Expenditure or "planned expenditure"

must always equal Y (so I've ignored E so far)

$$E = C + I + G = E(Y, r, G, T)$$

($+$ on Y; $(-)$ on r; $+$ $(-)$ on G, T)

r: $i - \pi^e$

where $0 < E_Y < 1$

Stories for this:

1) Current income affects C

$\frac{\partial C}{\partial Y} > 0$ ← "Marginal propensity to consume"

2) Current sales affect I "Accelerator"

- current sales affect expectations of future product demand
- some firms are liquidity-constrained, must finance investment out of earnings

$$Y = E(Y, r, G, T)$$

Y: affects relation between Y and r, G, T

Example: what is $\partial Y / \partial i$? — inverse of slope of IS curve

From above,

$$\frac{\partial Y}{\partial i} = E_Y \frac{\partial Y}{\partial i} + E_r$$

Solve for $\frac{\partial Y}{\partial i}$:

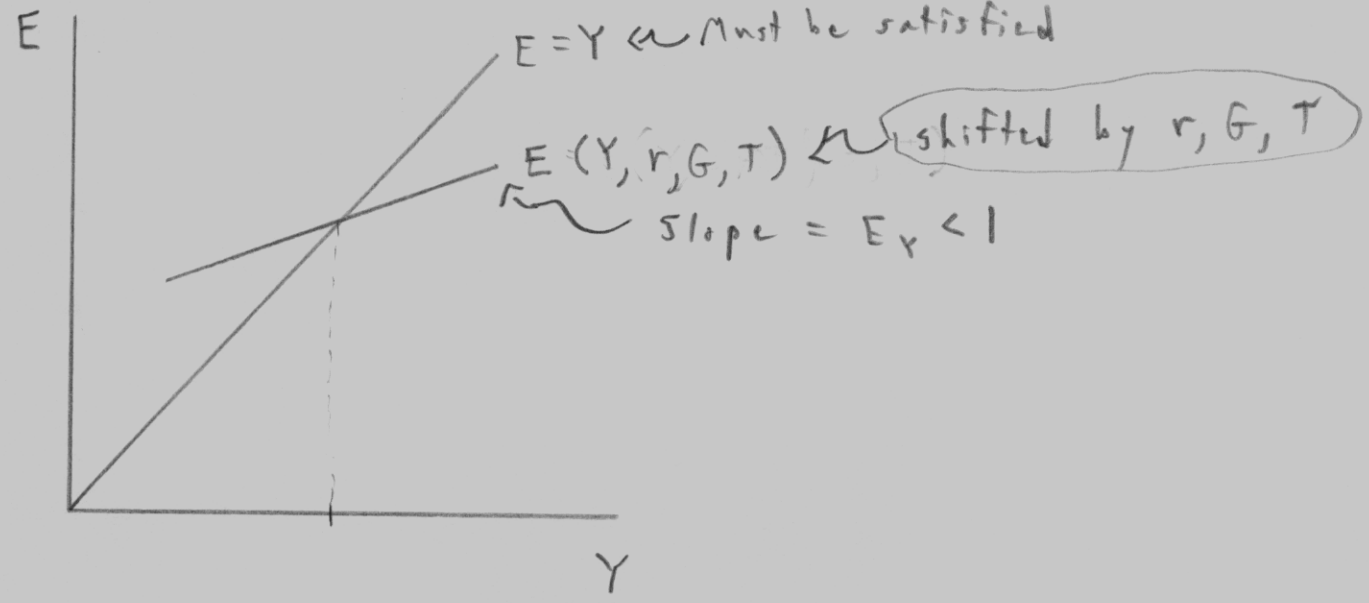
$$\frac{\partial Y}{\partial i} = \frac{E_r}{1 - E_Y} < 0$$

Note: $\frac{1}{1 - E_Y}$ is "the spending multiplier"

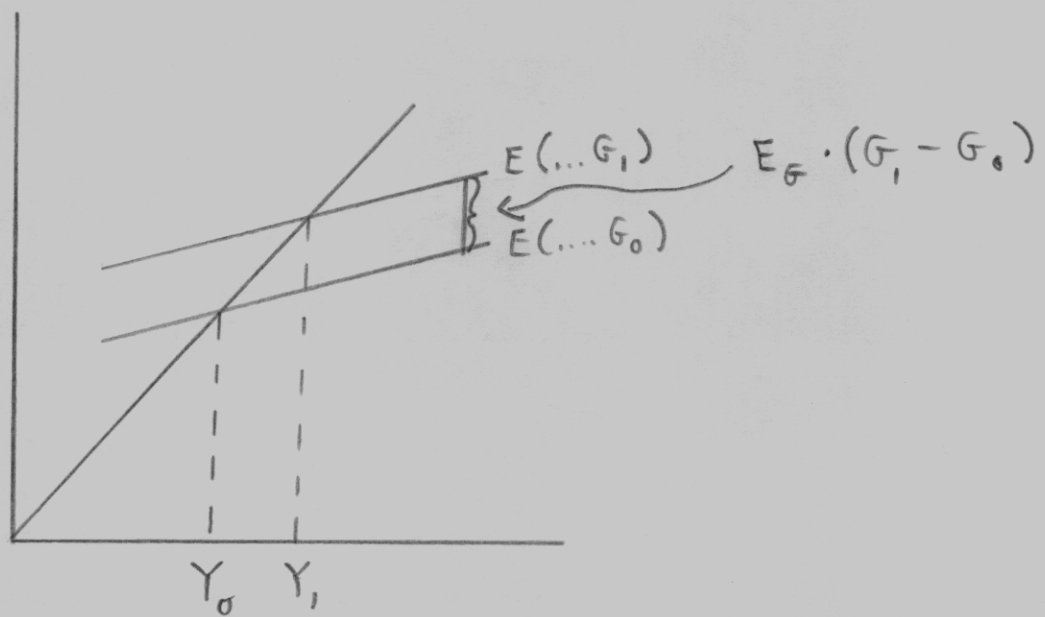
IS-Curve

Keynesian Cross (cont.)

Graph



A change in G $\quad G_1 > G_0$



$$Y_1 - Y_0 = E_G(G_1 - G_0)\underbrace{\frac{1}{1-E_Y}}_{\text{spending multiplier}} > E_G(G_1 - G_0)$$

IS curve (cont.)

Equation of IS curve

$$Y = E(Y, r, G, T, \ldots)$$

Solve out for Y to get expression corresponding to IS curve

$$Y(r, G, T, \ldots)$$

Inverse of this is function plotted as IS curve

$$r(Y, \ldots)$$

(Y) horizontal axis

(r) vertical axis

Note From $Y = E(Y, r, G, T \ldots)$

$$\frac{\partial Y}{\partial r} = E_Y \frac{\partial Y}{\partial r} + E_r \Rightarrow \partial Y/\partial r = \frac{E_r}{(1 - E_y)}$$

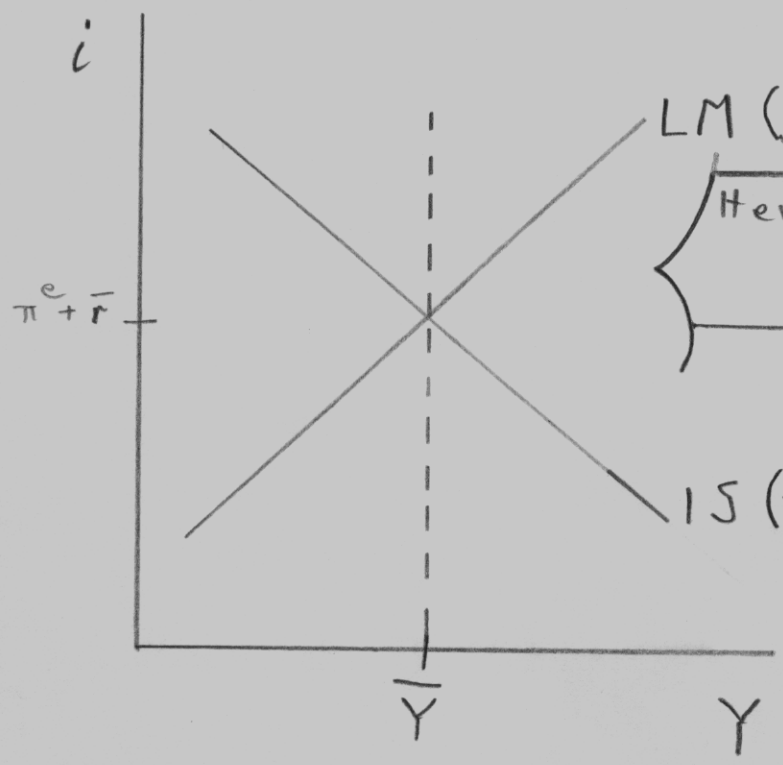
slope of IS $\rightsquigarrow \frac{\partial r}{\partial Y} = \frac{(1 - E_y)}{E_r}$

From $Y(r, G, T \ldots)$

$$\frac{\partial Y}{\partial r} = Y_r$$

IS-LM

IS-LM Graph

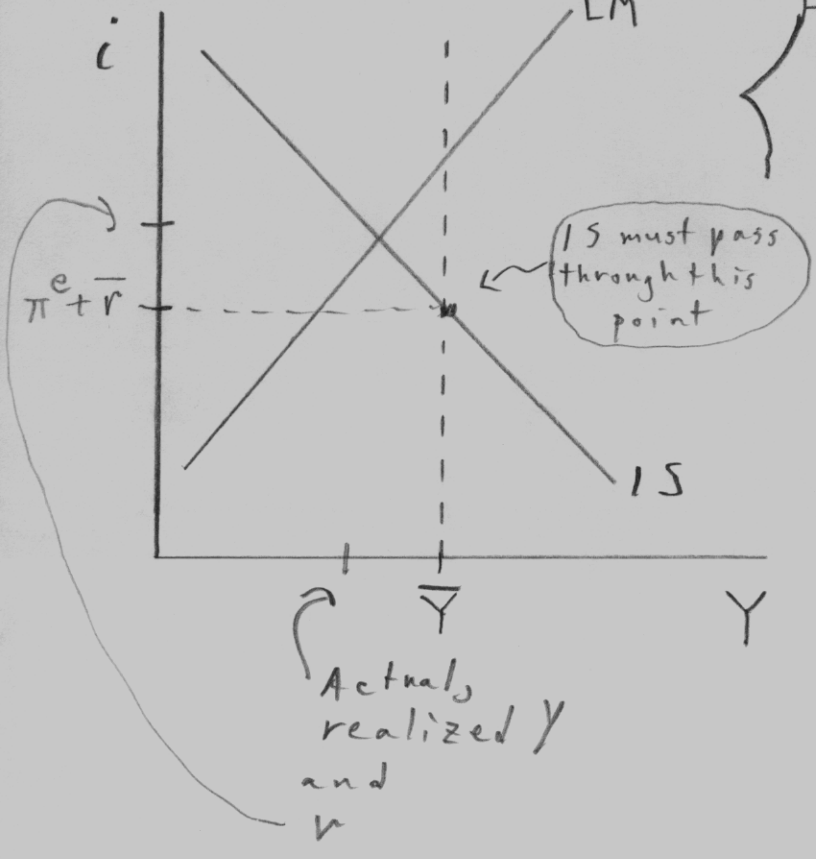


shifted by...

$LM\left(\overset{+}{\frac{M}{P}}\right)$ Story about "money supply & demand"

Here, M P π^e G & T are such that $Y = \bar{Y}$

$IS(\overset{+}{\pi^e}, \overset{+}{G}, \overset{-}{T}, \text{etc.})$ Story about real interest rate & spending, including $\bar{r}$



Here, M P π^e G & T are such that $Y < \bar{Y}$

Mechanically, Y can be increased to equal $\bar{Y}$ by any of the following:

$\downarrow P$
$\uparrow M$ } equivalent

$\pi^e \uparrow$ shifts IS up

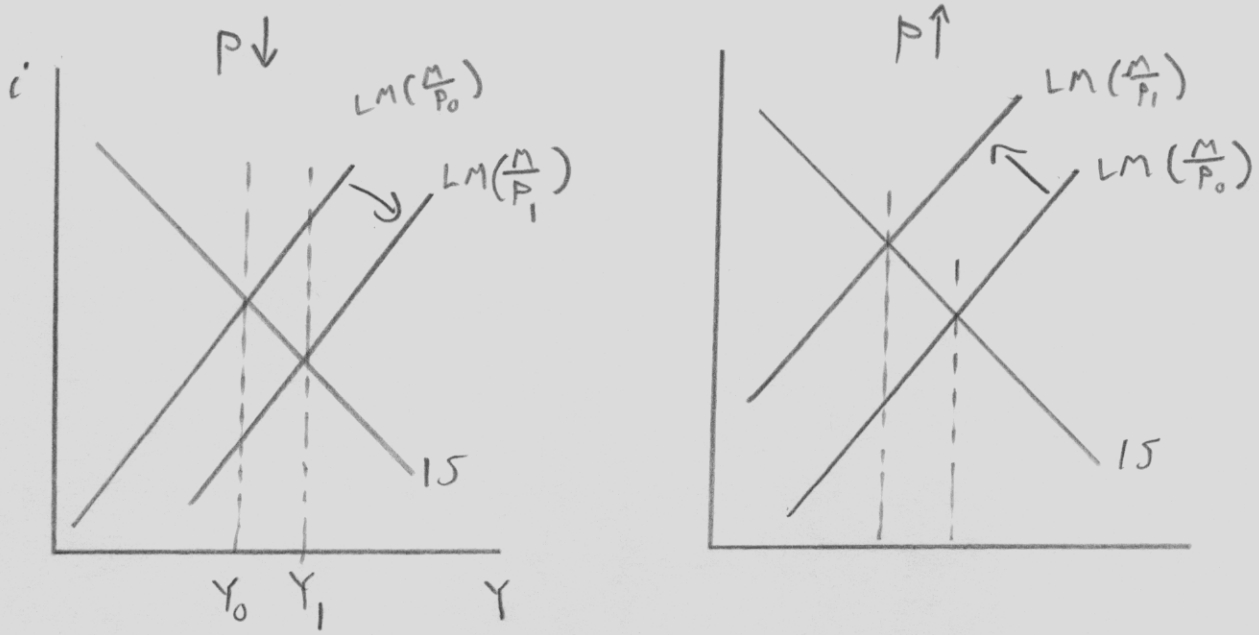
IS-LM (closed economy)

Aggregate Demand Curve

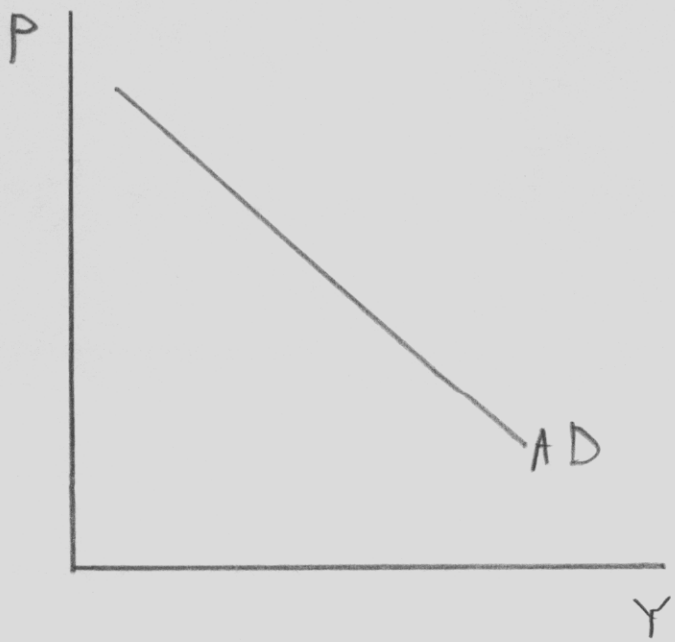
Hold M Fixed, vary P: what must happen to Y?

For Fixed M, $P\downarrow \rightarrow \frac{M}{P}\uparrow$

LM shifts out $\rightarrow Y\uparrow$



Aggregate Demand Curve



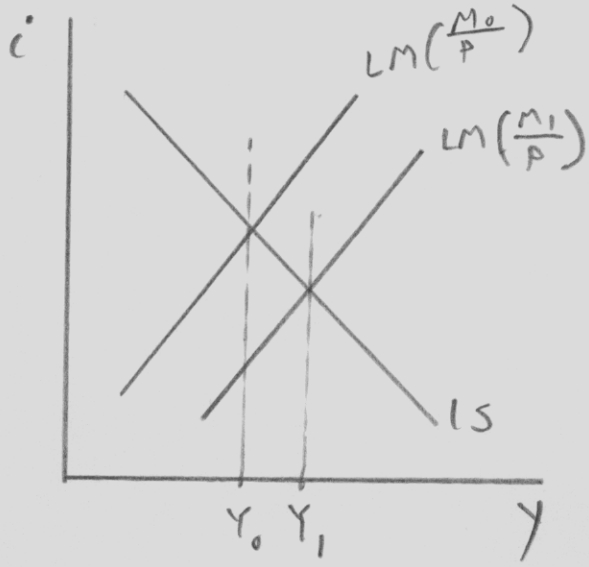
Given M, G, T, & π^e;
plot out P vs. Y

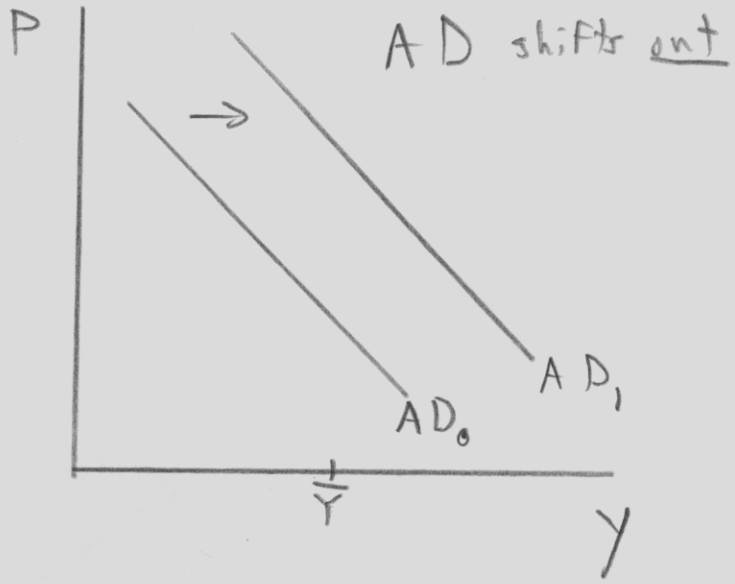
IS-LM (closed economy)

Aggregate Demand Curve (cont.)

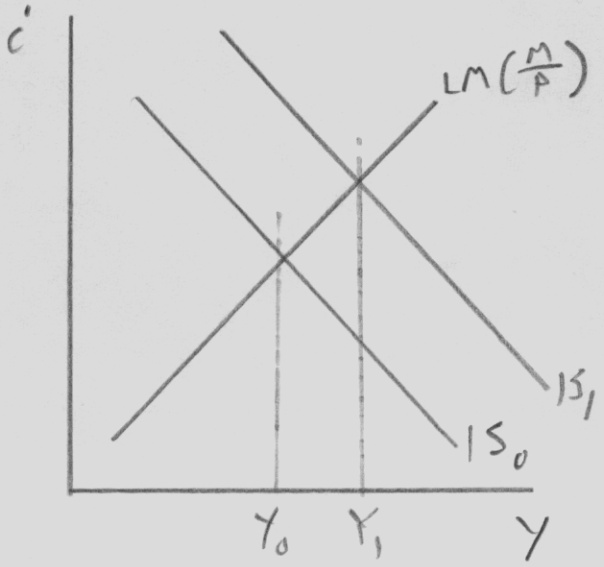
Changes in M, G, T, π^e shift AD curve

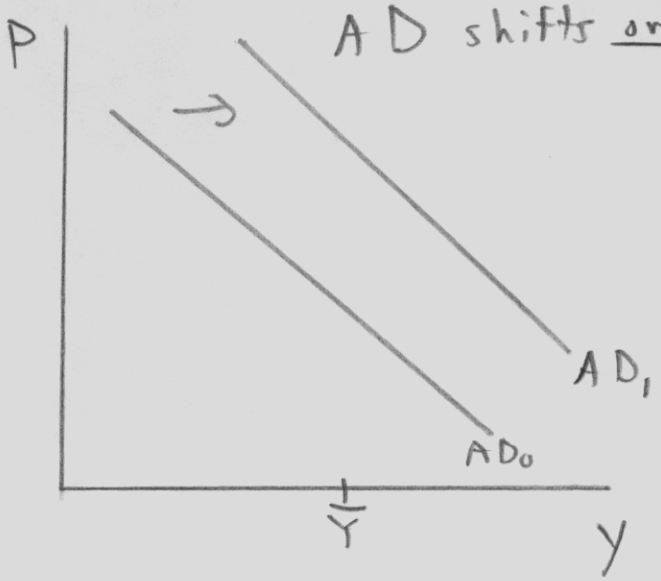
$M\uparrow$: For any P, Y↑





$G\uparrow, T\downarrow$: For any P, Y↑





NOTE: As short-run P ... [illegible]

15 - LM (closed economy)

Aggregate Demand Curve (cont.)

Derive slope of AD curve using $Y(r, G, T)$

What is $\frac{\partial Y}{\partial P}$ holding M (& other stuff) fixed?

$$Y = Y(r, G, T)$$

$$\frac{\partial Y}{\partial P} = Y_r \frac{\partial i}{\partial P}$$

$$\frac{M}{P} = L(i, Y)$$

$$-\frac{M}{P^2} = L_i \frac{\partial i}{\partial P} + L_Y \frac{\partial Y}{\partial P}$$

$$\frac{\partial i}{\partial P} = -\frac{1}{L_i}\frac{M}{P^2} - \frac{L_Y}{L_i}\frac{\partial Y}{\partial P}$$

$$\frac{\partial Y}{\partial P} = Y_r\left(-\frac{1}{L_i}\frac{M}{P^2} - \frac{L_Y}{L_i}\frac{\partial Y}{\partial P}\right)$$

$$\left(1 + \frac{Y_r L_Y}{L_i}\right)\frac{\partial Y}{\partial P} = -\frac{Y_r}{L_i}\frac{M}{P^2}$$

$- \frac{+}{+}$

$= + +$

$$\frac{\partial Y}{\partial P} = \frac{-\frac{M}{P^2}}{\frac{L_i}{Y_r}\left(1 + \frac{Y_r L_Y}{L_i}\right)} = \frac{-\frac{M}{P^2}}{\frac{L_i}{Y_r} + L_Y} < 0$$

Note: AD is steeper (ΔP has less effect on Y)

for:

- L_i big (money demand sensitive to i)
- L_Y big (money demand sensitive to Y)
- Y_r small (output insensitive to r)

15 - LM (closed economy)

Aggregate Demand curve (cont.)

Derive slope of AD curve using $E(Y, r, G, T)$

holding M etc. Fixed

$$Y = E(Y, r, G, T)$$

$$\frac{\partial Y}{\partial P} = E_Y \frac{\partial Y}{\partial P} + E_r \frac{\partial i}{\partial P}$$

$$(1 - E_Y)\frac{\partial Y}{\partial P} = E_r \frac{\partial i}{\partial P}$$

$$\frac{\partial Y}{\partial P} = \frac{E_r}{(1-E_Y)} \frac{\partial i}{\partial P}$$

$$\frac{M}{P} = L(i, Y)$$

$$-\frac{M}{P^2} = L_i \frac{\partial i}{\partial P} + L_Y \frac{\partial Y}{\partial P}$$

$$\frac{\partial i}{\partial P} = -\frac{1}{L_i}\frac{M}{P^2} - \frac{L_Y}{L_i}\frac{\partial Y}{\partial P}$$

$$\frac{\partial Y}{\partial P} = \frac{E_r}{(1-E_Y)}\left(-\frac{1}{L_i}\right)\left(\frac{M}{P^2} + L_Y \frac{\partial Y}{\partial P}\right)$$

rearrange to get

$$\frac{\partial Y}{\partial P} = -\frac{\frac{M}{P^2}}{\frac{(1-E_Y)L_i}{E_r} + L_Y} < 0$$

This is inverse of function plotted as AD curve

IS-LM (closed economy)

Review: What the math of IS-LM does

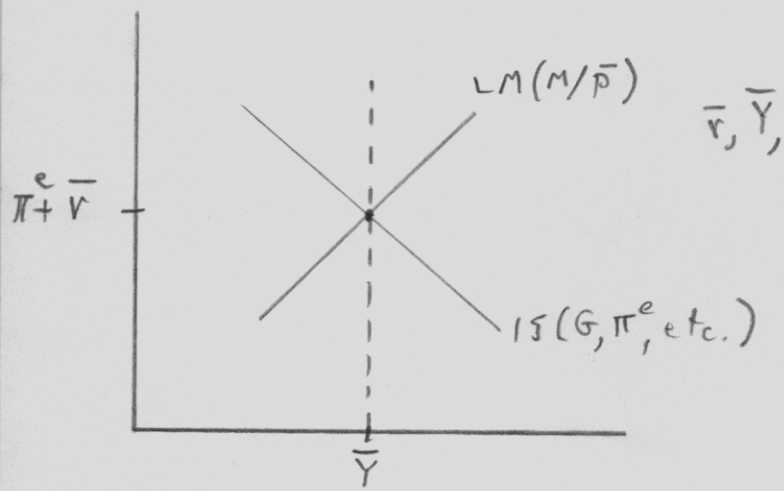
By itself, IS-LM describes relations between M, P, π^e, i, Y, G etc. that must hold if

① $Y(\overset{-}{r}, \overset{+}{G}, \ldots)$

② $\frac{M}{P} = L(\overset{-}{i}, \overset{+}{Y})$

What's endogenous/exogenous? Depends on assumptions

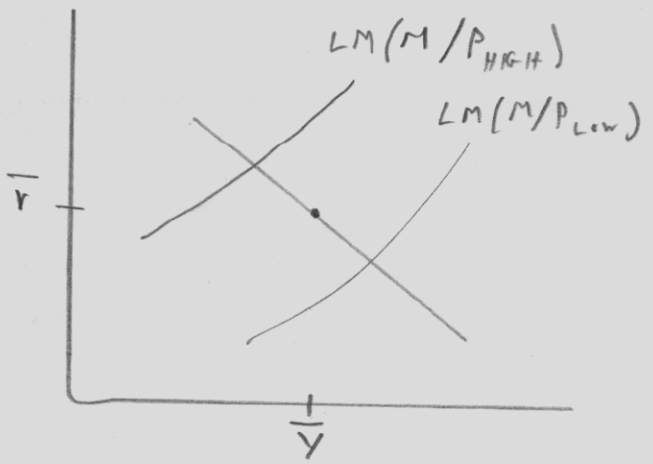
- Assuming $Y = \bar{Y}$ (output must equal natural rate)



Exogenous

$\bar{r}, \bar{Y}, M, \pi^e, G$ etc. $\longrightarrow \bar{P}$

- Assuming $P = P_0$ (fixed price level)



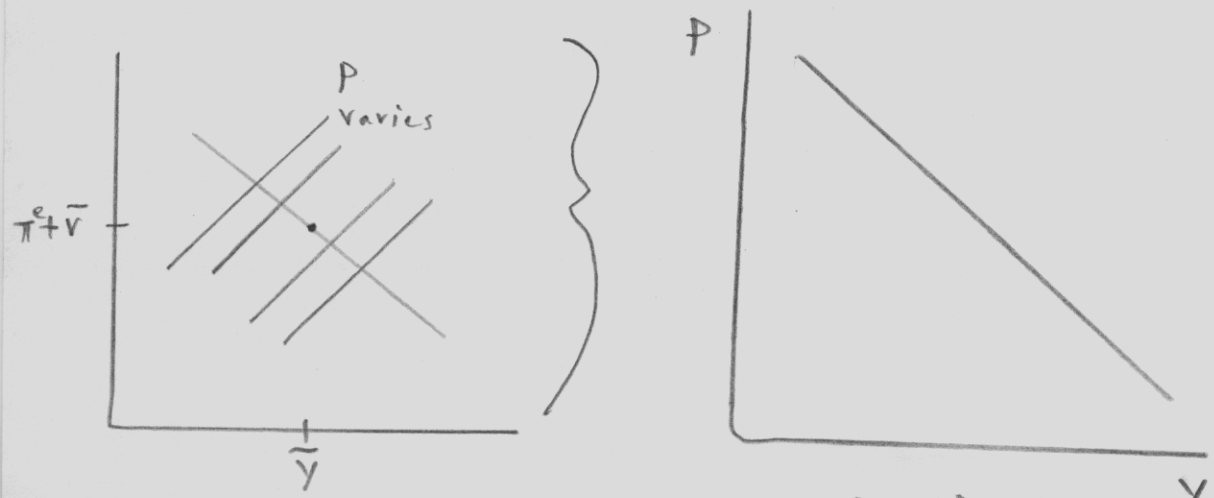
P, M, π^e, G, etc. $\longrightarrow Y, i, r$

$i = r + \pi^e$

IS-LM (closed economy)

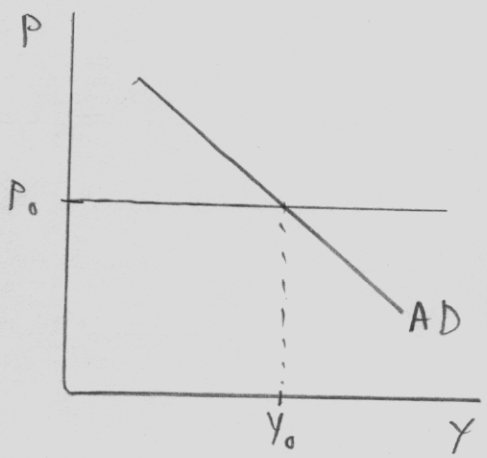
Review: math of IS-LM (cont.)

- Assuming M fixed, P & Y can vary: AD curve



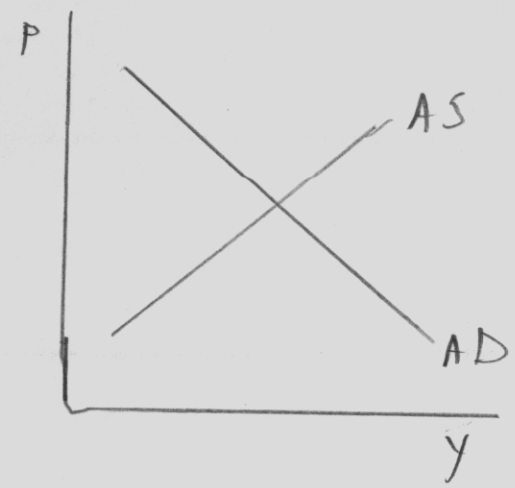
M, π^e, G etc. $\longrightarrow$ P(Y) or Y(P)

To determine P & Y, need "aggregate supply" curve



Fixed P: equivalent to IS-LM with Fixed P

implies i



Upward-sloping AS curve.

Note: anything that shifts AD out ($M\uparrow$, $\pi^e\uparrow$, $G\uparrow$ etc.) raises Y <u>and</u> P

IS-LM (closed economy)

Monetary policy: fix i or M?

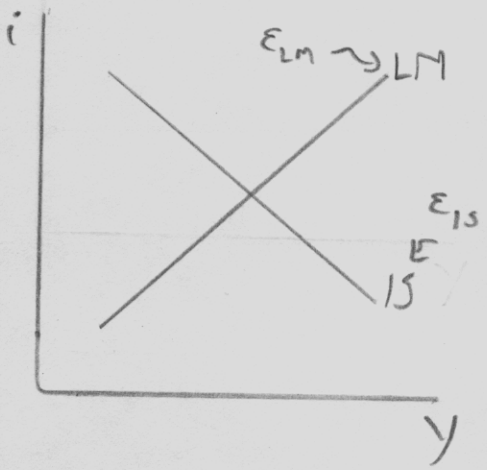
Alternative ways to do monetary policy:

What Central Bank can do

Central Bank (policy committee) can set M_t or i_t
based on information available at time t (or $t-1$)
given IS stuff $Y = Y(r, G, \text{etc.}) + \varepsilon_{IS}$ ← "IS shock"
LM stuff $M/P = L(i, Y) + \varepsilon_{LM}$ ← "LM shock"

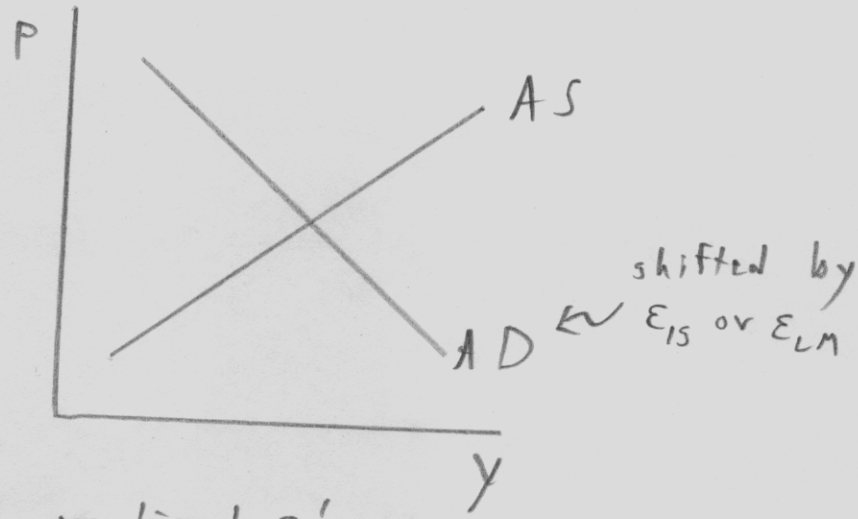
Fix M

Given M_t,
Fixed P,



realized ε_{IS}
ε_{LM}
determine i, Y

Endogenous P with AS



realized ε's
determine i, Y, and P (given AS)

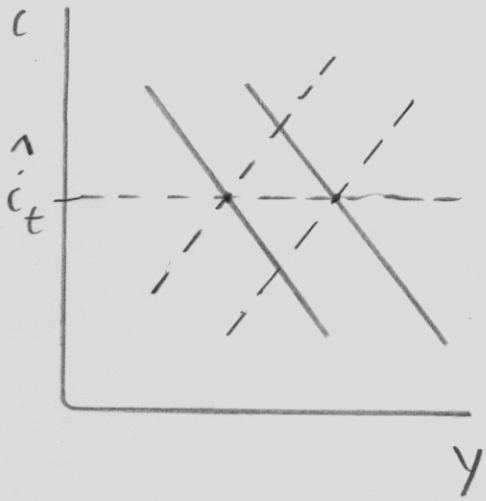
If Central Bank can't see $\varepsilon_{IS}, \varepsilon_{LM}$ when setting M,
realizations of ε's affect Y
whether P fixed or AS/AD

IS-LM

Monetary policy
Alternative ways (cont.)

Fix i_t

Vary M_t to hold i_t at a fixed value $\hat{i}_t$



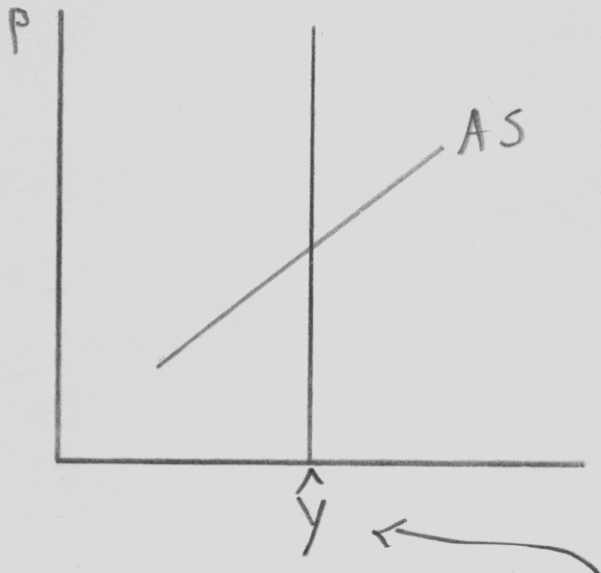
ε_{LM} is irrelevant

ε_{IS} affects Y

No relation between P & Y!

(Any effect of ΔP on $\frac{M}{P}$ is counteracted by ΔM)

How would we describe AD?



AD is vertical at $\hat{Y}$

$\hat{Y}$ shifted by π^e, G, ..., ε_{IS}

$$\hat{Y} = Y(\hat{i}_t + \pi^e, G, \text{etc.}) + \varepsilon_{IS}$$

? Should Central Bank Fix i or M?