

Economics 501

Problem set on inflation-forecast targeting

1) A central bank sets the real interest rate to affect the output gap y with a “control lag” of one period:

$$y_{t+1} = -\beta r_t + x_t + \varepsilon_{t+1}$$

where r denotes the real interest rate *minus* the natural rate of interest, and x_t is a factor affecting spending that can be observed at time t . ε is a mean-zero, i.i.d. random variable that cannot be observed at time t . The expectations-augmented Phillips curve is:

$$\pi_{t+1} = \pi_{t+1}^e + \alpha y_{t+1}$$

The central bank acts to minimize a loss function:

$$L = \frac{1}{2} E_t [(\pi_{t+1} - \pi^*)^2]$$

a) First consider what happens taking π_{t+1}^e as given, that is *not* assuming that the economy is necessarily in rational expectations equilibrium. This is analogous to Svensson, who assumes a Phillips curve with “adaptive expectations.”

i) What value will the central bank choose for r_t ?

ii) What will be the realized value of y_{t+1} and π_{t+1} ?

iii) What is the forecast for y_{t+1} and π_{t+1} as of time t ? That is, what are $E_t[y_{t+1}]$ and $E_t[\pi_{t+1}]$?

b) Now consider the rational expectations equilibrium.

i) What is π_{t+1}^e ?

ii) What will be the realized value of y_{t+1} and π_{t+1} ?

iii) What is the forecast for y_{t+1} and π_{t+1} as of time t ? That is, what are $E_t[y_{t+1}]$ and $E_t[\pi_{t+1}]$?

2) Assume that the economy has the same structure as in 1), except that the central bank acts to minimize a loss function:

$$L = \frac{1}{2} E_t [a y_{t+1}^2 + (\pi_{t+1} - \pi^*)^2]$$

a) First consider what happens taking π_{t+1}^e as given, that is *not* assuming that the economy is necessarily in rational expectations equilibrium.

i) What value will the central bank choose for r_t ?

ii) What will be the realized value of y_{t+1} and π_{t+1} ?

iii) What is the forecast for y_{t+1} and π_{t+1} as of time t ? That is, what are $E_t[y_{t+1}]$ and $E_t[\pi_{t+1}]$?

b) Now consider the rational expectations equilibrium.

i) What is π_{t+1}^e ?

ii) What will be the realized value of y_{t+1} and π_{t+1} ?

iii) What is the forecast for y_{t+1} and π_{t+1} as of time t ? That is, what are $E_t[y_{t+1}]$ and $E_t[\pi_{t+1}]$?

3) As in 2), assume that the central bank acts to minimize a loss function

$$L = \frac{1}{2} E_t [\alpha y_{t+1}^2 + (\pi_{t+1} - \pi^*)^2]$$

but now there are no disturbances to spending:

$$y_{t+1} = -\beta r_t$$

Instead there are disturbances to the inflation equation - “supply shocks” - as:

$$\pi_{t+1} = \pi_{t+1}^e + \alpha y_{t+1} + z_t + \varepsilon_{t+1}$$

z_t is a factor affecting $(t+1)$ inflation that can be observed at time t . ε is a mean-zero, i.i.d. random variable that cannot be observed at time t .

a) First consider what happens taking π_{t+1}^e as given, that is *not* assuming that the economy is necessarily in rational expectations equilibrium.

i) What value will the central bank choose for r_t ?

ii) What will be the realized value of y_{t+1} and π_{t+1} ?

iii) What is the forecast for y_{t+1} and π_{t+1} as of time t ? That is, what are $E_t[y_{t+1}]$ and $E_t[\pi_{t+1}]$?

b) Now consider the rational expectations equilibrium.

i) What is π_{t+1}^e ?

ii) What will be the realized value of y_{t+1} and π_{t+1} ?

iii) What is the forecast for y_{t+1} and π_{t+1} as of time t ? That is, what are $E_t[y_{t+1}]$ and $E_t[\pi_{t+1}]$?