Economics 501
Problem set on rational expectations

1) Suppose aggregate demand in a economy follows

$$y_t = a\ (m_t - p_t)$$

(recall this can be derived from IS/LM assuming expenditure depends on the nominal interest rate)
and the economy has a Friedman-Phelps Phillips curve:

$$\pi_t = {}_{t-1}\pi_t^e + fy_t$$

where y denotes the log of the output gap (y is zero if output is equal to the natural rate of output).

Expectations are model-consistent rational expectations (what you get in rational expectations equilibrium).

Everyone knows the money supply evolves according to a random walk:
$$m_t = m_{t-1} + \varepsilon_t$$

where ε is a mean-zero, i.i.d. random variable with variance σ_ε^2. As of time $(t-1)$, no one knows what the *realized* value of ε_t will be.

a) Derive ${}_{t-1}p_t^e$ in terms of m_{t-1}.

b) Derive p_t in terms of m_{t-1} and ε_t.

c) Derive π_t in terms of m_{t-1}, p_{t-1} and ε_t.

d) Derive y_t in terms of ε_t.

e) What is the variance of y?

2) Suppose aggregate demand in a economy follows

$$y_t = a\,(m_t - p_t) + \varepsilon_t$$

where ε is a mean-zero, i.i.d. random variable with variance σ_ε^2. As of time *(t-1)*, no one knows what the *realized* value of ε_t will be. The economy has a Friedman-Phelps Phillips curve:

$$\pi_t = {}_{t-1}\pi_t^e + fy_t$$

where *y* denotes the log of the output gap (*y* is zero if output is equal to the natural rate of output).

Expectations are model-consistent rational expectations (what you get in rational expectations equilibrium).

Everyone knows the money supply is always equal to a fixed value $\overline{m}$.

a) Derive ${}_{t-1}p_t^e$ in terms of $\overline{m}$.

b) Derive p_t in terms of $\overline{m}$ and ε_t.

c) Derive π_t in terms of $\overline{m}$, p_{t-1} and ε_t.

d) Derive y_t in terms of ε_t.

e) What is the variance of *y*?

3) Suppose an economy has an IS curve corresponding to:
$$y_t = -ar_t$$

and a Friedman-Phelps Phillips curve:
$$\pi_t = {}_{t-1}\pi_t^e + fy_t$$

where y denotes the log of the output gap (y is zero if output is equal to the natural rate of output) and r denotes the difference between the real interest rate and the natural rate of interest.

The central bank sets interest rates, not the money supply, and follows an interest-rate rule:
$$r_t = by_t + c(\pi_t - \bar{\pi})$$

where $\bar{\pi}$ is a fixed value. Note that this interest-rate rule is consistent with the general form $r(\pi,y)$ assumed for the IS/MP model.

Expectations are model-consistent rational expectations (what you get in rational expectations equilibrium).

Solve for π_t.

4) Suppose an economy has an IS curve corresponding to:
$$y_t = -ar_t + \varepsilon_t$$

and a Friedman-Phelps Phillips curve:
$$\pi_t = {}_{t-1}\pi_t^e + fy_t$$

where y denotes the log of the output gap (y is zero if output is equal to the natural rate of output) , r denotes the difference between the real interest rate and the natural rate of interest, and ε is a mean-zero, i.i.d. random variable with variance σ_ε^2.

The central bank sets interest rates, not the money supply, and follows an interest-rate rule:
$$r_t = by_t + c(\pi_t - \bar{\pi})$$

where $\bar{\pi}$ is a fixed value. Expectations are model-consistent rational expectations (what you get in rational expectations equilibrium).

a) Solve for ${}_{t-1}\pi_t^e$.

b) Derive π_t in terms of ε_t and $\bar{\pi}$.

c) Derive y_t in terms of ε_t .

d) What is the variance of y?