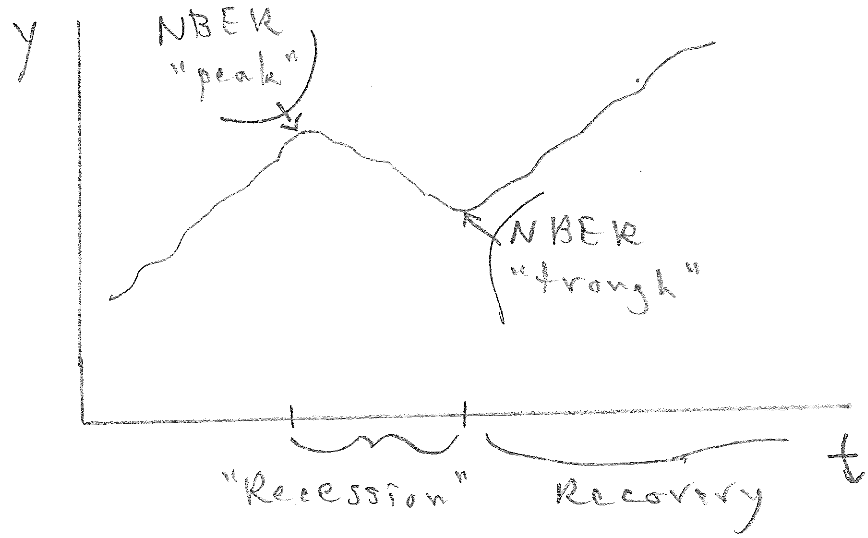
SOME FACTS ABOUT MACROECONOMIC FLUCTUATIONS

Usually called "business cycles," but they do <u>not</u> occur at regular intervals (e.g. every 7 years) so we do <u>not</u> think they are actually cycles (where downturn eventually causes upturn & vice-versa).

Rather, they seem to be result of randomly-occurring shocks + propagation mechanisms.

A "recession" is a fall in absolute <u>level</u> of Y, not just a slowdown in growth.



Recessions vary in length, from 3 months to 1½ years

When Y falls, L (labor hours) falls

Employment↓ Most L↓ due to employment↓

Hours/Worker↓

C (consumption) falls some.

I (investment) falls a lot.

Imports↓, NX↑

Some Facts (contd.)

A variable is

"Procyclical": falls in recessions (+ corr w/ Y)

"Countercyclical": rises in recessions (− corr w/ Y)

"Acyclical"

Real wages W/P acyclical or procyclical

Inflation π in P or W procyclical

Unemployment rate countercyclical

Aggregate TFP or "Solow residual" (from growth accounting) is procyclical.

$$Y = AK^{\alpha}L^{1-\alpha}$$

$$y = a + \alpha k + (1-\alpha) l$$

in recession, falls

falls a tiny bit

falls less than y

Real Business Cycle Theory

Apparent A$\downarrow$ in recession is an exogenous negative "productivity shock" that causes a recession as households choose to work less, consume more leisure in response to temporarily-low productivity.

What causes decline in A? Technology gets worse?

(Alternative explanation for A$\downarrow$ in recessions, not RBC theory: input of L & K to production is mismeasured:

"labor hoarding"

"capital utilization")

REAL BUSINESS CYCLE THEORY

Big idea

B cycles are Pareto-optimal response of a competitive economy, with perfectly rational agents, to "real" shocks, not to "monetary" shocks (shocks to $\$M^D$ or $\$M^S$).

Structure of models

RCK + variable endogenous L

No M^D, M^S or P

Exogenous stochastic shocks to TFP, + maybe other shocks

Shut down shocks, model converges to LRSS "balanced-growth" path that looks like RCK

Response to a shock is temporary fluctuations of endogenous variables (Y, L, K, $\frac{W}{P}$ etc.) from balanced-growth path.

$$\ln X_t = \underbrace{\bar{X} + g_x t}_{\text{balanced-growth path}} + \tilde{X}_t$$

$\tilde{X}_t$: deviation of ln of variable from balanced-growth path

RBC THY (cont.)

Romer's "baseline" RBC model

Two shocks

TFP $\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \varepsilon_{At}$

i.i.d

Govt. purchases of output

$$\tilde{G}_t = \rho_G \tilde{G}_{t-1} + \varepsilon_{Gt}$$

G is output taken by "goverment" and thrown away (doesn't enter prodn. or util. functions)

Preview: what we get from the model

A set of loglinear equations that give endogenous variables as functions of $\tilde{A}$ & $\tilde{G}$ current or lagged endogenous variables,

$$\left.\begin{aligned} \tilde{C}_t &\simeq a_{CK}\tilde{K}_t + a_{CA}\tilde{A}_t + a_{CG}\tilde{G}_t \\ \tilde{L}_t &\simeq a_{LK}\tilde{K}_t + a_{LA}\tilde{A}_t + a_{LG}\tilde{G}_t \\ \tilde{K}_{t+1} &\simeq b_{KK}\tilde{K}_t + b_{KA}\tilde{A}_t + b_{KG}\tilde{G}_t \end{aligned}\right\}$$

linear approximations around LRSS path

$$\tilde{Y}_t = \alpha\tilde{K}_t + (1-\alpha)\tilde{L}_t + (1-\alpha)\tilde{A}_t$$

From $Y = K^{\alpha}(AL)^{1-\alpha}$

What do these equations mean?

RBC THY

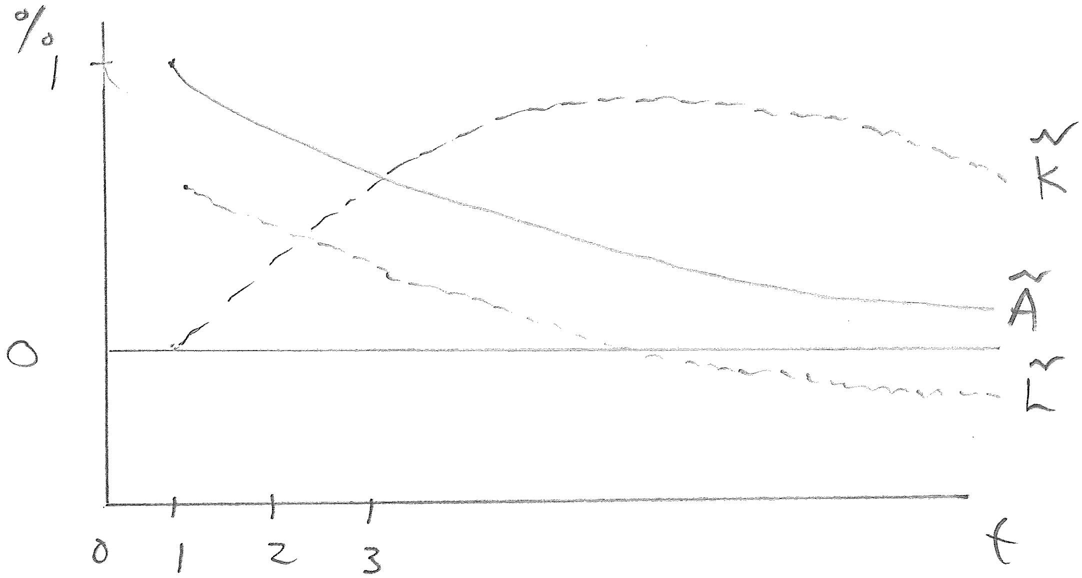
.... baseline model

preview (cont.)

Imagine Excel spreadsheet

t	ε_A	ε_G	$\tilde{A}$	$\tilde{G}$	$\tilde{C}_t$	$\tilde{L}_t$	etc.
0	0	0	0				
1	1 SD	0	1 SD	equations give values for these			
2	0	0	ρ_A 1 SD				
3	0	0	ρ_A^2 1 SD				
4	0	0	etc.				
5	0	0					

Make time-series plots of $\tilde{X}$'s



Compare with data from a "typical" business cycle.
Do they look alike?

Do same for ε_G.

RBC THY

Model assumptions

Prodn. Fn, etc.

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} \qquad (5.1)$$

$$Y_t = C_t + I_t + G_t \qquad \text{closed economy}$$

$$K_{t+1} = K_t + I_t - \delta K_t \qquad (5.2)$$
$$= K_t + Y_t - C_t - G_t - \delta K_t$$

G_t = lump-sum tax on household (see below)
(Recall "Ricardian equivalence")

$\ln N_t = \bar{N} + nt$ Exogenous rate of popn. growth

$L_t = N_t \ell_t$ where $0 \le \ell \le 1$

ℓ_t: fraction of potential labor hours (=1) supplied as labor by a person

"Real wage" $w_t = MPL = (1-\alpha) K_t^{\alpha}(A_t L_t)^{-\alpha} A_t \qquad (5.3)$

$$= (1-\alpha)(K_t/A_t L_t)^{\alpha} A_t$$

Real interest rate $r_t = MPK - \delta = \alpha(K_t/A_t L_t)^{-(1-\alpha)} - \delta$

Note: if (K/AL) fixed, as it is in LRSS,
r is fixed
real wage grows at same rate as A

RBC
Model assumptions (cont.)
Utility function

$$U = E\left[\sum_{t=0}^{\infty} e^{-\rho t} u(c_t, 1-l_t)\frac{N_t}{H}\right] \quad (5.5)$$

$$\& \; u(c_t, 1-l_t) = \ln c_t + b\ln(1-l_t) \quad 0<b<1 \quad (5.7)$$

c_t ← (consumption per person

"Felicity," "instantaneous" or "Flow" or "momentary" utility

Note:

— In this expression, Romer is sloppy: uses t to denote horizon looking forward from a point in time, as well as time periods.

Should be:

$$U_t = E_t\left[\sum_{\tau=0}^{\infty} e^{-\rho\tau} u(c_{t+\tau}, 1-l_{t+\tau})\frac{N_t}{H}\right]$$

t "calendar time" (see King & Rebelo p. 943)

τ "Planning time"

— Agent is household not person

— Population grows by new people in fixed number of households

— Other ways to write subjective discounting:

$$e^{-\rho\tau} = \left(\frac{1}{1+\rho}\right)^{\tau} = \beta^{\tau} \leftarrow (0<\beta<1)$$

How are they equivalent? Take logs!

$$-\rho\tau = \tau\ln\left(\left(\frac{1}{1+\rho}\right)\right) = -\tau\ln(1+\rho) \approx -\tau\rho$$

RBC

"Benthamite" utility function

$$U = \sum e^{-\rho t}\left(\ln c_t + b(\ln(1-l_t))\right)\frac{N_t}{H}$$

Means population growth increases felicity & utility

Requires $n < \rho$ (otherwise U is infinite, can't deal with that).

Why assume this?
1) Need to allow for popn. growth
2) With popn. growth, Benthamite utility simplifies Euler equation etc.

Without Benthamite:

$$c_t = e^{\rho} c_{t+1}(1+r_t)^{-1}(1+n)$$

With:

$$c_t = e^{\rho} c_{t+1}(1+r_t)$$

simpler

Derive Euler equation

Generally, utility maximation requires

$$\partial U/\partial c_t = \partial U/\partial c_{t+1} \cdot \frac{\partial c_{t+1}}{\partial c_t}$$

where $\partial c_{t+1}/\partial c_t = (1+r)(1+n)^{-1}$

Why? $C_{t+1} = (1+r)(Y-C)_t + Y_{t+1}$ this is big C

$$\partial C_{t+1} = -(1+r)\,\partial C_t$$

Little c: $\partial c_{t+1} = -(1+r)\,\partial c_t \frac{1}{1+n}$

From this we get Euler equation.

RBC

"Benthamite" utility

Derive Euler equation (cont.)

$$\partial U/\partial c_t = \partial U/\partial c_{t+1} (1+r)(1+n)^{-1}$$

Without Benthamite

$$U = \ln c_t + b(1-l_t) + e^{-\rho}[\ln c_{t+1} + b(1-l_{t+1})] + \ldots$$

$$\frac{\partial U}{\partial c_t} = \frac{1}{c_t} \qquad \frac{\partial U}{\partial c_{t+1}} = e^{-\rho}\frac{1}{c_{t+1}}$$

Put that into above, get previous page.

With Benthamite:

$$U = N_t(\ln c_t + b(1-l_t)) + e^{-\rho}N_{t+1}[\ln c_{t+1} + b(1-l_{t+1})] + \ldots$$

$$\frac{\partial U}{\partial c_t} = N_t\frac{1}{c_t} \qquad \frac{\partial U}{\partial c_{t+1}} = N_{t+1}e^{-\rho}\frac{1}{c_{t+1}}$$

Put that into above, get ...

RBC TH7

Model assumptions

Utility function (cont.)

$$U = \sum e^{-\rho t}\left(\ln c_t + b\ln(1-l_t)\right)\frac{N_t}{H}$$

Utility is "additively separable" across periods

$$U = u(c_t, 1-l_t) + e^{-\rho}u(c_{t+1}, 1-l_{t+1}) + e^{-2\rho}(\quad)\ldots$$

so $\frac{\partial U}{\partial c_t}, \frac{\partial U}{\partial l_t}$ unaffected by c_{t-1} l_{t-1}, i.e. $\frac{\partial U}{\partial c_t \partial c_{t-1}} = 0$

How much I ate, worked last period does not directly affect what I want to do today.

Example of not separable across periods:

$$U = \sum e^{-\rho t}\left(\ln(c_t - j c_{t-1}) + \ldots\right.$$

so $\frac{\partial U}{\partial c_{t+1}} = \frac{1}{c_{t+1} - j\, c_t}$

See: $c_t \uparrow \rightarrow \frac{\partial U}{\partial c_{t+1}} \uparrow \quad \frac{\partial U}{\partial c_t \partial c_{t-1}} > 0$

The more I ate yesterday, the more I want to eat today.

A function of this form is called "habit formation" in consumption.

Utility Function (cont.)

$$u(c_t, 1-l_t) = \ln c_t + b \ln (1-l_t)$$

Felicity is "additively separable" in c, l

so $\frac{\partial u}{\partial c_t}$ unaffected by l and vice-versa.

Example of not separable:

a function

$$u(c_t, 1-l_t) = \frac{1}{1-\sigma}\left\{\left[c_t v(l_t)\right]^{1-\sigma} - 1\right\}$$

where $0 < \sigma < 1$

$$\frac{\partial u}{\partial c_t} = \frac{1}{1-\sigma}(1-\sigma)(c_t v(l_t))^{-\sigma} v(l_t) = \frac{v(l_t)^{1-\sigma}}{c_t^{-\sigma}}$$

etc. (See King & Rebelo p. 945)

RBC THY

Household's problem

Representative household owns capital, gets paid $r_t K_t$ and $w_t \ell_t \frac{N_t}{H}$ in period t.

Pays lump-sum tax equal to $\frac{G_t}{H}$.

Can borrow at r_t if he wants (but in equilibrium he won't).

<u>Ricardian equivalence:</u> it makes no difference to him if govt. doesn't tax this period but instead borrows to pay for G_t now and taxes him $(1+r_t)G_t$ to repay the "govt bonds" in $t+1$.

Household's budget constraint:

$$\underbrace{\sum_{\tau=0}^{\infty} \frac{1}{(1+r_{t+\tau})^{\tau}} c_{t+\tau} \frac{N_{t+\tau}}{H}}_{\text{present discounted value of consumption}} \leq \underbrace{\frac{K_t}{H}}_{\text{wealth today}} + \sum_{\tau=0}^{\infty} \frac{1}{(1+r_{t+\tau})^{\tau}} \Big(\underbrace{\ell_{t+\tau} \frac{N_{t+\tau}}{H} w_{t+\tau}}_{\text{p.d.v. wage income}} - \underbrace{\frac{G_{t+\tau}}{H}}_{\text{p.d.v. lump-sum tax}} \Big)$$

Household takes as given current & expected w, r, G and existing wealth $(K/H)_t$

and chooses c_t, ℓ_t.

Note: N_t is exogenous, so when L falls, that's just $\ell \downarrow$

RBC THY

Solving model as a "Social planner"

"First fundamental theorem of welfare economics" says outcome of perfect competition, etc. is Pareto optimal.

Thus we can assume outcome of this economy maxxes utility of rep. agent given constraints on economy as a whole, which we get by maxxing $E_t[U]$ s.t. K_t, G_t and prodn. fn.

You can solve model either way.

Dynamic Programming

1) Define "value function" V_t (value of maximand (here, $E[U]$), looking forward from time t, as function of variables taken as given at time t (here, $K_t, \tilde{A}_t, \tilde{G}_t$) assuming control variables (C, ℓ) are set at optimums

2) Write down Bellman equation

3) Write down f.o.c.'s

— "Intratemporal" relationships between time-t values

— "Intertemporal" relationships between values at t and at $t+1$

RBC Thy

Bellman Equation

Describes intertemporal optimization problem as a 'problem to be solved each period, looking just one step ahead, taking as given variables determined by past history that matter for now & future. Here, K_t and A_t (and G_t if we include govt. in model).

fixed population

For simplicity, & to avoid Benthamite issue, set $N = 1$ $(n = 0)$

$$V(K_t, A_t) = \underset{c,\,l}{\operatorname{Max}} \left\{ U_c(C_t) + U_{1-l}(1-l_t) + e^{-\rho} E_t\left[V(K_{t+1}, A_{t+1}) \right] \right\}$$

F.O.C.'s

Take derivatives w.r.t. choice variables, set $\frac{\partial V}{\partial m} = 0$

Note: choice of c & l will affect K_{t+1} but not A_{t+1}

$$K_{t+1} = K_t + Y_t - C_t - \delta K_t = K_t + F(K_t, A\, l_t) - C_t - \delta K_t$$

hence $\frac{\partial K_{t+1}}{\partial C_t} = -1$ $\qquad \frac{\partial K_{t+1}}{\partial l_t} = \frac{\partial Y}{\partial l} = w$ MPL

w.r.t. consumption:

$\frac{\partial V}{\partial K}$

$$\frac{\partial V}{\partial C_t} = 0 = U_c'(C_t) + e^{-\rho} E_t\left[V_K(K_{t+1}, A_{t+1}) \frac{\partial K_{t+1}}{\partial C_t} \right]$$

$$= U_c'(C_t) + e^{-\rho} E_t\left[V_K(\quad)(-1) \right]$$

w.r.t. l:

$$\frac{\partial V}{\partial l_t} = 0 = U_{1-l}'(-1) + e^{-\rho} E_t\left[V_K(\quad) \frac{\partial K_{t+1}}{\partial l_t} \right]$$

$$= U_{1-l}'(-1) + e^{-\rho} E_t\left[V_K(\quad)\, w \right]$$

LOOK! $E_t[V_K(\quad)]$ is in both expressions!

RBC Thy

F.O.C.'s (cont.)

From $\frac{\partial V}{\partial l_t} = 0$, we have

$$\frac{1}{w_t} U'_{1-l} = e^{-\rho} E_t[V_k(\quad)]$$

From $\frac{\partial V}{\partial C_t} = 0$, we have

$$U'_c(C_t) = e^{-\rho} E_t[V_k(\quad)]$$

hence $\frac{1}{w_t} U'_{1-l} = U'_c(C_t)$

$$U'_{1-l}(1-l_t) = w_t U'_c(C_t)$$

Romer calls this
"Intratemporal F.O.C."

What it means:
I have an endowment of time, which I can "eat" directly as leisure, or convert into consumption & "eat" more C. Rate of exchange between l and c is real wage w.

When I work one more hour, I lose $MU_{LEISURE} = U'_{1-l}$
I gain $MU_c w = U'_c w$

At optimum, can't be $MU_{LEISURE} > MU_c w$
work less!
can't be $MU_{LEISURE} < MU_c w$
work more!

so it must be...
But how do I determine $U'_c(C_t)$?

RBC Thy

Benveniste – Scheinkman equation

Recall

$$V(K_t, A_t) = \underset{c,\ell}{Max}\left\{U_c(C_t) + U_{1-\ell}(1-\ell_t) + e^{-\rho} E_t[V(K_{t+1}, A_{t+1})]\right\}$$

What is V_K? (which will tell us $E_t[V_K(\ )]$)

Recall the "envelope theorem":
if we are maximizing a function subject to constraints,
the effect of a marginal change in constraint
on the maximized value of the function
is equal to the marginal effect of choice variable
on optimand,
times marginal effect on the choice variable of the
change in constraint,
holding fixed other choice variables.

Example: Max $U(X_1, X_2)$ s.t. $P_1 X_1 + P_2 X_2 = Y$

U^* Maximized value of U

$$\frac{\partial U^*}{\partial Y} = U_{X_1} \frac{1}{P_1} = U_{X_2} \frac{1}{P_2}$$

$\frac{\partial X_1}{\partial Y}$ holding fixed X_2

Here,

$$V_K(K_t, A_t) = U_c'(C_t) \frac{\partial C_t}{\partial K_t}$$ holding fixed ℓ_t, K_{t+1}, etc.

RBC Thy

Benveniste-Scheinkman equation (cont.)

Recall $K_{t+1} = K_t + F(K_t, A\ell_t) - C_t - G_t - \delta K_t$

hence $C_t = -K_{t+1} + F(K_t, A\ell_t) - G_t + (1-\delta) K_t$

$$\frac{\partial C_t}{\partial K_t} = \underbrace{F_K(\quad)}_{\text{or } \frac{\partial Y}{\partial K}} + (1-\delta)$$

Recall $r \equiv \frac{\partial Y}{\partial K} - \delta$

hence $\frac{\partial C_t}{\partial K_t} = 1 + r_t$

hence

$$V_K(K_t, A_t) = U'_c(C_t)(1+r_t)$$ BS equation

what this means: value of a marginal unit of capital is MU of eating another unit of output times marginal effect of capital on this period's output <u>after</u> covering extra depreciation, to keep K_{t+1} unchanged.

BS equation will hold next period too, so:

$$E_t[V_K(K_{t+1}, A_{t+1})] = E_t[U'_c(C_{t+1})(1+r_{t+1})]$$

or

$$= E_t\left[\frac{1}{w_{t+1}} U'_{1-\ell}(1-\ell_t)(1+r_{t+1})\right]$$

RBC Thy

"Intertemporal F.O.C."

Recall

$$\frac{1}{w_t} U'_{1-l}(c_t) = e^{-\rho} E_t[V_K(K_{t+1}, A_{t+1})]$$

$$U'_c(c_t) = e^{-\rho} E_t[V_K(\quad)]$$

hence from BS eqn

$$U'_c(C_t) = e^{-\rho} E_t[U'_c(C_{t+1})(1+r_{t+1})]$$

$$U'_{1-l}(1-l_t) = w_t e^{-\rho} E_t[U'_c(C_{t+1})(1+r_{t+1})]$$

We could also write these in terms of F w_{t+1}, l_{t+1}

Assuming Romer's specific utility function:

Recall $U(c, 1-l_t) = \ln(c_t) + b\ln(1-l_t)$

hence $U_c(\ ) = \frac{1}{c_t}$ $\qquad U_{1-l}(\ ) = \frac{b}{1-l_t}$

Intratemporal F.O.C

$$U'_{1-l_t}(\ ) = w_t U'_c(c_t)$$

becomes $\frac{b}{1-l_t} = w_t \frac{1}{c_t}$ or $\frac{c_t}{1-l_t} = \frac{w_t}{b_t}$ (5.26)

Intertemporal F.O.C.

$$\frac{1}{c_t} = e^{-\rho} E_t\left[\frac{1}{c_{t+1}}(1+r_{t+1})\right] \quad (5.23)$$

Now what?

If we know $E_t\left[\frac{1}{c_{t+1}}(1+r_{t+1})\right]$, we know c_t

If we know c_t, we know l_t.

With c_t and l_t we know everything else.

But to know $E_t\left[\frac{1}{c_{t+1}}(1+r_{t+1})\right]$ we need to know

$E_t\left[E_{t+1}\left[\frac{1}{c_{t+2}}(1+r_{t+2})\right]\right]$ because intertemporal F.o.C. will hold at time t+1 too!

So we have to define expected values for all variables going into the infinite future!

How?

1) Assume $E_t[X_{t+\infty}] = X_{LRSS}$ (if no shocks)

make this the "Final period"

2) Solve back from there.

To begin, Figure out what LRSS looks like.

But note there's an approximation here.

$E_t[X_{t+\infty}] \neq X_{LRSS}$ (if no shocks).

because there <u>will</u> be shocks in the future!

RBC THY

LRSS without shocks ("balanced-growth path")

Without ε's, model is RCK with endogenous L.

$L_t = N_t \ell_t$

In LRSS, ℓ must be fixed (ℓ can't grow or shrink forever)

so L_t grows at rate n (   )

$\ln A_t = \ln \bar{A} + gt$

As in RCK,

Y grows at rate $n+g$

G grows at rate $n+g$ (otherwise $\frac{G}{Y}$ grows or shrinks)

$k = \frac{K}{AL} = k^*$ Fixed where $y = \frac{Y}{AL} = f(k) = k^\alpha$

$r = MPK - \delta = f'(k) - \delta = \alpha k^{*\alpha-1} - \delta$ Fixed

$w_t = MPL = (1-\alpha) k^{*\alpha} A_t$ grows at rate g

$c = \frac{C}{L} = A_t \underbrace{\left(f(k^*) - (n+g+\delta)k^* \right)}_{\text{Fixed}}$ grows at rate g

Note $e^g = \frac{A_{t+1}}{A_t} = \frac{c_{t+1}}{c_t} = \frac{w_{t+1}}{w_t}$

Why does felicity function take natural-log form?

$$u_t = \ln c_t + b \ln(1 - l_t) \qquad 0 < b \le 1$$

Note utility is "separable" in c vs. l

(i.e. $\frac{\partial u_t}{\partial c_t}$ unaffected by l_t)

Natural-log form is only thing that works if u is separable in c vs. l and U is separable across time

Recall $u'_c(c_t) = u'_{1-l}(1-l_t)\frac{1}{w_t}$

hence $$\frac{u'_c(c_{t+1})}{u'_c(c_t)} = \frac{u'_{1-l}(1-l_{t+1})}{u'_{1-l}(1-l_t)} \frac{w_t}{w_{t+1}}$$

In LRSS, l fixed, $c_{t+1} = e^g c_t$, $w_t / w_{t+1} = \frac{1}{e^g}$

So $u(\ )$ must be such that

$$\frac{u'_c(e^g x)}{u'_c(x)} = \frac{1}{e^g}$$

hence $u'_c(z) = \frac{1}{z}$ so $u_c(z) = \ln(z)$

RBC THY

LRSS without shocks

LRSS value of r^*, k^*

Euler equation: $u_c'(c_t) = e^{-\rho} E_t [u'(c_{t+1})(1+r_{t+1})]$

In nonstochastic LRSS with ln utility

$$\frac{1}{c_t} = e^{-\rho} \frac{1}{c_{t+1}} (1+r^*)$$

$$\Rightarrow r^* = \frac{c_{t+1}}{c_t} e^{\rho} = e^{g} e^{\rho} - 1 = e^{g+\rho} - 1$$

Note: if you have a number for r^* (trend value of r)
and g (trend TFP growth)
you can infer ρ

Also, since $r^* = f'(k^*) - \delta = \alpha k^{*\,\alpha-1} - \delta$

$$e^{g+\rho} - 1 = \alpha k^{*\,\alpha-1} - \delta$$

$$\Rightarrow k^* = \left(\frac{\alpha}{e^{g+\rho} + \delta - 1} \right)^{\frac{1}{1-\alpha}}$$

If you know δ (depreciation rate)
α (share of capital in income)

you can infer k^*.

RBC THY

LRSS w/o shocks (cont.)

LRSS ℓ^*

Intratemporal f.o.c.: $u'_{1-\ell}(1-\ell_t) = w\, u'_c(c_t)$

with $u_t = \ln c_t + b \ln(1-\ell_t)$

$$\frac{b}{1-\ell^*} = w_t \frac{1}{c_t} = \frac{A_t(1-\alpha)k^{*\alpha}}{A_t\left(k^{*\alpha} - (n+g+\delta)k^*\right)}$$

$$\text{recall } k^* = \left(\frac{\alpha}{e^{g+\rho}+\delta-1}\right)^{\frac{1}{1-\alpha}}$$

$$\left.\frac{b}{1-\ell^*} = \frac{1-\alpha}{1-\alpha\dfrac{n+g+\delta}{e^{g+\rho}+\delta-1}}\right\} \text{call this } Z$$

so

$$\ell^* = 1 - \frac{b}{Z}$$

$$b = \frac{1-\ell^*}{Z}$$

Recall ℓ^* is fraction of potential labor time devoted to work (not leisure)

so if you know values of other parameters, you can infer value of b from ℓ^*.

RBC THEORY

How do you choose parameter values?

Take a "period" to be $\frac{1}{4}$, matching NIPA data

1) Make model match LR trends in U.S. data.

$\alpha = 1/3 \quad (1-\alpha) = 2/3$ ~ {labor's share of national income in NIPAs}

$g = 0.5\%$ (Annual TFP growth of 2%)

$n = 0.25\%$ (Labor Force growth 1%)

$\delta = 2.5\%$ (Annual depreciation 10%)

2) Estimate ρ_G & $Var(\varepsilon_G)$

3) Infer quarterly values of A from data on Y, L, K & $\alpha = 1/3$.

From inferred quarterly A, estimate ρ_A & $Var(\varepsilon_A)$

4) A "reasonable" value for r^*: 1.5% (6% annually)

hence $f'(k^*) = r^* + \delta$ and k^*

5) ρ is determined by g, & r^*, given log utility

Recall $C_t = C_{t+1} \frac{1}{e^{-\rho}(1+r)}$

in LRSS $C_{t+1} = e^g C_t$

hence $e^g = e^{-\rho}(1+r)$ take logs of both sides

$$\rho = \ln(1+r) - g \approx r - g$$

6) Given all these, infer b from data on l^*

RBC Thy

Log-linear approximation of model

$$\tilde{X}_t = \text{Log}(X_t) - \text{Log}(X_t \text{ on balanced-growth path})$$

Aggregate production function:

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} = K_t^{\alpha}(A_t l_t N_t)^{1-\alpha}$$

$$\ln Y_t = \alpha \ln K_t + (1-\alpha)\ln A_t + (1-\alpha)\ln l_t + (1-\alpha)\ln N_t$$

doesn't deviate from LRSS path

$$\tilde{Y}_t = \alpha \tilde{K}_t + (1-\alpha)\tilde{A}_t + (1-\alpha)\tilde{l}_t$$

$\rho_A \tilde{A}_{t-1} + \varepsilon_{A,t}$ $\tilde{L}_t$

together with

$$\tilde{C}_t \approx a_{CK}\tilde{K}_t + a_{CA}\tilde{A}_t + a_{CG}\tilde{G}_t$$

$$\tilde{L} \approx a_{LK}\tilde{K}_t + \ldots$$

$$\tilde{K}_{t+1} \approx b_{KK}\tilde{K}_t + b_{KA}\tilde{A}_t + b_{KG}\tilde{G}_t$$

and $\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \varepsilon_{A,t}$ $\tilde{G}_t = \rho_G \tilde{G}_{t-1} + \varepsilon_{G,t}$

Feed in shocks for ε's, get paths over time for everything.

Note: ε_t affects $\tilde{X}_{t+1}$ through $\rho\,\varepsilon_t$ ← (serial correlation in shock

<u>and</u> b_{KA} ← (effect of $\tilde{A}_t$ on future capital stock

or

b_{KG}

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How parameters enter loglinear approximation

Simplified model

Consider simplified version of model with fixed l and no G

(Campbell, 1994)

$$\tilde{C}_t = a_{CK}\,\tilde{k}_t + a_{CA}\,\tilde{A}_t$$

$$\tilde{K}_{t+1} = b_{KK}\,\tilde{k}_t + b_{KA}\,\tilde{A}_t$$

$$b_{KK} = \lambda_1 + (1-\lambda_1-\lambda_2)\,a_{CK} \qquad b_{KA} = \lambda_2\,(1-\lambda_1-\lambda_2)\,a_{CA}$$

$$\lambda_1 = \frac{1+r}{1+g} \qquad \lambda_2 = \frac{\alpha(r+\delta)}{(1-\alpha)(1+g)} \qquad \lambda_3 = \frac{\alpha(r+\delta)}{1+r}$$

$$a_{CK} = \frac{1}{2Q_2}\left(-Q_1 - \sqrt{Q_1^2 - 4Q_0Q_2}\right)$$

$$a_{CA} = \frac{-a_{CA}\lambda_2 + \lambda_3(\rho_A - \lambda_2)}{\rho_A - 1 + (1-\lambda_1-\lambda_2)(a_{CK}+\lambda_3)}$$

$$Q_0 = \lambda_3\lambda_1$$

$$Q_1 = \lambda_1 - 1 + \lambda_3(1-\lambda_1-\lambda_2)$$

$$Q_2 = 1-\lambda_1-\lambda_2$$

Note: change any parameter $r, g, \alpha, \delta, \rho_A$

and all coeffs a's, b's change

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Complete model

Assuming "reasonable" values for $r, g, \alpha, \delta, \rho_A, \rho_G, b$ gives

$$\tilde{C}_t = 0.59\,\tilde{K}_t + 0.38\,\tilde{A}_t - 0.13\,\tilde{G}_t$$

$$\tilde{L}_t = -0.31\,\tilde{K}_t + 0.35\,\tilde{A}_t + 0.15\,\tilde{G}_t$$

$$\tilde{K}_{t+1} = 0.95\,\tilde{K}_t + 0.08\,\tilde{A}_t - 0.004\,\tilde{G}_t$$

Effect of $\tilde{K}_t$

$\tilde{K}_t \uparrow \rightarrow C_t \uparrow, L_t \downarrow, K_{t+1} \uparrow$

More K means we're richer, higher lifetime income

So $C \uparrow$, leisure $\uparrow$, labor $\downarrow$

Because MU_C is diminishing, we want to transfer some consumption to future by hiking K_{t+1}

Effect of $\tilde{G}_t$

$\tilde{G}_t \uparrow \Rightarrow C_t \downarrow, L_t \uparrow, K_{t+1} \downarrow$

More G means higher lifetime tax burden, lower lifetime after-tax income,

so $C \downarrow$, leisure $\downarrow$, labor $\uparrow$

Transfer some loss to future by $K_{t+1} \downarrow$

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Complete model (cont.)

Effect of $\tilde{A}_t$

$\tilde{A}_t \uparrow \rightarrow C_t \uparrow, L_t \uparrow, K_{t+1} \uparrow$

Higher productivity means higher lifetime income,
higher <u>current</u> real wage,
higher r_{t+1} for any given K_{t+1} because of persistence in A shock.

C_t rises because lifetime income is higher.
More persistent $\tilde{A}$ (ρ_A higher) <u>increases</u> a_{CA}

L_t rises because current real wage higher:

$$\underbrace{u'_{1-\ell}(1-\ell_t)}_{\text{For } \ell_t \uparrow, \text{ this must rise}} = w_t \underbrace{u'_c(C_t)}_{\text{but this falls because } C_t \uparrow}$$

so $w_t \uparrow$ allows $\ell_t \uparrow$.

More persistent $\tilde{A}$ <u>decreases</u> a_{LA} as it increases a_{CA}

K_{t+1} rises to spread some consumption to future periods, and because $r_{t+1} \uparrow$.

Note: apart from ρ_A, $\hat{A}_t$ has some effect on $t+1$, $t+2 \ldots$ through b_{KA}.

Complete model (cont.)

Look at Figures 5.2–5.4 (effects of ε_A)

5.5–5.7 (effects of ε_G)

See: effects of ε_A look like a business cycle.

" " ε_G don't.

Thus, the explanation of business cycles implied by RBC theory:

bad TFP shocks

"technology shocks"

RBC Thy

Labor Market

Recall in Romer

$$\frac{1}{c_t} = e^{-\rho} E_t\left[\frac{1}{c_{t+1}}(1+r_{t+1})\right]$$

hence

$$c_t = e^{\rho} \frac{1}{E_t\left[\frac{1}{c_{t+1}}(1+r_{t+1})\right]}$$

(G and expected future A affect C thru this.

Anything that is expected to

raise c_{t+1} raises c_t

raise r_{t+1} Lowers c_t

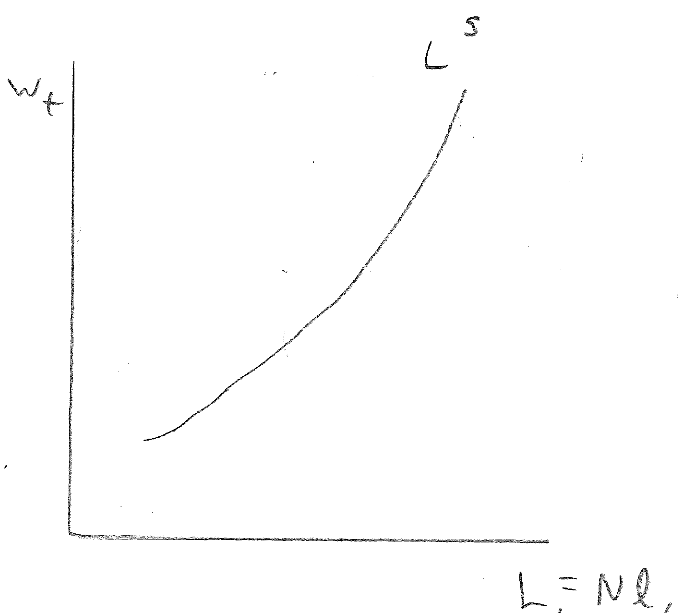
$$\frac{c}{1-l_t} = w_t / b$$

hence

$$w_t^s = b\, c_t \frac{1}{1-l^s}$$

Wage at which household is willing to supply l^s

("labor supply" shock comes along with c_t



L^s is shifted by anything that affects c_t, like G.

Anything that shifts L_t^s out (reduces leisure at a given real wage)

also causes c_t to fall

Anything that raises lifetime income raises c_t, shifts L_t^s back.

RBC thy

Labor Market (cont.)

Labor demand:

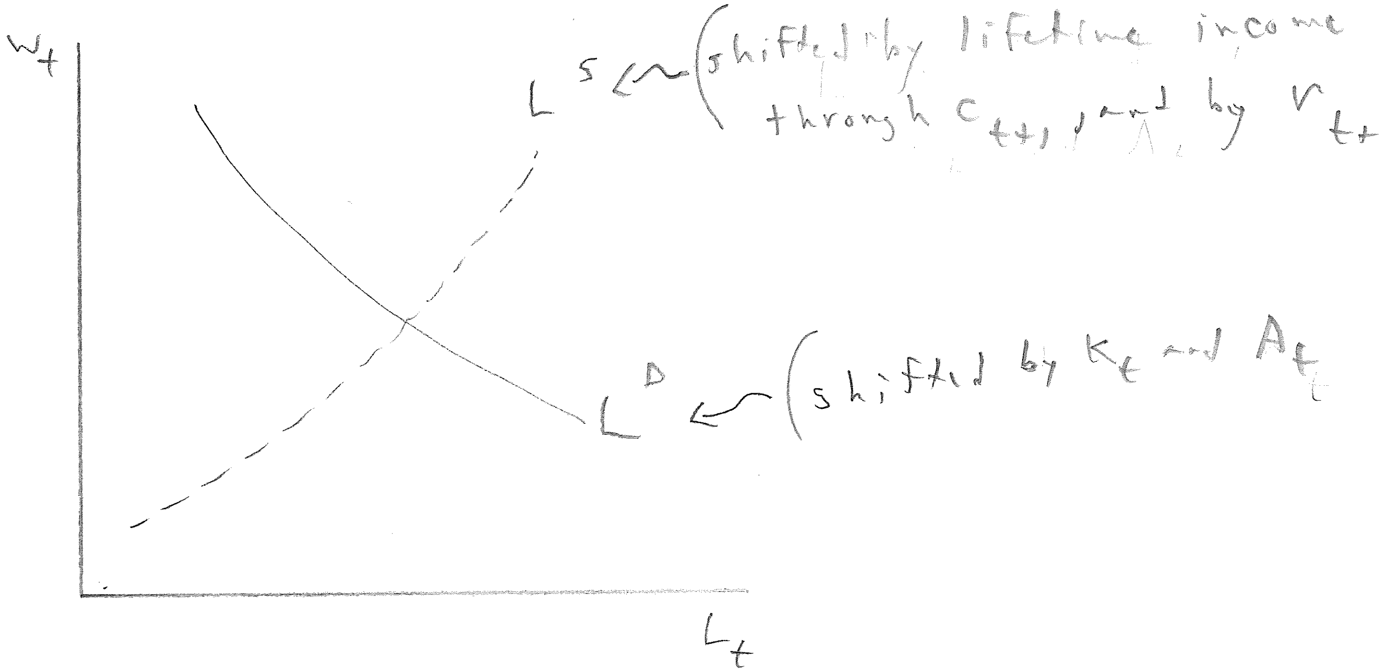
· Recall

$$w = MPL = (1-\alpha)\left(\frac{K}{AL}\right)^{\alpha} A_t$$

hence

$$w_t^D = (1-\alpha)\, K_t^{\alpha} A_t^{1-\alpha} (N \ell^D)^{-\alpha}$$

wage firms are willing to pay for ℓ^D



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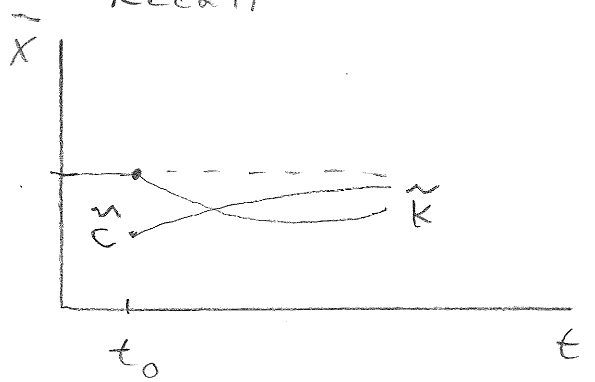
Labor Market (cont.)

Response to $\tilde{G}$ ↑

Recall

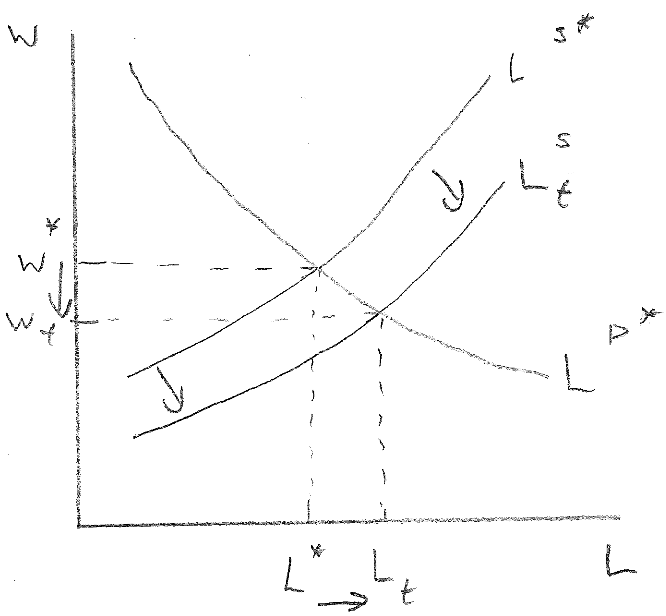


C jumps down, then gradually rises back to C^*.

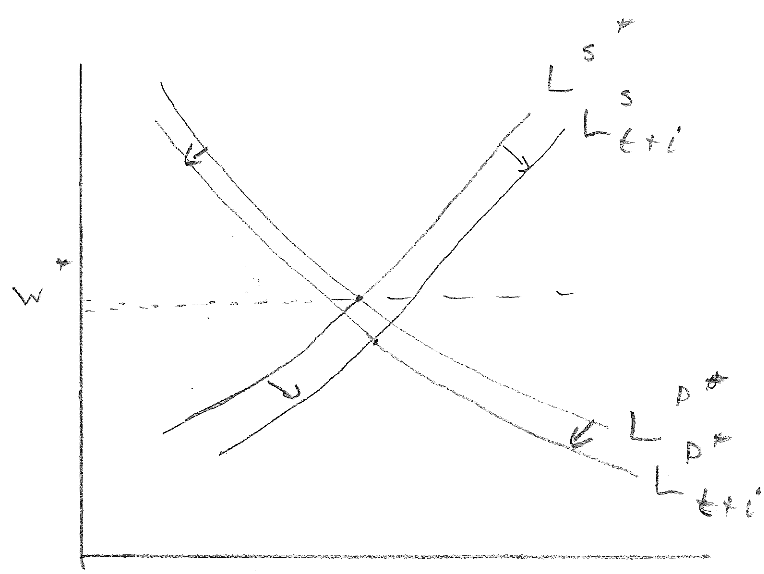
Hence LS curve jumps out, then gradually returns to original position.

Meanwhile, K gradually falls, then rises. Hence LD gradually shifts in, then gradually returns.

Just after $\tilde{G}$ ↑



A bit later on



Note: real wage jumps down, then gradually rises back to W^* as L and Y jump up, then....

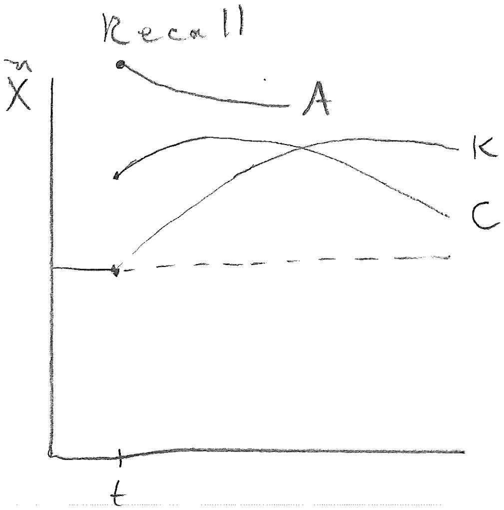
RW is <u>countercyclical</u>

(so is consumption)

RBC Thy

Labor Market (cont.)

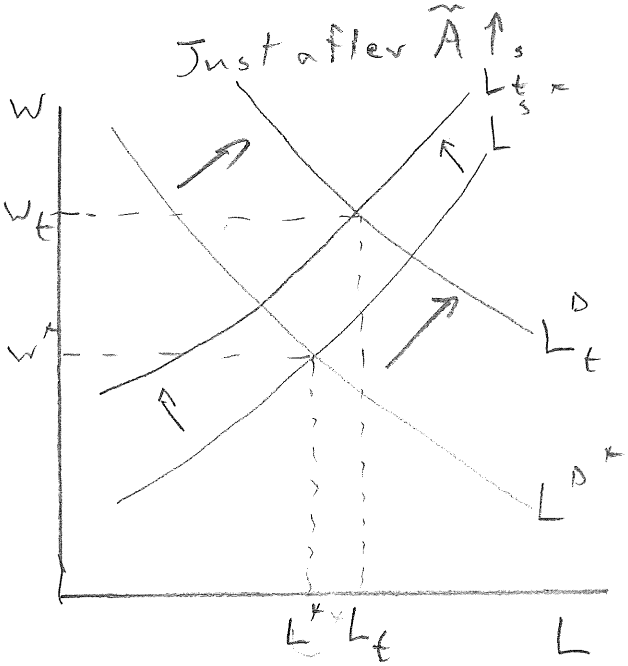
Response to $\tilde{A}\uparrow$



C jumps up, then rises a bit more before falling back.

Hence L^S jumps back, then slides back further before shifting out again.

Meanwhile L^D jumps out & remains out: A returns to LRSS, but K has been building up.



How do we know L^S shift is small enough relative to L^D shift that L rises?

Recall we chose utility function so that L is unaffected by permanent change in A (trend growth): for permanent change, "lifetime income" effect on labor supply (thru C) just balances labor demand / real wage effect.

For an expected-to-be temporary change in A, "lifetime income" effect is smaller, so we know L must rise in response to temporary change in A.

Note: RW is procyclical (so is consumption)