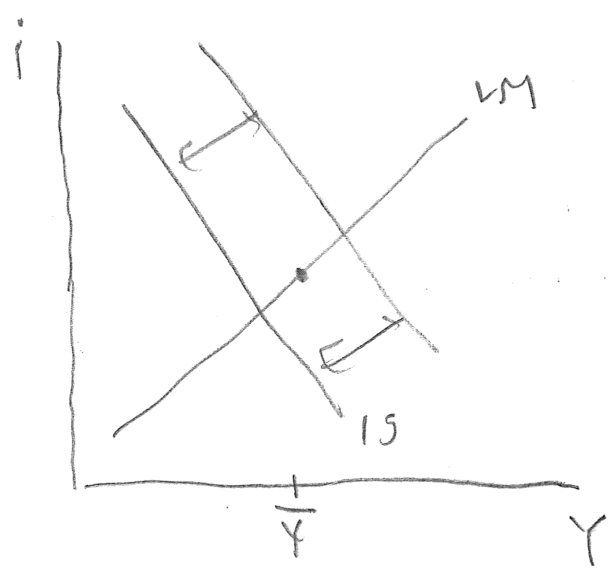
Economics 614, Advanced Macro, Spring 2022 Final examination

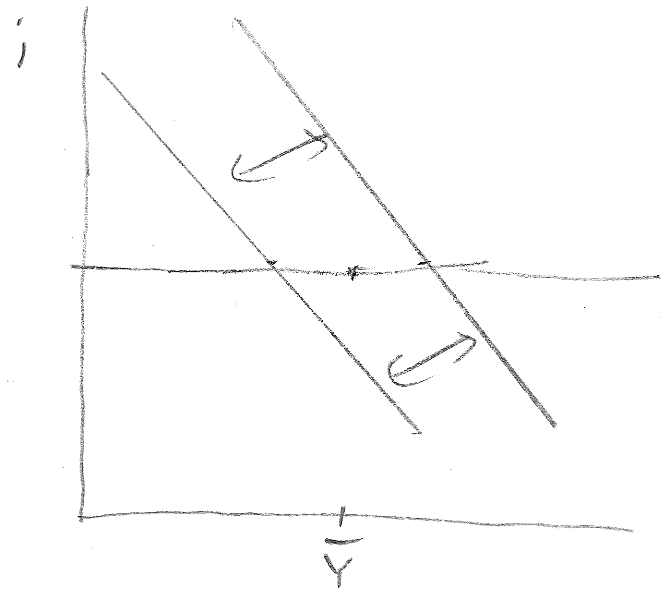
1) Recall the paper of Poole (1970, "Optimal Choice of Monetary Instruments..."). Using "IS/LM" graphs, reproduce the argument of the paper that it is better for a central bank's policy committee to set the money supply M, not the interest rate i, if the policymakers can forecast money demand pretty well but cannot forecast aggregate spending well at all.

10 pts. Extreme case: perfect M^D forecasts. Then:

results of setting M:



results of setting i:



Variance (to be minimized) is bigger at right.

2) Consider two economies, A and B, that have "new Keynesian Phillips curves" of general form $\pi_t = \beta E_t \pi_{t+1} + \lambda y_t$ where $0 < \beta < 1$. In both economies, this equation is derived from the structures assumed for the standard Calvo model (in which long-run steady state inflation is zero). Data show that in economy A the average "contract length" or "lifetime" of a price is one year. In B it is two years. Will the two economies' new Keynesian Phillips curves be different in any way? Explain. 10 pts

In Calvo model, $\beta = \frac{\alpha}{1-\alpha} \phi$ (from $p_i^* = p_t + \phi y_t$)

frequency of price adjustment, or probability ball goes off in a period,

Bigger α means shorter "contract length."

So α, hence β, bigger in A, smaller in B.

3) Consider the following "three-equation" model:

$y_t = E_t y_{t+1} - s r_t$

$\pi_t = E_t \pi_{t+1} + \kappa y_t$

$r_t = \phi \pi_t + u_t$ where $u_t = \rho u_{t-1} + \epsilon_t$ and ϵ is "white noise" (mean-zero i.i.d.) with variance $Var(\epsilon)$

y is the output gap. r is the gap between the real interest rate and the natural rate of interest. Expectations are rational. Assume there is a long-run steady state where $y = 0, \ \pi = 0$.

a) Derive equations that give y_t , π_t and r_t as functions of u_t .

9 pts. See notes

$$y_t = -sa\frac{1}{1-a\rho}u_t = -sa\frac{1}{1-a\rho}u_t \quad \left(\text{where } a = \frac{1}{1+\frac{s\phi\kappa}{1-\rho}} = \frac{1-\rho}{1-\rho+s\phi\kappa} \right.$$

$$= -\frac{s}{\frac{1}{a}-\rho}u_t$$

$$\pi_t = -\frac{1}{\frac{1-\rho}{s\kappa}+\frac{\phi}{1-\rho}}u_t = \kappa\frac{1}{1-\rho}y_t$$

$$r_t = \frac{\frac{1-\rho}{s\kappa\phi}+\frac{\rho}{1-\rho}}{\frac{1-\rho}{s\kappa\phi}+\frac{1}{1-\rho}}u_t$$

(or any other form, of course)

$$= \left(1 - \frac{1}{\frac{1-\rho}{s\kappa}+\frac{\phi}{1-\rho}}\right)u_t$$

b) Based on your answer to a),

i) Consider the correlation between r_t and y_t. Is it positive, negative or zero?

(3) See that $\frac{\partial r}{\partial u} > 0$, $\frac{\partial y}{\partial u} < 0$, so correlation is negative.

ii) Consider the correlation between r_t and y_{t-1} (the lagged output gap). Is it positive, negative or zero?

(3) See that $y_{t-1} = -sa\frac{1}{1-\alpha\rho}u_{t-1}$ & + correlation u_{t-1} & u_t, so correlation between u_{t-1} & r_t is negative.

c) Christiano, Eichenbaum and Evans (2005; "Nominal Rigidities and...") begin their paper with an empirical exercise that estimates the response of y and π to "monetary-policy shocks" in American data. Is the "model" for this question consistent with the "identifying assumptions" that they made for that empirical exercise? Explain.

They assumed r_t could respond to endogenous macro variables simultaneously. That holds here.

But they also assumed r_t affected macro variables only with a lag. That's <u>not</u> true here.

(9)

4) Consider the following model:

$y_t = E_t y_{t+1} - sr_t + u_t$ where $u_t = \rho u_{t-1} + \epsilon_t$ and ϵ is "white noise" (mean-zero i.i.d.) with variance $Var(\epsilon)$.

$\pi_t = E_t \pi_{t+1} + \kappa y_t$

y is the output gap. r is the gap between the real interest rate and the natural rate of interest. Expectations are rational. Assume there is a long-run steady state where $y = 0,\ \pi = 0$.

Every period a central bank chooses r_t to minimize a loss function:

$$L_t = E_t \sum_{\tau=0}^{\infty} \beta^\tau \frac{1}{2}\left(y_{t+\tau}^2 + \pi_{t+\tau}^2\right)$$

At the time the central bank chooses r_t, it does not know u_t but it *does* know u_{t-1} . The central bank has "discretion," that is, its choice of r_t in no way constrains its future behavior.

a) *Derive* the value of r_t that the central bank will choose, and the values of y_t and π_t that will result from the central bank's choice. Explain what you are doing, including any conjectures (provisional assumptions) involved.

10 pts

Recall "discretion" case in CGG. Applies here: minimizing L_t means minimizing $E_t \frac{1}{2}(y_t^2 + \pi_t^2)$.

Recall how we did this in notes: conjecture that $E_t y_{t+1} = 0$, $E_t \pi_{t+1} = 0$, & then confirm conjecture.

So, choose r_t to minimize

$$E_t \tfrac{1}{2}\left((-sr_t + \rho u_{t-1} + \varepsilon_t)^2 + (\kappa(-sr_t + \rho u_{t-1} + \varepsilon_t))^2\right)$$

$$= E_t \tfrac{1}{2}\left((1+\kappa)(-sr_t + \rho u_{t-1} + \varepsilon_t)^2\right)$$

Recall $E[x^2] = (E[x])^2 + Var[x]$

Here $E[\varepsilon_t] = 0$, $Var(-sr_t + \rho u_{t-1} + \varepsilon_t) = Var(\varepsilon)$

So:

$$= \tfrac{1}{2}(1+\kappa)\left((-sr_t + \rho u_{t-1})^2 + Var(\varepsilon)\right)$$

4 pts off for forgetting variance

Take f.o.c.:

$$\frac{\partial}{\partial r_t} = 0 = (1+k)\left(-s r_t + \rho u_{t-1}\right)(-s)$$

so $r_t^* = \frac{1}{s} \rho u_{t-1}$

$$y_t = \varepsilon_t$$

$$\pi_t = k \varepsilon_t$$

Now check conjecture: is $E_t y_{t+1} = E_t \pi_{t+1} = 0$? Yes.

b) Based on your answer to a),

i) Consider the correlation between r_t and y_t. Is it positive, negative or zero?

4 pts

Zero.

ii) Consider the correlation between r_t and y_{t-1} (the lagged output gap). Is it positive, negative or zero?

4 pts

$$y_{t-1} = \varepsilon_{t-1}$$

$$r_t^* = \frac{1}{s} \rho (\rho u_{t-2} + \varepsilon_{t-1})$$

so correlation is positive.

5) Consider a model in which a representative-agent household maximizes $E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\frac{1}{1-\theta} C_{t+\tau}^{1-\theta} - \frac{1}{1+\lambda} L_{t+\tau}^{\;1+\lambda} \right]$

The agent can hold capital K and choose the capital utilization rate u_t. The agent earns a nominal rental rate R for each unit of "effective capital" $(uK)_t$. The agent's nominal wealth is A. At time t, the agent takes as given A_t, the wage W_t, the nominal interest rate i_t and the price level P_t. She chooses consumption, labor, investment I_t (purchases of more capital, or sales in which case I is negative) and u_t, among other things. The price of a unit of capital is always equal to the price of a unit of consumption. The effective-capital rental rate is determined in a competitive market. For a firm, capital is a "variable factor" like labor. The agent's nominal wealth evolves as:

$$A_{t+1} = (1+i_t)[A_t + W_t L_t + R_t u_t K_t - P_t(C_t + I_t + u_t^2 K_t)] + P_{t+1}[I_t + (1-\delta)K_t]$$

The term u^2 represents a current cost of running a unit of capital harder. δ is the depreciation rate (a fixed parameter). Starting from the Bellman equation, derive the value of u_t that the agent will set for period t.

$$V_t = \frac{1}{1-\theta} C_t^{1-\theta} + \frac{1}{1+\lambda} L_{t+T}^{1+\lambda} + \beta E_t V_{t+1}$$

F.O.C.:

$$0 = \beta \frac{\partial E V_{t+1}}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial u_t} = \ldots [(1+i)(R_t K_t - P_t 2 u_t K_t)]$$

gives $u_t = \frac{1}{2}\left(\frac{R}{P}\right)_t$

10 pts

6) Suppose an economy's representative household is infinitely lived and maximizes the expected value of:

$$U_t = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left(\ln C_{t+\tau} - \frac{1}{2}\theta L_{t+\tau}^2 + \frac{1}{1-\sigma}(M/P)_{t+\tau}^{1-\sigma} \right) \qquad where \ 0<\beta<1$$

subject to $Z_{t+1} = \frac{P_t}{P_{t+1}} \left[\frac{M_t}{P_t} + (1+i_t)\left(Z_t - \frac{M_t}{P_t} \right) + W_t L_t - C_t \right]$

where Z_{t+1} is real wealth entering period *(t+1)*. The other variables are as usual. At time *t* the household takes Z_t as given and chooses consumption, labor and real money balances for the period. Assume "certainty equivalence" holds (a household treats expected future values as being equivalent to known future values - this is our usual assumption).

a) Starting from the value function, derive the quantity of real money balance $(M/P)_t$ that a household will choose to hold in a period, as a function of consumption C_t and the nominal interest rate i_t .

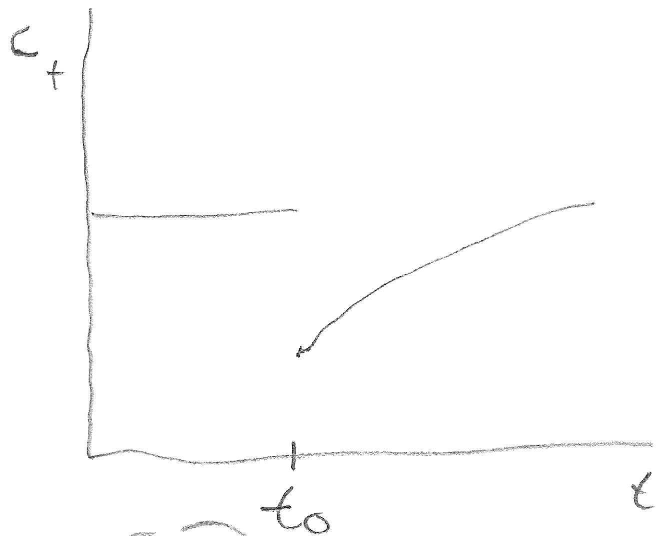
$$V_t = \ln C_t + \frac{1}{2}\theta L_t^2 + \frac{1}{1-\sigma}(M/P)_t^{1-\sigma} + \beta E_t V_{t+1}$$

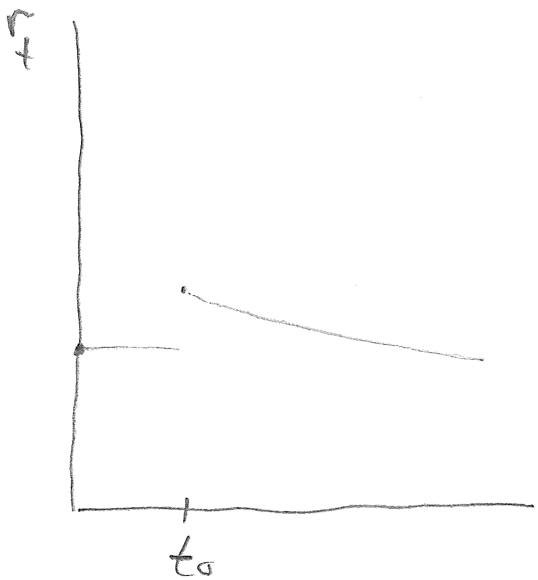
See problem set

$$(M/P)_t = (C_t / i_t)^{1/\sigma}$$

5 pts

b) Suppose this representative household lives in a simple real business cycle model like the one described in Romer's textbook. Just as in that model, there are shocks to government purchases of goods and services, such that $\tilde{G}_t = \rho_G \tilde{G}_{t-1} + \epsilon_{G,t}$ *where* $0 < \rho_G < 1$ and $\epsilon_{G,t}$ is mean-zero i.i.d. To simplify things assume there is *no trend growth* in population or productivity, so steady-state output is stable. Based on what you know about the textbook model, how would consumption and the real interest rate in this economy evolve in response to a *positive* realization of $\epsilon_{G,t}$? Make two graphs to describe this response, one with the real interest rate on the vertical axis, the other with consumption on the vertical axis, and time on the horizontal axis. Use "t_0" to denote the time the shock hits. Note: I am not asking you to derive anything here; just use your memory of the textbook model.

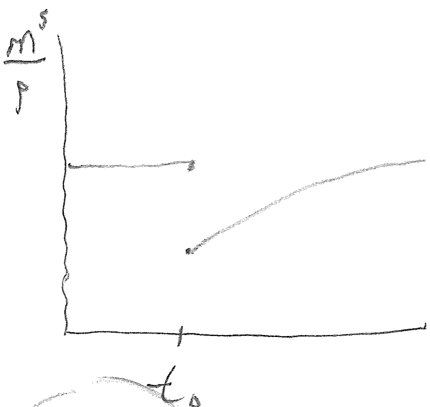


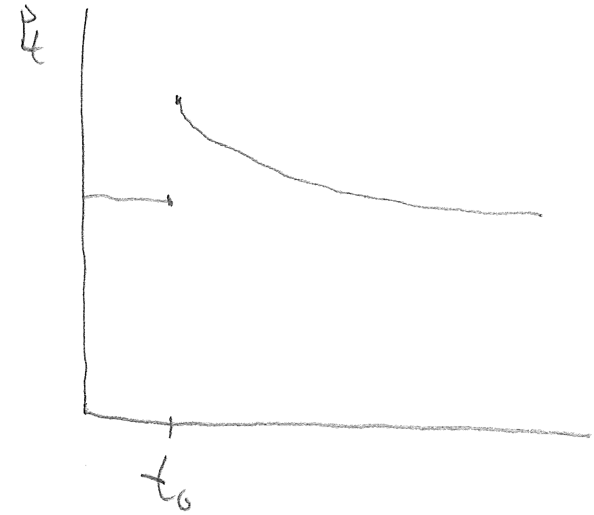


4 pts

c) Now put your answers to a) and b) together. Assume that the money supply M^S is fixed and the price level adjusts as needed to keep real variables equal to the values that come out of the simple RBC model. Also *assume expected inflation is always equal to zero.* On a graph draw the path the price level *P* must follow in response to a positive realization of $\epsilon_{G,t}$. Again use "t_0" to denote the time the shock hits.

If $E\pi = 0$, i follows same path as r. So, in response to ϵ_G, i rises, C falls. Hence money demand falls. Hence Holding M^s fixed,





4 pts

d) I said to assume that expected inflation is always zero. Would this be a *rational* expectation? Explain.

No, Starting at t_0, P is falling, & people can expect this. So starting at t_0, $E\pi$ should be < 0. Note that initial jump in P is not the problem; the jump can be unexpected.

4 pts