Econ 614, Advanced Macro, Spring 2022, Hanes. First Midterm

IMPORTANT! I do not expect you to answer all the questions on this exam. Look over the whole exam and find the questions that you think you know how to answer, or how to partially answer. Answer those questions first.

1) Consider a model like Romer's "baseline" RBC model. To simplify it,
- there are no taxes or government spending
- there is no population growth
- there is no trend growth in total factor productivity.

A representative household has one unit of time which it divides between leisure and work. It maximizes

$$U_t = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left(\ln C_{t+\tau} + b \ln\, (1 - l_{t+\tau}) \right) \qquad where\ \ 0 < \beta < 1\,,\ 0 < b < 1$$

where C is household consumption per household and l is the fraction of household time devoted to labor.

The household saves by holding capital (it can turn one unit of real income into one unit of capital). The capital it will hold in period $t+1$ is $K_{t+1} = Y_t - C_t + (1-\delta)K_t$ where δ is the depreciation rate and Y is (real) household income.

One part of household income is (real) labor income. The household earns a real wage w for every unit of labor supplied, so real labor income in period t is $w_t l_t$.

The other part of household income is income from renting out capital. The "real interest rate" r is equal to the marginal product of capital minus the depreciation rate. Thus (real) income from renting out capital in period t is $(r_t + \delta)K_t$.

a) Write down the value function and the intertemporal budget constraint. **3 pts.**

$$V_t = \max_{K_t} \left[\ln C_t + b \ln(1 - l_t) + E_t V(K_{t+1}) \right]$$

$$\text{s.t. } K_{t+1} = w_t l_t + (r_t + \delta) K_t - C_t + (1-\delta) K_t$$

$$= w_t l_t + (1 + r_t) K_t - C_t$$

b) Using the value function, derive the "intratemporal first-order condition" that relates labor supply l_t to current consumption C_t and the current real wage w_t. **3 pts.**

Note: you know the answer should say

Real Wage · MUC = MU Leisure.

F.O.C. with respect to consumption:

From $K_{t+1} = w_t l_t + \cdots$

$$\frac{\partial V_t}{\partial C_t} = \frac{1}{C_t} + \beta E_t \frac{\partial V(K_{t+1})}{\partial K_{t+1}} \cdot \frac{\partial K_{t+1}}{\partial C_t}$$

$$= \frac{1}{C_t} + \beta E_t \frac{\partial V(K_{t+1})}{\partial K_{t+1}} \cdot (-1)$$

so $\frac{1}{C_t} = \beta E_t \cdots\cdots$

F.O.C. with respect to labor:

$$\frac{\partial V_t}{\partial l_t} = b \frac{1}{1-l_t}(-1) + \beta E_t \frac{\partial V(K_{t+1})}{\partial K_{t+1}} \cdot \frac{\partial K_{t+1}}{\partial l_t}$$

so $\frac{1}{(1-l_t)} \frac{b}{w_t} = \beta E_t \cdots$

put them together, get

$$w_t \cdot \frac{1}{C_t} = \frac{b}{1-l_t}$$

Real wage · MUC = MU Leisure

c) Using the value function, derive current consumption C_t as a function of the representative agent's beliefs about future consumption C_{t+1} and the future real interest rate r_{t+1}. **3 pts.**

To answer this question, you must know what $E_t \frac{\partial V(K_{t+1})}{\partial K_{t+1}}$ *is. This is where the envelope theorem (or Benveniste-Scheinkman condition) comes in. Assuming the agent will optimize in the upcoming period t+1, the utility value of relaxing the intertemporal budget constraint by a marginal bit will be equal to the utility value of spending all of that extra wealth on consumption within t+1, carrying none of it forward into the future period. Thus:*

(holding K_{t+2} fixed)

$$E_t \frac{\partial V(K_{t+1})}{\partial K_{t+1}} = E_t\left[\frac{1}{C_{t+1}} \cdot \frac{\partial C_{t+1}}{\partial K_{t+1}}\right]$$

From $K_{t+2} = W_{t+1} l_{t+1} (1+r_{t+1}) K_{t+1} - C_{t+1}$.

$$\left.\frac{\partial C_{t+1}}{\partial K_{t+1}}\right|_{\partial K_{t+2}=0} = (1+r_{t+1})$$

So

$$E_t \frac{\partial V(K_{t+1})}{\partial K_{t+1}} = E_t\left[\frac{1}{C_{t+1}}(1+r_{t+1})\right]$$

Note that I didn't say "use certainty equivalence."

So from f.o.c. for C_t,

$$\frac{1}{C_t} = \beta E_t[\quad]$$

$$C_t = \frac{1}{\beta} \frac{1}{E_t[\quad]}$$

d) Now consider the "nonstochastic long-run steady state." Assume that there is no trend growth in total factor productivity.

i) What is the steady state real interest rate r^*? **3 pts.**

Using the answer to c), $c^* = \frac{1}{\beta} \frac{1}{\frac{1}{c^*}(1+r^*)}$

so $r^* = \frac{1}{\beta} - 1$

ii) Let C^* denote steady-state consumption per household and w^* denote steady-state real wage. In terms of C^* and w^*, what is steady-state labor supply l^*? **3 pts**

Using the answer to b),

$$w^* \frac{1}{c^*} = \frac{b}{1-l^*}$$

$$l^* = 1 - b \frac{c^*}{w^*}$$

2) Suppose demand for log real money balances is given by $(m-p)^d = y - bi$ where y is log output and i is the nominal interest rate. The money supply m^S is fixed. The natural rate of output has been growing at a low steady rate g_L. Unexpectedly, at time t_0, the natural rate of output begins growing at a higher rate g_H. The natural rate of interest does not change; it remains at the same stable value before and after t_0.

There is one path for the log price level p that will allow output to remain equal to the natural rate at all points in time. Draw this path on a graph. Put time on the horizontal axis of the graph. Mark time t_0 on the horizontal axis. Explain your answer. **9 pts.**

Before t_0, system is solved by $i_0 = r^* + E\pi = r^* + \pi = r^* - g_L$

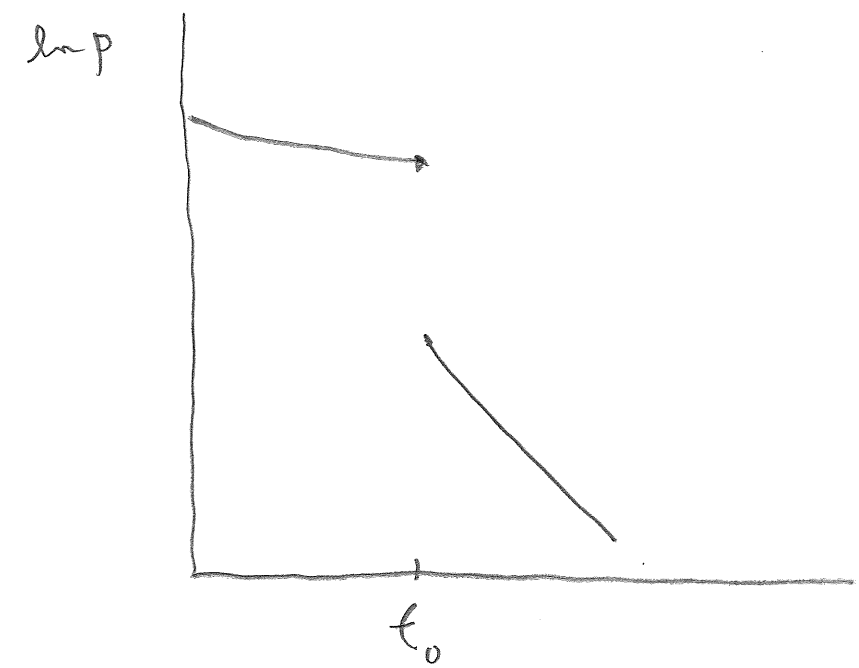
$\pi = -g_L$ to let $(m-p)^d$ grow at g_L

After t_0, system is solved by $i_1 = r^* - g_H < i_0$

$\pi = -g_H$

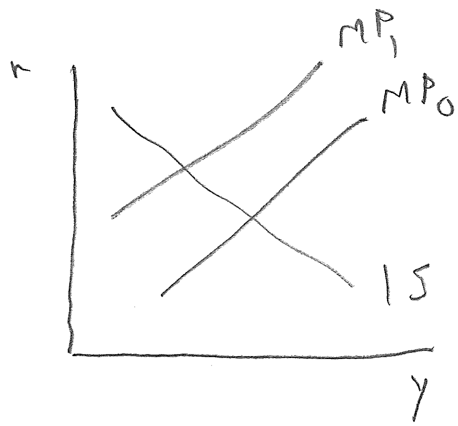
Because $i_1 < i_0$, $(m-p)_1 > (m-p)_0$

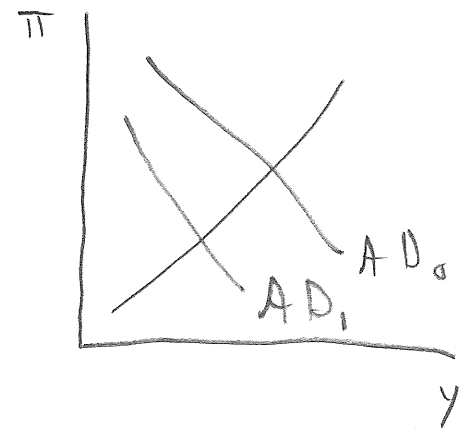
How do you make $(m-p)\uparrow$ at time t_0? $p \downarrow$.



3) Consider an economy where the central bank follows an interest-rate rule $r(\pi, y)+\epsilon$ where $r_{\pi}>0$, $r_y>0$. ϵ represents "shocks" to the central bank's behavior (times when the central bank sets an r different from the one specified by the usual interest-rate rule). "Aggregate supply" can be described by $\pi=\pi^e+\beta y$. Demand for real money balances is given by the usual function $(M/P)^D=L(i,y)$ where $L_i<0$, $L_y>0$. Assume *expected future inflation is always equal to zero*. Consider how the economy will respond to a positive realization of ϵ .

a) Using the appropriate graphs, show what will happen to output, the real interest rate and inflation. Label your graphs clearly. **5 pts.** *Output falls, real interest rate rises, inflation falls.*





b) What must happen to the supply of real money balances $(M/P)^S$ in response to a positive realization of ϵ ? Increase, decrease, no effect? Explain. **5 pts.** *Since output falls and the real interest rate rises, demand for real money balances falls. To keep the real interest rate at the desired value, the supply of real money balances has to fall accordingly.*

4) Consider an economy whose economy can be described by the "Lucas supply function" model. Recall that the expectations-augmented Phillips curve must be $\pi_t=E_{t-1}\pi_t+(1/b)y_t$ where b is the b coefficient we derived in class: $y=b(p-E[p])=\dfrac{b}{1-b}(m-E[m])$.

In the past, the country has been subject to many random, unpredictable money-supply shocks. At time t_0 the country unexpectedly adopts a new monetary regime under which there are fewer unpredictable money-supply shocks. The change in regime is immediately communicated to the public, and understood by them. At time t_0 , does b increase, decrease, or remain the same? Explain. 9 pts. *Fewer unpredictable money-supply shocks means a decrease in V_m . A decrease in V_m means an increase in b (see notes and textbook). Intuitively, when a household knows that unobservable, surprise money-supply shocks are less frequent, it will believe that an observed increase in the price of the thing it sells is more likely to be due to an increase in its relative price than due to a money-supply shock (general increase in the price level), and will respond more strongly - bigger increase in labor supply. That means labor supply and output will respond more strongly to a money-supply shock when it occurs. That means b is bigger.*

5) Suppose the economy is described by the Fischer model with staggering. That is, using information from time *t-1* "odd" pricesetters set their prices for periods *t* and *t+1*. Using information from time *t-2* "even" pricesetters set their prices for periods *t-1* and *t*. And so on. When you solve this model, you find that:

$$y_t = \frac{1}{1+\phi}\left(E_{t-1}m_t - E_{t-2}m_t\right) + (m_t - E_{t-1}m_t)$$

$$p_t = E_{t-2}m_t + \frac{\phi}{1+\phi}\left(E_{t-1}m_t - E_{t-2}m_t\right)$$

Assume that nominal aggregate demand evolves as $m_t = m_{t-1} + g + u_t$ where g is a constant and u is mean-zero i.i.d. The public knows this is the case. Expectations are rational.

Derive y_t and p_t in terms of m_{t-2} and (if relevant) u_{t-1}, u_t and g. **10 pts.**

$$E_{t-1}m_t = m_{t-1} + g + E_{t-1}u_t = m_{t-1} + g = m_{t-2} + u_{t-1} + g$$

$$E_{t-2}m_t = m_{t-2} + g + g + E_{t-2}u_{t-1} + E_{t-2}u_t = m_{t-2} + 2g$$

$$m_t = m_{t-2} + 2g + u_{t-1} + u_t$$

so $E_{t-1}m_t - E_{t-2}m_t = u_{t-1}$

$$m_t - E_{t-1}m_t = u_t$$

$$y_t = \frac{1}{1+\phi}u_{t-1} + u_t$$

$$p_t = m_{t-2} + 2g + \frac{\phi}{1+\phi}u_{t-1}$$

6) Consider a model in which a representative-agent household maximizes

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\theta} C_t^{1-\theta} - \frac{1}{1+\lambda} L_t^{1+\lambda} \right]$$

The agent's nominal wealth A evolves as:

$$A_{t+1} = (1+i_t)[A_t + W_t L_t + R_t u_t K_t - P_t(C_t + I_t)] + P_{t+1}[I_t + (1-\delta_t)K_t]$$

This expression comes from the following assumptions and notation. i is the nominal interest rate. C is consumption. L is labor. The agent can hold capital K and choose the capital utilization rate u. The agent earns a nominal rental rate R for each unit of "effective capital" uK. I is investment, that is purchases of new capital. P is the price level, that is the price of a unit of consumption or a unit of capital. δ is the depreciation rate. Running capital harder raises the depreciation rate: $\delta_t = \theta u_t^2$. Thus the equation for nominal wealth becomes:

$$A_{t+1} = (1+i_t)[A_t + W_t L_t + R_t u_t K_t - P_t(C_t + I_t)] + P_{t+1}[I_t + (1-\theta u_t^2)K_t]$$

At time t, the agent takes as given A_t, R_t, W_t, i_t, the price level P_t and the expected future price level $E_t P_{t+1}$. She chooses C_t, L_t, I_t and capital utilization u_t.

Note: this question is related to the King and Rebelo (1999) model. See notes ("RBC Theory developments", pp. 9-10).

a) Use the Bellman equation and the equation for the evolution of nominal wealth to derive the agent's optimal value of u_t in terms of R_t, i_t, *and* $E_t P_{t+1}$. **8 pts.**

$$V_t = \max\left[\frac{1}{1-\theta} C_t^{1-\theta} - \frac{1}{1+\lambda} L_t^{1+\lambda} + \beta E_t\{V(A_{t+1})\}\right]$$

s.t. $A_{t+1} = \ldots$

F.O.C. with respect to u:

$$0 = \frac{\partial V_t}{\partial u_t} = \beta E_t \frac{\partial V(A_{t+1})}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial u_t}$$

$$= \beta E_t \ldots\ldots \underbrace{\left[(1+i_t) R_t K_t + P_{t+1}(-\theta 2 u_t K_t)\right]}_{\text{so this is zero,}}$$

$$E_t P_{t+1} \theta 2 u_t K_t = (1+i_t) R_t K_t$$

$$u_t = \frac{(1+i_t)}{2\theta} \frac{R_t}{E_t P_{t+1}}$$

b) Starting from your answer to a), put the optimal value for u_t in terms of the real interest rate r using the usual approximation that $(1+r)_t \approx (1+i_t)/(1+E_t\pi_{t+1})$ where π is the inflation rate. Remember that $E_t P_{t+1} = P_t E_t (1+\pi_{t+1})$. **8 pts.**

$$E_t P_{t+1} = P_t (1 + E_t \pi_{t+1})$$

So $$u = \frac{1+i_t}{2\theta} \frac{R_t}{P_t(1+E_t \pi_{t+1})} = \frac{1}{2\theta}\left(\frac{R}{P}\right)_t \frac{1+i_t}{1+E_t \pi_{t+1}}$$

$$= \frac{1}{2\theta}\left(\frac{R}{P}\right)_t (1+r_t)$$

7) In Romer's discussion of the menu cost model we examined in class, he concludes that a menu cost would have to be implausibly large to keep firms from adjusting their prices in response to an aggregate demand shock. Tp put it another way, only an implausibly large menu cost can maintain a "fixed price equilibrium" in the fact of a plausibly large aggregate demand shock. he says "The source of difficulty lies in the labor market." What does he mean when he says that "The source of difficulty lies in the labor market"? **9 pts.**

See textbook p. 279. Don't understand what Romer means? Think of the real-rigidity equation $p_i^* - p = c + \phi y$. *This equation tells you how a firm would like to set its price if there were no menu cost. The magnitude of the coefficient* ϕ *tells you how much a firm would like to cut(raise) its price in response to a recession (boom), assuming other firms hold their prices fixed, if there were no menu cost. If* ϕ *is small enough, then a small menu cost can suffice to maintain a fixed-price equilibrium (because a firm wants to adjust its price just a little anyway). If* ϕ *is big, then the menu cost has to be big to maintain a fixed-price equilibrium (because a firm would really, really like to adjust its price). And what does the value of* ϕ *depend on? The elasticity of labor supply! And why does* ϕ *depend on the elasticity of labor supply? Because, in the model, the labor market is perfectly competitive ("the labor market clears").*