Econ 614 Hanes Another problem set on Clarida, Gali & Gertler

1) Consider the following model:

$y_t = E_t y_{t+1} - r_t$

$\pi_t = E_t \pi_{t+1} + y_t + u_t$ where $u_t = \rho u_{t-1} + \epsilon_t$ and ϵ_t is "white noise" (mean-zero i.i.d.).

y is the output gap. r is the gap between the real interest rate and the natural rate of interest. Expectations are rational. Assume there is a long-run steady state where $y = 0$, $\pi = 0$.

Every period a central bank chooses r_t to minimize a loss function $L_t = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{1}{2}\left(y_{t+\tau}^2 + \pi_{t+\tau}^2\right)$

The central bank has "discretion," that is, its choice of r_t in no way constrains its future behavior.

At the time the central bank chooses r_t, it knows u_t and the public's $E_t \pi_{t+1}$.

a) Solve this model for π_t , y_t and r_t . Hint one: this model is similar to the one in Clarida, Gali and Gertler, "The Science of Monetary Policy: a New Keynesian Perspective." Solve it in the same way.

Because discretion, choice of r_t affects only this period.
Because central bank has no uncertainty about effect of r on y & π, skip step of choosing r. Choose y_t to minimize

$\frac{1}{2}\left(y_t^2 + (y_t + E\pi_{t+1} + u_t)^2\right)$

gives $y_t = -\frac{1}{2}E\pi - \frac{1}{2}u_t$. Put into equation for π_t, gives

$\pi_t = \frac{1}{2}E_t\pi_{t+1} + \frac{1}{2}u_t$ where u_t is AR(1). So solving back from LRSS gives

$\pi_t = \frac{1}{2}\frac{1}{1-\frac{1}{2}\rho}u_t = \frac{1}{2-\rho}u_t$. Now, get y_t.

From $y_t = -\frac{1}{2}E\pi - \frac{1}{2}u_t$,

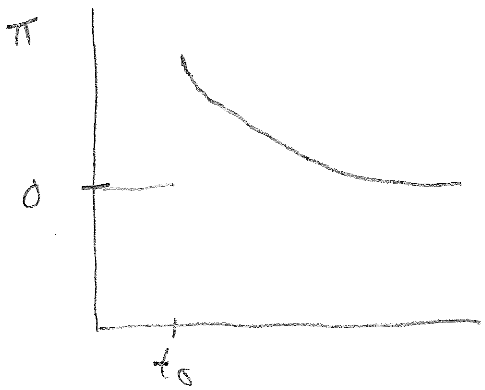
$y_t = -\frac{1}{2}\rho\frac{1}{2-\rho}u_t - \frac{1}{2}u_t = -\frac{1}{2}\left(\frac{\rho}{2-\rho}+1\right)u_t = -\frac{1}{2}\left(\frac{\rho+(2-\rho)}{2-\rho}\right)u_t$

$= -\frac{1}{2-\rho}u_t$

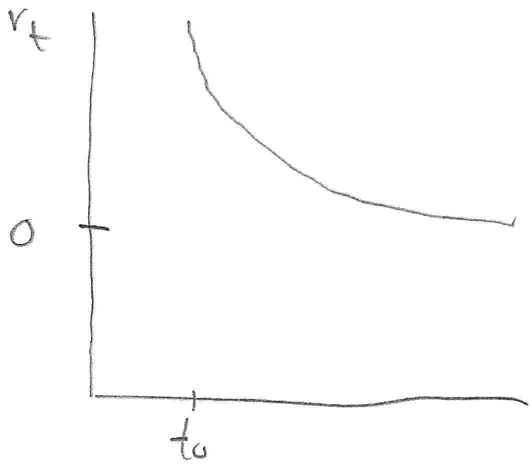
To get r_t, from $y_t = E_t y_{t+1} - r_t$,

$r_t = E_t y_{t+1} - y_t = -\frac{\rho}{2-\rho}u_t + \frac{1}{2-\rho}u_t = \frac{1-\rho}{2-\rho}u_t$

b) Draw a graph with time on the horizontal axis and π on the vertical axis. On the graph draw a line that shows how π_t evolves over time in response to a positive realization of ϵ at time t_0, assuming that just before t_0, the variables y, π and u were all zero.



c) Draw a graph with time on the horizontal axis and r on the vertical axis. On the graph draw a line that shows how r_t evolves over time in response to a positive realization of ϵ at time t_0, assuming that just before t_0, the variables y, π and u were all zero.



d) Would it be possible to reproduce the behavior of this central bank with an "interest rate rule" of the form $r_t = \phi \pi_t$? If your answer is yes, what would be the coefficient ϕ ?

$$\phi = \frac{\partial r / \partial u}{\partial \pi / \partial u} = \frac{\frac{1-\rho}{2-\rho}}{\frac{1}{2-\rho}} = 1-\rho$$

2) Consider the following model:

$y_t = E_t y_{t+1} - r_t + g_t$ where $g_t = \rho g_{t-1} + \epsilon_{gt}$ and ϵ_{gt} is "white noise" (mean-zero i.i.d.).

$\pi_t = E_t \pi_{t+1} + y_t$

y is the output gap. r is the gap between the real interest rate and the natural rate of interest. Expectations are rational. Assume there is a long-run steady state where $y = 0$, $\pi = 0$.

Every period a central bank chooses r_t to minimize a loss function $L_t = E_t \sum_{\tau=0}^{\infty} \beta^\tau \frac{1}{2}\left(y_{t+\tau}^2 + \pi_{t+\tau}^2\right)$

The central bank has "discretion," that is, its choice of r_t in no way constrains its future behavior.

At the time the central bank chooses r_t, it knows g_t and the public's $E_t \pi_{t+1}$.

a) Solve this model for π_t , y_t and r_t. Hint one: this model is similar to the one in Clarida, Gali and Gertler, "The Science of Monetary Policy: a New Keynesian Perspective." Solve it in the same way.

Choose y_t to minimize

$\frac{1}{2}\left(y_t^2 + (y_t + E\pi_{t+1})^2\right)$

gives $y_t = 0$. True for all t, so solving back from LRSS,

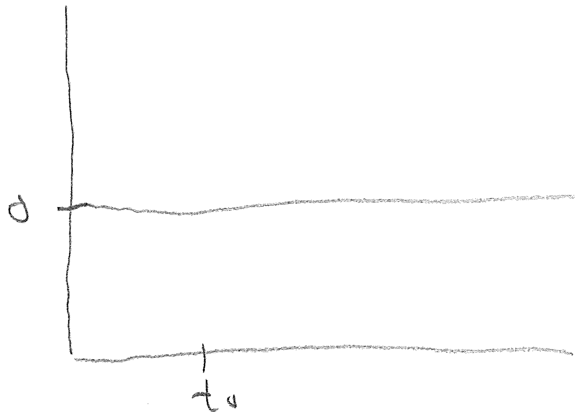
$\pi_t = 0$.

To get r_t, from $y_t = Ey_t - r_t + g_t$,

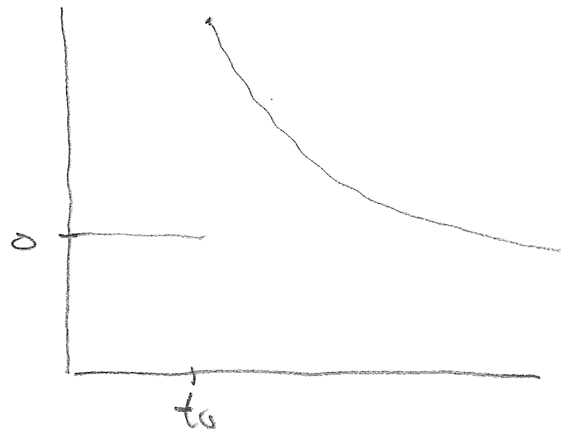
$0 = 0 - r_t + g_t$

$\Rightarrow r_t = g_t$.

b) Draw a graph with time on the horizontal axis and y on the vertical axis. On the graph draw a line that shows how y_t evolves over time in response to a positive realization of ϵ at time t_0, assuming that just before t_0, the variables y, π and g were all zero.



c) Draw a graph with time on the horizontal axis and r on the vertical axis. On the graph draw a line that shows how π_t evolves over time in response to a positive realization of ϵ at time t_0, assuming that just before t_0, the variables y, π and g were all zero.



d) Would it be possible to reproduce the behavior of this central bank with an "interest rate rule" of the form $r_t = \phi y_t$? If your answer is yes, what would be the coefficient ϕ ?

No.