Answers to "Problem set on the New Keynesian Phillips Curve"

1)

a) An expression for π_t as a function of y_t

Start with $\pi_t = \beta \pi^e_{t+1} + \kappa y_t$, work backward from LRSS to get π_t as determined by expected future output gaps, as in notes p. 5.

LRSS means $\pi = 0$, $y = 0$

$$\pi_\infty = 0, \; y_\infty = 0$$

$$\pi_{\infty-1} = 0 + \kappa y_{\infty-1}$$

$$\pi_{\infty-2} = 0 + \beta \kappa y_{\infty-1} + \kappa y_{\infty-2}$$

$$\pi_{\infty-3} = 0 + \beta^2 \kappa y_{\infty-1} + \beta \kappa y_{\infty-2} + \kappa y_{\infty-3}$$

Starting from present & going forward,

$$\pi_t = \kappa y_t + \beta \kappa y_{t+1} + \beta^2 \kappa y_{t+2} + \beta^3 \kappa y_{t+3} + \cdots$$

$$\pi_t = \kappa \sum_{j=0}^{\infty} \beta^j y_{t+j}$$

or, putting in E[ ],

$$\pi_t = \kappa \sum_{j=0}^{\infty} \beta^j E_t[y_{t+j}]$$

Now, what is $E_t[y_{t+j}]$?

Answers... Phillips curve

1)

a) Cont.

$E_t[y_{t+j}]$ comes from $y_t = \rho y_{t-1} + \varepsilon_t$:

$E_t[y_{t+1}] = \rho E_t[y_t] + E_t[\varepsilon_{t+1}]$

$= \rho y_t + 0$

$E_t[y_{t+2}] = \rho E_t[y_{t+1}] + E_t[\varepsilon_{t+1}]$

$= \rho \rho y_t + 0 = \rho^2 y_t$

so $E_t[y_{t+j}] = \rho^j y_t$

so $\pi_t = \kappa \sum_{j=0}^{\infty} \beta^j \rho^j y_t = \kappa \sum_{j=0}^{\infty} (\beta\rho)^j y_t = \kappa \frac{1}{1-\beta\rho} y_t$

b) One would observe a positive relationship between π_t & y_t, which is the original Phillips curve.

Answers... Phillips curve

2)

a)

Here $p_t = \alpha x_t + (1-\alpha)(p_{t-1} + \bar{\pi})$

$= \alpha x_t + p_{t-1} - \alpha p_{t-1} + (1-\alpha)\bar{\pi}$

$p_t - p_{t-1} =$

$\pi_t = \alpha x_t - \alpha p_{t-1} + (1-\alpha)\bar{\pi}$

$x_t = \frac{1}{\alpha}(\pi_t - (1-\alpha)\bar{\pi}) + p_{t-1}$

$E_t p_t$

which means

$E_t x_{t+1} = \frac{1}{\alpha}(E_t \pi_{t+1} - (1-\alpha)\bar{\pi}) + p_t$

b)

$x_t = [1-\beta(1-\alpha)] p_t^* + \beta(1-\alpha) E_t x_{t+1} - \beta(1-\alpha)\bar{\pi}$

Substitute in above stuff:

$\frac{1}{\alpha}(\pi_t - (1-\alpha)\bar{\pi}) + p_{t-1} =$

$[1-\beta(1-\alpha)] p_t^* + \beta(1-\alpha)\left(\frac{1}{\alpha}(E_t \pi_{t+1} - (1-\alpha)\bar{\pi}) + p_{t+1}\right)$

$- \beta(1-\alpha)\bar{\pi}$

we want to get rid of p_t^* & p_{t+1} & get y_t in there.

To do that, subtract p_t from both sides. You would guess this because it's what we did in other derivations

Answers....

2)
b.) Cont.

$$\frac{1}{\alpha}(\pi_t-(1-\alpha)\bar{\pi})+p_{t-1}-p_t$$

$$=[1-\beta(1-\alpha)]p_t^* + \beta(1-\alpha)\frac{1}{\alpha}E_t\pi_{t+1} - \beta(1-\alpha)\frac{1-\alpha}{\alpha}\bar{\pi}$$
$$+\beta(1-\alpha)p_t - \beta(1-\alpha)\bar{\pi} - \underbrace{(1-\beta(1-\alpha)+\beta(1-\alpha))}_{\text{this is one, as in notes}}p_t$$

$$=[1-\beta(1-\alpha)]\overbrace{(p_t^*-p_t)}^{ky_t} + \beta(1-\alpha)\frac{1}{\alpha}E_t\pi_{t+1} - \beta\frac{(1-\alpha)^2}{\alpha}\bar{\pi}$$
$$+\underbrace{(\beta(1-\alpha)-\beta(1-\alpha))}_{\text{zero}}p_t - \beta(1-\alpha)\bar{\pi}$$

$$\frac{1}{\alpha}\pi_t - \frac{1-\alpha}{\alpha}\bar{\pi} - \pi_t = [1-\beta(1-\alpha)]ky_t + \beta\frac{1-\alpha}{\alpha}E_t\pi_{t+1}$$
$$-\left[\beta\frac{(1-\alpha)^2}{\alpha}+\beta(1-\alpha)\right]\bar{\pi}$$

$$\frac{1-\alpha}{\alpha}(\pi_t-\bar{\pi}) = [1-\beta(1-\alpha)]ky_t + \beta\frac{1-\alpha}{\alpha}E_t\pi_{t+1}$$
$$-\beta(1-\alpha)\underbrace{\left[\frac{1-\alpha}{\alpha}+1\right]}_{=\frac{1-\alpha}{\alpha}+\frac{\alpha}{\alpha}=1/\alpha}\bar{\pi}$$

$$\frac{1-\alpha}{\alpha}(\pi_t-\bar{\pi}) = [1-\beta(1-\alpha)]ky_t + \beta\frac{1-\alpha}{\alpha}E_t\pi_{t+1} - \beta\frac{1-\alpha}{\alpha}\bar{\pi}$$

Answers....

2)
(b) cont.

$$\frac{1-\alpha}{\alpha}(\pi_t-\bar{\pi}) = [1-\beta(1-\alpha)]ky_t + \beta\frac{1-\alpha}{\alpha}(E_t\pi_{t+1}-\bar{\pi})$$

$$\pi_t = \bar{\pi} + \frac{\alpha}{1-\alpha}[1-\beta(1-\alpha)]ky_t + \beta(E_t\pi_{t+1}-\bar{\pi})$$

If $\bar{\pi}=0$, is this (7.60)? Yes.

c) Looks like original Phillips curve (as long as $\bar{\pi}$ remains fixed).

Answers... Phillips curve

Where did $x_t = [1-\beta(1-\alpha)]p_t^* + \beta(1-\alpha)E_t x_{t+1} - \beta(1-\alpha)\bar{\pi}$ come from?

1) Write down loss function
2) Take f.o.c., find optimal x_t
3) Define $E_t x_{t+1}$
4) Find $E_t x_{t+1}$ within the expression for x_t

1) Loss function

$$\hat{x}_{t+j} = x_t + j\bar{\pi}$$

$$Z = E\left[\sum_{j=0}^{\infty} \beta^j q_{t+j}\left(\hat{x}_{t+j} - p_{i,t+j}^*\right)^2\right] = E\left[\sum_{j=0}^{\infty} \beta^j (1-\alpha)^j \left(x_t + j\bar{\pi} - p_t^*\right)^2\right]$$

2) Take f.o.c., Find x_t

Ignoring $E[\ ]$ & variance terms,

$$0 = \frac{\partial Z}{\partial x_t} = \beta^0 (1-\alpha)^0 (x_t - p_t^*)$$
$$+ \beta(1-\alpha)(x_t + \bar{\pi} - p_{t+1}^*)$$
$$+ \beta^2 (1-\alpha)^2 (x_t + 2\bar{\pi} - p_{t+2}^*)$$
$$+ \ldots$$

Answers... Phillips curve?

Where did $x_t = \ldots$ come from?

2) ..., Find x_t

$$0 = [\beta^0(1-\alpha)^0 + \beta(1-\alpha) + \beta^2(1-\alpha)^2 + \ldots]x_t$$
$$+ \beta^0(1-\alpha)^0(0 - p_t^*) + \beta^1(1-\alpha)^0(\bar{\pi} - p_t^*) + \beta^2(1-\alpha)^2(2\bar{\pi} - p_t^*) \ldots$$

$$\sum_{j=0}^{\infty} \beta^j (1-\alpha)^j x_t = \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j (p_{t+j}^* - j\bar{\pi})$$

$$\frac{1}{1-\beta(1-\alpha)} x_t = \ldots$$

$$x_t = [1-\beta(1-\alpha)] \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j (p_{t+j}^* - j\bar{\pi})$$

3) Define $E_t x_{t+1}$

$$x_{t+1} = E_{t+1}\left[\sum_{j=0}^{\infty} \beta^j (1-\alpha)^j (p_{t+1+j}^* - j\bar{\pi})\right)$$

$$E_t x_{t+1} = \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j (E_t p_{t+1+j}^* - j\bar{\pi})$$

used "law of iterated projections"

Answers... Phillips curves?

Where did x_t .... come from?

4) Find $E_t x_{t+1}$ within expression for x_t

Can we find $[1-\beta(1-\alpha)]\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j(E_t p^*_{t+1+j}-j\bar{\pi})$ within

$$x_t = [1-\beta(1-\alpha)]\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j(E_t p^*_{t+1+j}-j\bar{\pi}) \; ?$$

Start by pulling out first period of x_t:

$$x_t = [1-\beta(1-\alpha)]\beta^0(1-\alpha)^0 p^*_t + [1-\beta(1-\alpha)]\sum_{j=1}^{\infty}\beta^j(1-\alpha)^j(p^*_{t+j}-j\bar{\pi})$$

Look! $\sum_{j=1}^{\infty}\beta^j(1-\alpha)^j(p^*_{t+j}-j\bar{\pi}) = \beta(1-\alpha)\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j(p^*_{t+1+j}-(j+1)\bar{\pi})$

$$=\beta(1-\alpha)\left(\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j(p^*_{t+1+j}-j\bar{\pi}) + \sum_{j=0}^{\infty}\beta^j(1-\alpha)^j(-\bar{\pi})\right)$$

$$=\beta(1-\alpha)\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j(p^*_{t+1+j}-j\bar{\pi}) - \beta(1-\alpha)\frac{1}{1-\beta(1-\alpha)}\bar{\pi}$$

so

$$x_t = [1-\beta(1-\alpha)]p^*_t + \beta(1-\alpha)\overbrace{[1-\beta(1-\alpha)]\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j(p^*_{t+1+j}-j\bar{\pi})}^{E_t x_{t+1}}$$

$$-\beta(1-\alpha)\underbrace{[1-\beta(1-\alpha)]\frac{1}{1-\beta(1-\alpha)}}_{\text{one}}\bar{\pi}$$

so $x_t = [1-\beta(1-\alpha)p^*_t + \beta(1-\alpha)E_t x_{t+1} - \beta(1-\alpha)\bar{\pi}$