

1) Recall the old-Keynesian Friedman-Phelps expectations-augmented Phillips curve  $\pi_t = E_{t-1} \pi_t + \kappa y_t$ , and also recall the new Keynesian expectations-augmented Phillips curve  $\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$  assuming  $\beta$  is practically equal to one.  $y$  is the output gap. Assuming rational expectations, neither of these equations is consistent with the actual behavior of output and inflation. Explain, using equations. (10 pts)

Friedman-Phelps,  $\pi_t = E_{t-1} \pi_t + \epsilon_t$  where  $\epsilon_t$  is error in expectation.

$$E_{t-1} \pi_t = \pi_t - \epsilon_t \text{ so } \pi_t = \pi_t - \epsilon_t + \kappa y_t. \text{ So } \epsilon_t = \kappa y_t,$$

$y_t = \frac{1}{\kappa} \epsilon_t$ . Rational expectations means  $\epsilon_t$  is uncorrelated with anything observable at time  $(t-1)$ , including  $y_{t-1}$ . So, the equation implies  $y_t$  is uncorrelated with  $y_{t-1}$  - no serial correlation in output gap. False. Also, rational expectations means  $E_t \epsilon_{t+1} = 0$ , so the equation implies  $E_t y_{t+1} = 0$  - expected future output gap is always zero. Also False.

New Keynesian,  $\pi_{t+1} = E_t \pi_{t+1} + \epsilon_{t+1}$  where  $\epsilon_{t+1}$  is error in expectation.  $E_t \pi_{t+1} = \pi_{t+1} - \epsilon_{t+1}$

$$\text{so } \pi_t = \pi_{t+1} - \epsilon_{t+1} + \kappa y_t. \text{ So } \pi_{t+1} - \pi_t = -\kappa y_t + \epsilon_{t+1}$$

Rational expectations means  $\epsilon_{t+1}$  is uncorrelated with  $y_t$ .

So if you regress  $\Delta \pi_{t+1}$  on  $y_t$ , coefficient should be zero. False.

2) Consider a model with a competitive labor market with a market-clearing nominal wage  $W$  per unit of labor. A representative-agent household maximizes

$$E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{1-\theta} C_t^{1-\theta} + \frac{1}{1-\nu} (M_t / P_t)^{1-\nu} - \frac{1}{1+\lambda} L_t^{1+\lambda} \right]$$

The agent's nominal wealth evolves as  $A_{t+1} = M_t + (A_t + W_t L_t - P_t C_t - M_t)(1+i_t)$

At time  $t$ , the agent takes as given  $A_t$ , the wage  $W_t$ , and the price level  $P_t$ , and chooses consumption, labor and his real money balance. Assume "certainty equivalence" holds, so that in the agent's optimization problem you take expected values of future variables to be equivalent to actual known values of future variables.

a) Write down the Bellman equation for the agent's problem.

$$V_t = \text{Max} \left[ \frac{1}{1-\theta} C_t^{1-\theta} + \frac{1}{1-\nu} (M/P)_t - \frac{1}{1+\lambda} L_t^{1+\lambda} + \beta E_t V_{t+1} \right]$$

note  $E_t V_{t+1}$  must be inside the brackets

s.t.  $A_{t+1} = \dots$

b) Derive an equation that gives the quantity of labor the household chooses to supply at time  $t$ ,  $L_t^s$ , as a function of the real wage  $(W/P)_t$ , and consumption  $C_t$ .

$$0 = \frac{\partial V_t}{\partial L_t} = -L_t^{\lambda} + \beta E_t \frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial L_t} = \dots W_t (1+i_t)$$

$$0 = \frac{\partial V_t}{\partial C_t} = C_t^{-\theta} + \beta E_t \frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial C_t} = \dots (- (1+i_t) P_t)$$

$$L_t^{\lambda} \frac{1}{W_t} = \beta E_t \frac{\partial V}{\partial A_{t+1}} (1+i_t) = C_t^{-\theta} \frac{1}{P_t}$$

$$L_t = C_t^{-\frac{\theta}{\lambda}} (W/P)_t^{\frac{1}{\lambda}}$$

c) What is the elasticity of labor supply?

$$\ln L_t = -\frac{\theta}{\lambda} \ln C_t + \frac{1}{\lambda} \ln (W/P)_t$$

$$\frac{\partial \ln L_t}{\partial \ln (W/P)_t} = \frac{1}{\lambda} \text{ elasticity}$$

d) Using the usual approximations and definitions, derive an equation that gives  $C_t$  as a function of  $E_t C_{t+1}$  and the "real interest rate"  $r_t = i_t - E_t \pi_{t+1}$ .

From a),  $0 = \frac{\partial V}{\partial C_t} = C_t^{-\theta} + \beta E_t \frac{\partial V}{\partial C_{t+1}} (- (1+i_t) P_t)$

Envelope theorem (or Benveniste-Scheinkman condition) says

$$\frac{\partial V}{\partial A_{t+1}} = \frac{\partial W}{\partial C_{t+1}} \cdot \frac{\partial C_{t+1}}{\partial A_{t+1}} = C_{t+1}^{-\theta} \frac{1}{P_{t+1}} \quad \text{so}$$

$$0 = C_t^{-\theta} + \beta E_t C_{t+1}^{-\theta} \frac{1}{E_t P_{t+1}} (- (1+i_t) P_t) \quad \text{using certainty equivalence}$$

$$\Rightarrow C_t = E_t C_{t+1} \left[ \beta (1+i_t) \frac{P_t}{E_t P_{t+1}} \right]^{-\frac{1}{\theta}}$$

$$= E_t C_{t+1} \left[ \beta (1+i_t) \frac{1}{1+E_t \pi_{t+1}} \right]^{-\frac{1}{\theta}}$$

Approximation  $\frac{1+i_t}{1+E_t \pi_{t+1}} \approx 1+r_t$

$$\Rightarrow C_t = E_t C_{t+1} \left[ \beta (1+r_t) \right]^{-\frac{1}{\theta}}$$

e) Suppose you draw a labor supply curve for this model in log terms, that is a graph with the log of the real wage  $(w-p)_t$  on the vertical axis and the log quantity of labor per household  $l_t$  on the horizontal axis, assuming that that  $E_t C_{t+1} = \bar{C}$ , where  $\bar{C}$  denotes the long-run steady state level of consumption. What happens to this labor supply curve in as there is an increase in the real interest rate  $r_t$ ? Explain how you know. (5)

$$\text{From } L_t = C_t^{-\frac{\theta}{\lambda}} (w/p)_t^{\frac{1}{\lambda}}$$

$$\text{and } C_t = E_t C_{t+1} [\beta(1+r_t)]^{-\frac{\theta}{\lambda}}$$

$$\text{we have } L_t = \left( \bar{C}, \beta(1+r_t) \right)^{-\frac{\theta}{\lambda}} (w/p)_t^{\frac{1}{\lambda}}$$

$$= \bar{C}^{-\frac{\theta}{\lambda}} \beta^{-\frac{\theta}{\lambda}} (1+r_t)^{\frac{\theta}{\lambda}} (w/p)_t^{\frac{1}{\lambda}}$$

See that a decrease in  $C_t$  increase  $L^S$  at any given  $w/p$ . That means out/down shift in  $L^S$  curve

Take logs

$$l_t = -\frac{\theta}{\lambda} \bar{C} - \frac{1}{\lambda} \ln \beta + \frac{1}{\lambda} \ln(1+r_t) + \frac{1}{\lambda} (w-p)_t$$

Approximation  $\ln(1+r_t) \approx r_t$

$$l_t = -\frac{\theta}{\lambda} \bar{C} - \frac{1}{\lambda} \ln \beta + \frac{1}{\lambda} r_t + \frac{1}{\lambda} (w-p)_t$$

Equation of labor supply curve:

$$\frac{1}{\lambda} (w-p)_t = l_t - \frac{\theta}{\lambda} \bar{C} - \frac{1}{\lambda} \ln \beta - \frac{1}{\lambda} r_t$$

$$(w-p)_t = \lambda l_t - \theta \bar{C} - \ln \beta - r_t$$

So  $r_t \uparrow$  shifts  $L^S$  curve down (out).

f) Consider drawing a new Keynesian IS curve for this model with the real interest rate on the vertical axis and the log of output on the horizontal axis, by assuming that output  $Y_t$  is equal to consumption  $C_t$  and  $E_t C_{t+1} = \bar{C}$ . Suppose there are two economies described by this model. In economy A, the representative agent is extremely risk averse. In economy B, the representative agent is less risk-averse. Is this IS curve steeper for economy A, steeper for economy B, or do both curves have the same slope? Explain. (5)

With  $Y_t = C_t$  -  $\frac{1}{\theta}$

$$Y_t = E_t C_{t+1} [\beta(1+r_t)]$$

With  $E_t C_{t+1} = \bar{C}$ , -  $\frac{1}{\theta}$

$$Y_t = \bar{Y} [\beta(1+r_t)]$$

Log's:

$$\ln Y_t = \ln \bar{Y} - \frac{1}{\theta} \ln \beta - \frac{1}{\theta} \ln(1+r_t)$$

with  $\ln(1+r_t) \approx r_t$ ,

$$\ln Y_t = \ln \bar{Y} - \frac{1}{\theta} \ln \beta + \frac{1}{\theta} r_t$$

Equation of IS curve:

$$r_t = \theta \ln \bar{Y} - \theta \ln Y_t - \ln \beta$$

Slope is  $-\theta$ .

More risk averse means  $\theta$  bigger (MU of C diminishes faster) so IS curve is steeper in economy A.

3) Consider the following "three-equation" model:

(i)  $y_t = E_t y_{t+1} - r_t$

(ii)  $\pi_t = E_t \pi_{t+1} + y_t + u_t$  where  $u_t = \rho u_{t-1} + \epsilon_t$  and  $\epsilon_t$  is mean-zero i.i.d.

(iii)  $r_t = \phi \pi_t$

where  $y$  is the output gap and  $r$  is the gap between the real interest rate and the natural rate of interest.

Expectations are rational.

Assume there is a long-run steady state where  $y = 0$ ,  $\pi = 0$ .

a) Derive equations that give  $\pi_t$  and  $y_t$  as functions of  $u_t$ .

(10)

See notes, "effect of  $u$ "

Conjecture  $E_t \pi_{t+1} = \rho \pi_t$

$y_t = E_t y_{t+1} - r_t = E_t y_{t+1} - \phi \pi_t$

Apply math trick

$y_t = -\frac{\phi}{1-\rho\pi} \pi_t$

Substitute into (ii)

$\pi_t = E_t \pi_{t+1} - \frac{\phi}{1-\rho\pi} \pi_t + u_t$

$(1 + \frac{\phi}{1-\rho\pi}) \pi_t = E_t \pi_{t+1} + u_t$

$\pi_t = a E_t \pi_{t+1} + a u_t$  where  $a = \frac{1}{1 + \frac{\phi}{1-\rho\pi}}$

Apply math trick.

$\pi_t = \frac{a}{1 - a\rho\pi} u_t$  See that conjecture was correct, and  $\rho\pi = \rho$

$\pi_t = \frac{1}{\frac{1}{a} - \rho} = \frac{1}{1 + \frac{\phi}{1-\rho} - \rho} u_t$

$y_t = -\frac{\phi}{1-\rho} \frac{1}{1-\rho + \frac{\phi}{1-\rho}} u_t = -\frac{\phi}{(1-\rho)^2 + \phi} = -\frac{1}{\frac{(1-\rho)^2}{\phi} + \frac{1}{1-\rho}}$

b) Derive an equation that gives the nominal interest rate  $i_t$  as a function of  $u_t$  and the natural rate of interest  $\bar{r}_t$ .

(5)

$$\begin{aligned} \tilde{i}_t &= \bar{r}_t + v_t + E_t \pi_{t+1} = \bar{r}_t + \phi \pi_t + E_t \pi_{t+1} \\ &= \bar{r}_t + \phi \frac{1}{1-\rho + \frac{\delta}{1-\rho}} u_t + \frac{1}{1-\rho + \frac{\delta}{1-\rho}} E_t u_{t+1} \end{aligned}$$

$$E_t u_{t+1} = \rho u_t \quad \text{so}$$

$$\begin{aligned} \tilde{i}_t &= \bar{r}_t + \frac{\phi}{1-\rho + \frac{\delta}{1-\rho}} u_t + \frac{1}{1-\rho + \frac{\delta}{1-\rho}} \rho u_t \\ &= \bar{r}_t + \frac{\phi + \rho}{1-\rho + \frac{\delta}{1-\rho}} u_t \end{aligned}$$

c) Suppose you had time-series data from this economy. You regress inflation  $\pi_t$  (left-hand side) on output  $y_t$  (right-hand side). Would the estimated coefficient be positive, negative or zero? (3)

$$\text{From a), see that } \frac{\partial \pi_t}{\partial u_t} > 0, \quad \frac{\partial y_t}{\partial u_t} < 0,$$

$$\text{so } \frac{\partial \pi_t}{\partial y_t} < 0,$$

So estimated coefficient is negative.

d) Consider an alternative model in which (i) and (ii) hold, but (iii) does not. Instead of (iii), there is a central bank that chooses  $r_t$  to minimize:

$$L = \sum_{\tau=0}^{\infty} \beta^{\tau} E \left[ \frac{1}{2} y_{t+\tau}^2 + \frac{1}{2} \pi_{t+\tau}^2 \right]$$

after observing  $u_t$ ,  $E_t \pi_{t+1}$  and  $E_t y_{t+1}$ . Derive equations that give  $\pi_t$  and  $y_t$  as functions of  $u_t$ . Hint: remember Clarida, Gali and Gertler, "The Science of Monetary Policy: A New Keynesian Perspective," and note two things. First, the central bank has "discretion" in setting  $r_t$ . Second, the central bank knows with certainty the values of  $y_t$  and  $\pi_t$  that will result from any given  $r_t$ , because it knows  $u_t$ ,  $E_t \pi_{t+1}$  and  $E_t y_{t+1}$ .

I expected you to remember that under discretion minimizing that loss function is equivalent to minimizing component for current period, and that choosing  $r_t$  with certainty about resulting  $\pi$  &  $y$  is equivalent to choosing  $y$  or  $\pi$ . I'll take f.o.c. with respect to  $y$ , following CGG.

$$L = \frac{1}{2} y^2 + \frac{1}{2} (\pi^e + y + u)$$

$$0 = \frac{\partial L}{\partial y} = \frac{1}{2} 2y + \frac{1}{2} 2(\pi^e + y + u) \cdot 1 = y + \pi^e + y + u$$

$$y = -\frac{1}{2} (u + \pi^e)$$

$$\pi = \pi^e - \frac{1}{2} (u + \pi^e) + u = \frac{1}{2} \pi^e + \frac{1}{2} u$$

Look! Equation for  $\pi$  can be solved by math trick!  
 Math trick: soln. to  $x_t = \alpha x_{t+1} + \eta u_t$  is  $x_t = \eta \frac{1}{1-\alpha\rho} u_t$

so here

$$\pi_t = \frac{1}{2} \frac{1}{1-\frac{1}{2}\rho} u_t = \frac{1}{2-\rho} u_t$$

Or you can think about solving back from  $t \rightarrow \infty$ .

$$\pi_{\infty-1} = \frac{1}{2} u_{\infty-1}$$

$$\pi_{\infty-2} = \frac{1}{2} \cdot \frac{1}{2} u_{\infty-1} + \frac{1}{2} u_{\infty-2}$$

$$\pi_{\infty-3} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} u_{\infty-1} + \frac{1}{2} \cdot \frac{1}{2} u_{\infty-2} + \frac{1}{2} u_{\infty-3}$$



[3] d) cont.

$$\text{so } \pi_t = \frac{1}{2} \sum_{\tau=0}^{\infty} \frac{1}{2}^{\tau} u_{t+\tau} = \frac{1}{2} \sum_{\tau=0}^{\infty} \frac{1}{2}^{\tau} \rho^{\tau} u_t = \frac{1}{2} \frac{1}{1 - \frac{1}{2}\rho} u_t$$

What about  $\gamma_t$ ?

$$\gamma_t = -\frac{1}{2} (u_t + \pi_{t+1}^e) = -\frac{1}{2} u_t + \frac{1}{2} \frac{1}{2-\rho} u_{t+1} = -\frac{1}{2} u_t - \frac{1}{2-\rho} \rho u_t$$

$$\gamma_t = -\left(\frac{1}{2} + \frac{\rho}{2-\rho}\right) u_t$$

e) Consider the economy described by part d). Suppose you had time-series data from this economy. You regress inflation  $\pi_t$  (left-hand side) on output  $y_t$  (right-hand side). Would the estimated coefficient be positive, negative or zero?

See that  $\frac{\partial \pi_t}{\partial u_t} > 0$ ,  $\frac{\partial \gamma_t}{\partial u_t} < 0$ , so  $\frac{\partial \pi_t}{\partial \gamma_t} < 0$ .

So estimated coefficient is negative.

3

4) In Romer's textbook static (one-period) model of imperfect competition, each household  $i$  operates a monopoly firm and supplies labor to a perfectly competitive labor market. A household-firm does not use its own labor in production, but instead hires labor from the perfectly competitive labor market at a market-clearing nominal wage  $W$  per unit of labor. Each household acts to maximize:

$U_i = C_i - \frac{1}{\gamma} L_i$  where  $C_i$  is a function of the household's consumption of individual goods. Each household-firm's

production function is  $Y_i = H_i^\gamma$ , where  $H$  is the number of labor units the household-firm hires from the labor market.

Demand for the good produced by a household-firm is  $Y_i^D = (P_i/P)^{-\gamma} Y$  where  $P$  is the price level and  $Y$  is average real income or real GDP per household.

a) From profit-maximization on the part of the firm, derive an equation that gives an individual firm's profit-maximizing price  $P_i^*$  as a function of the market nominal wage  $W$ . (5)

$$\begin{aligned} \text{Max profit } R &= P_i Y_i - W L_i = P_i (P_i/P)^{-\gamma} Y - W (P_i/P)^{-\gamma} Y \\ R &= P_i^{1-\gamma} P^\gamma Y - W P_i^{-\gamma} P^\gamma Y = P^\gamma Y (P_i^{1-\gamma} - W P_i^{-\gamma}) \\ 0 &= \frac{\partial R}{\partial P_i} = P^\gamma Y ((1-\gamma) P_i^{-\gamma} - W(-\gamma) P_i^{-\gamma-1}) \\ (\gamma-1) P_i^{-\gamma} &= \gamma W P_i^{-\gamma-1} \Rightarrow (\gamma-1) = \gamma P_i^{-1} W \Rightarrow P_i = \frac{\gamma}{\gamma-1} W \\ &= \frac{1}{1-1/\gamma} W \end{aligned}$$

b) From utility-maximization on the part of the household, derive an equation that gives labor supply per household  $L_i^S$  as a function of the real wage  $W/P$ . (5)

$$\begin{aligned} \text{Max } U_i &= C_i - \frac{1}{\gamma} L_i = \frac{W}{P} L_i - \frac{1}{\gamma} L_i^\gamma \\ 0 &= \frac{\partial U}{\partial L_i} = \frac{W}{P} - L_i^{\gamma-1} \\ \Rightarrow L_i &= (W/P)^{\frac{1}{\gamma-1}} \end{aligned}$$

c) From your answers to a) and b), derive what I called the "real rigidity equation," which gives the log of a firm's profit-maximizing price as a function of the log price level and the log of real GDP. (5)

$$\begin{aligned} Y_i &= L_i \text{ so } Y = (W/P)^{\frac{1}{\gamma-1}} \text{ so } W = P Y^{\gamma-1} \\ P_i &= \frac{\gamma}{\gamma-1} W = \frac{\gamma}{\gamma-1} P Y^{\gamma-1} \end{aligned}$$

Take logs

$$P_i = \ln\left(\frac{\gamma}{\gamma-1}\right) + p + (\gamma-1) y$$

5) Consider the "expectations-augmented Phillips curve." For each of the following models, write down the corresponding expectations-augmented Phillips curve, and state the key assumptions of the model (the assumptions that differ from the assumptions of the other models).

- i) Lucas supply function model.
- ii) Fischer model.
- iii) Taylor model.
- iv) Rotemberg model.

i)  $\pi_t = E_{t-1} \pi_t + s y_t$  (4)

Perfect competition, Producer-households can see price of good they sell, Can't see prices of goods they buy (price level).

ii) Fischer (4) sorry! We didn't get to Fischer model Phillips curve. (It's  $\pi_t = E_{t-1} \pi_t + s y_t$ ).

Firm sets price for two periods, can set different prices for the two periods, Staggering.

iii)  $\pi_t = \frac{1}{2} (E_{t-1} \pi_t + E_t \pi_{t+1}) + s (y'_{t, even})$  (4)

Firm sets price for two periods, same price for both staggering.

iv)  $\pi_t = E_t \pi_{t+1} + s y_t$  (4)

Firm sets price subject to menu cost that increases with the amount of the change in price.

6) Consider a new Keynesian model in which prices are "indexed" to the long-run steady-state inflation rate  $\bar{\pi}$ . The fixed probability that a firm can fully reoptimize is price in a period is  $\alpha$ . Derive an equation that gives  $\pi_t$  in terms of  $x_t$ ,  $p_{t-1}$  and  $\bar{\pi}$ . (10)

$$p_t = \alpha x_t + (1 - \alpha) (\bar{\pi} + p_{t-1})$$

$$= \alpha x_t + (1 - \alpha) \bar{\pi} + p_{t-1} - \alpha p_{t-1}$$

$$p_t - p_{t-1} = \alpha x_t + (1 - \alpha) \bar{\pi} - \alpha p_{t-1}$$

$$\pi_t = (1 - \alpha) \bar{\pi} + \alpha (x_t - p_{t-1})$$

7) Consider a model in which a representative-agent household maximizes  $E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[ \frac{1}{1-\theta} C_{t+\tau}^{1-\theta} - \frac{1}{1+\lambda} L_{t+\tau}^{1+\lambda} \right]$

The agent can hold capital  $K$  and choose the capital utilization rate  $u_t$ . The agent earns a nominal rental rate  $R$  for each unit of "effective capital" ( $uK$ ). The agent's nominal wealth is  $A$ . At time  $t$ , the agent takes as given  $A_t$ , the wage  $W_t$ , the nominal interest rate  $i_t$  and the price level  $P_t$ . She chooses consumption, labor, investment  $I_t$  (purchases of more capital, or sales in which case  $I$  is negative) and  $u_t$ , among other things. The price of a unit of capital is always equal to the price of a unit of consumption. The effective-capital rental rate is determined in a competitive market. For a firm, capital is a "variable factor" like labor.

a) Suppose that the agent's nominal wealth evolves as:

$$A_{t+1} = (1+i_t)[A_t + W_t L_t + R_t u_t K_t - P_t(C_t + I_t + u_t^2 K_t)] + P_{t+1}[I_t + (1-\delta)K_t]$$

The term  $u^2$  represents a current cost of running a unit of capital harder.  $\delta$  is the depreciation rate (a fixed parameter). Starting from the Bellman equation, derive the value of  $u_t$  that the agent will set for period  $t$ .

$$V_t = \text{Max} \left[ \frac{1}{1-\theta} C_t^{1-\theta} - \frac{1}{1+\lambda} L_t^{1+\lambda} + \beta E_t V_{t+1} \right] \text{ s.t. } A_{t+1} = \dots$$

$$0 = \frac{\partial V_t}{\partial u_t} = \beta \frac{\partial E V_{t+1}}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial u_t}$$

$$= \beta \frac{\partial E V_{t+1}}{\partial A_{t+1}} \left( (1+i_t) [R_t K_t - P_t 2 u_t K_t] \right)$$

$$0 = R_t K_t - P_t 2 u_t K_t$$

$$u_t = \frac{1}{2} (R/P)_t$$

b) Now suppose that the agent's nominal wealth evolves as:

$$A_{t+1} = (1+i_t)[A_t + W_t L_t + R_t u_t K_t - P_t(C_t + I_t)] + P_{t+1}[I_t + (1 - \theta u_t^2)K_t]$$

Here there is no current cost of running capital harder, but the depreciation rate increases with  $u_t$ .

Starting from the Bellman equation, derive the value of  $u_t$  that the agent will set for period  $t$  in terms of the real interest rate  $r$ , using the usual approximation that  $(1+r)_t \approx (1+i_t)/(1+E_t\pi_{t+1})$  where  $\pi$  is the inflation rate. (5)

$$V_t = \dots$$

$$0 = \frac{\partial V_t}{\partial u_t} = \beta \frac{\partial E V_{t+1}}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial u_t}$$

$$0 = \beta \frac{\partial E V_{t+1}}{\partial A_{t+1}} \left( (1+i_t) R_t K_t + P_{t+1} (-2\theta u_t^2) K_t \right)$$

$$(1+i_t) R_t K_t = P_{t+1} 2 u_t^2 \theta K_t$$

$$u_t = \frac{1}{2\theta} \frac{1}{P_{t+1}} (1+i_t) R_t$$

Want to get it in terms of  $r_t$  where  $1+r_t = \frac{1+i_t}{1+\pi_{t+1}}$

I see  $P_{t+1}$  in there.  $P_{t+1} = P_t (1+\pi_{t+1})$

$$\text{So } u_t = \frac{1}{2\theta} \frac{1}{P_t (1+\pi_{t+1})} (1+i_t) R_t = \frac{1}{2\theta} \frac{1+i_t}{1+\pi_{t+1}} \frac{R_t}{P_t}$$

$$u_t = \frac{1}{2\theta} \left( \frac{R}{P} \right)_t (1+r_t)$$

c) In the model of King and Rebelo, "Resuscitating Real Business Cycles," capital utilization is variable.

i) Which of the two cases above does the model resemble?

ii) Why did the authors include this feature in their model? What purpose did it serve?

(5)

i) Case b, because cost of  $u_t \uparrow$  is greater depreciation.

ii) It allowed for cyclical downturn ( $Y \downarrow$ ) to occur without an absolute decrease in TFP.

When productivity growth is below trend, not necessarily negative, then  $r < \bar{r}$ , reducing  $u_t$ , which reduces labor demand etc.

d) In the model of Christiano, Eichenbaum and Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," capital utilization is variable.

i) Which of the two cases above does the model resemble?

ii) Why did the authors include this feature in their model? What purpose did it serve?

(5)

i) Case a, because cost of  $u_t \uparrow$  is deduction from current output.

ii) In effect, it makes for greater real rigidity, so that inflation is less procyclical.

When output falls and demand for capital services falls, supply of capital services falls too, so  $(r/P)_t$  falls less than it would if supply of capital services were fixed.