Economics 614, Advanced Macro, Spring 2020 Midterm examination

Before you begin to answer a question, read *all* of the question.

1) Consider two economies that can be described by Romer's baseline RBC model, where the deviation from trend in the log of government purchases of output is $\tilde{G}_t = \rho_G \tilde{G}_{t-1} + \epsilon_{G,t}$. In economy I, $\rho_G = 0.1$. In economy II, $\rho_G = 0.5$. Otherwise the economies are identical.
Think about the immediate response of output to a government spending shock (a positive realization of ϵ_t), that is, the response within the period that the shock hits. Suppose a positive shock of exactly the same size hits both economies.
10 pts total.

Note that the immediate change in G (that is, the value of $\tilde{G}$ within the period that the shock hits) is the SAME in both economies.

a) Is the immediate increase in output bigger in economy I, in economy II, or the same size in both economies? Explain.

See the textbook p. 216, which says that the effect of $\tilde{G}$ on employment (a_{LG}) is smaller if ρ_G is smaller. Since a shock to G does not affect productivity, and has no immediate effect on the capital stock, this means the effect on output is smaller too. Reason: see discussion on p. 214 about ρ_A and transfer logic to G.

b) Is the immediate decrease in the real wage bigger in economy I, in economy II, or the same size in both economies? Explain.

In economy II, since the effect on employment is smaller, the decrease in the real wage must be smaller, because there will be a smaller decrease in the marginal product of labor.

2) Consider a model with a competitive labor market with a market-clearing nominal wage W per unit of labor. A representative-agent household maximizes

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\theta} C_t^{1-\theta} + \frac{1}{1-\nu} (M_t / P_t)^{1-\nu} - \frac{1}{1+\lambda} L_t^{1+\lambda} \right]$$

The agent's nominal wealth evolves as $A_{t+1} = M_t + (A_t + W_t L_t - P_t C_t - M_t)(1+i_t)$
At time t, the agent takes as given A_t , the wage W_t and the price level P_t , and chooses consumption, labor and his real money balance. Assume "certainty equivalence" holds, so that in the agent's optimization problem you take expected values of future variables to be equivalent to actual known values of future variables.

Look at problem set: "problem on money demand and consumption." Note that here I have defined the budget constraint differently.

a) Write down the Bellman equation for the agent's problem. *2 pts.*

$$V_t = \max \left[\frac{1}{1-\theta} C_t^{1-\theta} + \frac{1}{1-\nu} (M/P)_t^{1-\nu} - \frac{1}{1+\lambda} L_t^{1+\lambda} + \beta E_t V_{t+1} \right]$$

s.t. $A_{t+1} = \ldots$

b) Derive an equation that gives the quantity of labor the household chooses to supply at time t, L_t^S, as a function of the real wage $(W/P)_t$ and consumption C_t. *3 pts.*

$$0 = \frac{\partial V_t}{\partial L_t} = -L_t^{\lambda} + \beta E_t \frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial L_t} \qquad = W_t(1+i_t)$$

$$0 = \frac{\partial V_t}{\partial L_t} = C_t^{-\theta} + \beta E_t \frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial C_t} \qquad = -(1+i_t)P_t$$

$$\text{so} \quad L_t^{\lambda} \frac{1}{W_t} = \beta E_t \frac{\partial V}{\partial A_{t+1}} (1+i_t) = C_t^{-\theta} \frac{1}{P_t}$$

solve for L

$$L_t = C_t^{-\frac{\theta}{\lambda}} (W/P)_t^{\frac{1}{\lambda}}$$

c) What is the elasticity of labor supply? *3 pts.Remember: elasticity of X to Y is $\partial(LnX)/\partial(LnY)$. So take logs.*

$$\ln L_t = -\frac{\theta}{\lambda} \ln C_t + \frac{1}{\lambda} \ln (W/P)_t$$

$$\frac{\partial \ln L_t}{\partial \ln (W/P)_t} = \frac{1}{\lambda}$$

d) Using the Bellman equation and the equation that describes evolution of wealth, derive an equation that gives the agent's demand for real money balance $(M/P)_t$ as a function of consumption C_t and the nominal interest rate i_t. *3 pts.*

$$\frac{\partial V}{\partial (M/P)_t} = (M/P)^{-\nu} + \beta E_t \frac{\partial V}{\partial A_{t+1}} (P - P(1+i_t)) \qquad \text{Note: } \frac{\partial M}{\partial (M/P)} = P$$

$$= \cdots \quad P(-i_t)$$

Using $\partial V/\partial C_t$ from a),

$$C_t^{-\theta} \frac{1}{1+i_t} = \beta E_t \frac{\partial V}{\partial A} P_t = \left(\frac{M}{P}\right)^{-\nu} \frac{1}{i_t}$$

Solve for $(M/P)_t$

$$(M/P)_t = C_t^{\theta/\nu} \left(\frac{i}{1+i}\right)_t^{-\frac{1}{\nu}}$$

e) Using the Bellman equation, derive C_t as a function of $E_t C_{t+1}$, i_t, P_t and $E_t P_{t+1}$. *2 pts.*

From a), $0 = \frac{\partial V}{\partial C_t} = C_t^{-\theta} + \beta E_t \frac{\partial V}{\partial A}(-(1+i_t))P_t$

Use "envelope theorem" (Benveniste–Scheinkman condition),

$$\frac{\partial V}{\partial A_{t+1}} = \frac{\partial u}{\partial C_{t+1}} \cdot \frac{\partial C_{t+1}}{\partial A_{t+1}}$$ holding fixed A_{t+2}, which is $\frac{1}{P_{t+1}}$

With certainty equivalence,

$$0 = C_t^{-\theta} + \beta E_t C_{t+1}^{-\theta} \frac{1}{E_t P_{t+1}} (-(1+i_t)) P_t$$

solve for C_t

$$C_t = E_t C_{t+1} \left[\beta(1+i_t)\frac{P_t}{E_t P_{t+1}}\right]^{-\frac{1}{\theta}}$$

f) Using $P_{t+1} = (1+\pi_{t+1})P_t$ and an approximation, derive an equation that gives C_t as a function of $E_t C_{t+1}$ and the "real interest rate" $r_t = i_t - E_t \pi_{t+1}$. *2 pts.*

Replacing $E_t P_{t+1}$ above with $(1+\pi_t)P_t$ gives

$$C_t = E_t C_{t+1} \left[\beta(1+i_t)\frac{1}{1+E_t \pi_{t+1}}\right]^{-\frac{1}{\theta}}$$

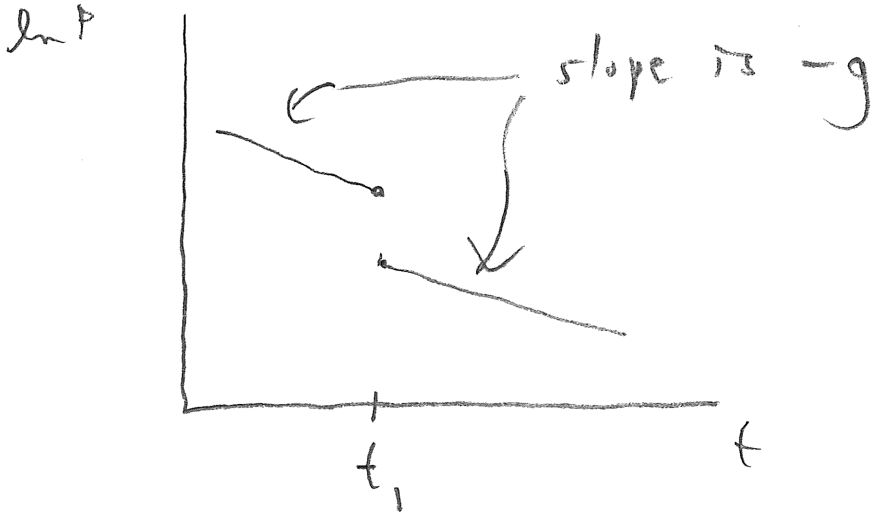
"An approximation" is $\frac{1+i_t}{1+E_t\pi_{t+1}} \approx 1 + (i_t - E_t\pi_{t+1}) = 1 + r_t$

so

$$C_t = E_t C_{t+1} \left[\beta(1+r_t)\right]^{-\frac{1}{\theta}}$$

3) Consider an economy in which the money supply m is fixed and demand for real money balances, in logs, is: $(m-p)_t^D = y_t - \lambda i_t$ where i is the log of output. The natural rate of output grows at a constant rate g. Suppose that at time t_1 the natural rate of interest unexpectedly decreases from a higher value $\bar{r}_0$ to a higher value $\bar{r}_1$. Consider the path of the price level that will allow output to remain at the natural rate throughout. Plot the log of this price level on a graph with time on the horizontal axis and the log of the price level on the vertical axis. Clearly mark the point in time t_1. *10 pts.*

Look at notes "Money" page 5. See that before and after time t_1, the price level must be falling at rate g. So at time t_1 the nominal interest rate must fall. So demand for real money balances must increase at that moment. So the SUPPLY of real money balances must increase at that moment. So the price level must fall at that moment.



4) Recall King and Rebelo's version of a real business cycle model in "Resuscitating Real Business Cycles" with "variable capacity utilization." Simplifying that model a bit, the production function is:

$Y_t = (Z_t K_t)^{\alpha} L_t^{1-\alpha}$ where Z is capacity utilization.

Capital depreciation δ increases with Z, and depreciation affects next period's capital stock:

$$K_{t+1} = K_t + Y_t - C_t - \delta(Z_t)K_t$$

The real interest rate is $r_t = \partial Y / \partial K - \delta(Z_t)$.

The representative agent's "felicity" (single-period utility) function can be written generally as $u_C(C_t) + u_{(1-l)}(1-l_t)$.

Assume specifically that $\delta_t = \frac{1}{2} Z_t^2$.

a) Write down a Bellman equation for the social planner. (2 pts)

$$V(K_t, \ldots) = \max_{c,\, l,\, Z} \{ u_c(C_t) + u_{(1-l)}(1-l_t) + E_t V(K_{t+1}, \ldots) \}$$

b) Z is a choice variable for the social planner. Using the Bellman equation, the production function and the equation above that gives K_{t+1} as a function of K_t and other stuff, write down the first-order condition that defines the optimal value of Z_t in terms of K_t and L_t. In the equation, let $\hat{Z}$ denote the optimal value ~~of Z.~~ You do *not* have to solve the equation for $\hat{Z}$. (3 pts)

$$0 = E_t \frac{\partial V(K_{t+1}, \ldots)}{\partial Z_t} \frac{\partial K_{t+1}}{\partial Z_t}$$

$$\frac{\partial K_{t+1}}{\partial Z_t} = \alpha Z_t^{\alpha-1} K_t^{\alpha} L_t^{1-\alpha} - Z_t K_t$$

so f.o.c. defines $\hat{Z}$:

$$0 = \alpha \hat{Z}^{\alpha-1} K^{\alpha} L^{1-\alpha} - \hat{Z} K$$

c) Take your answer to b) and rearrange it ~~so that~~ $\hat{Z}^2$ is alone on the left-hand side of the equation: that is, get an equation that looks like $\hat{Z}^2 = \ldots\ldots\ldots\ldots$ (2 pts)

From above,

$$\hat{Z}^2 = \alpha Z^{\alpha} K^{\alpha-1} L^{1-\alpha}$$

d) Let a denote the marginal product of capital, that is $a = \partial Y / \partial K$. Using the production function, derive an equation that gives a_t (on the left-hand side) as a function of K_t and L_t. (2 pts)

$$a_t = \alpha Z_t^{\alpha} K_t^{\alpha-1} L_t^{1-\alpha}$$

Note this is equal to answer to c)!

e) Using your answers to c) and d), and the definition of the real interest rate, derive an equation that gives $\hat{Z}_t$ as a function of r_t. (3 pts)

Defn of r: $r_t = a_t - \delta_t = a_t - \frac{1}{2} Z^2$

so $a_t = r_t + \frac{1}{2} Z^2$

From d) & c), $a_t = \hat{Z}^2$

so

$$\hat{Z}^2 = r_t + \frac{1}{2}\hat{Z}^2$$

$$\hat{Z}_t^2 - \frac{1}{2}\hat{Z}_t^2 = r_t$$

$$\frac{1}{2}\hat{Z}_t^2 = r_t$$

See: $r_t \uparrow \rightarrow \hat{Z}_t \uparrow$

5) Consider an economy similar to the Taylor model. A firm's "desired" log price for a period is $p^*_{it} = p_t + \phi y_t$ where p is the log price level and y is the output gap (deviation of log output from the natural rate). "Aggregate demand" is $y_t = m_t - p_t$. As in the Taylor model, each firm i in the economy must fix its price at the same value for two periods, and each firm can wait until it has observed m_t to set its price $x_{it} = p_{it} = p_{it+1}$. But there is *no staggering*. All firms do their pricesetting in the *same* period. (That is, all firms set prices in periods *t, t+2, t+4,...*; no firms set prices in periods *t+1, t+3, t+5,...* Thus $x_{it} = p_t = p_{t+1}$.

a) Write down x_{it} as a function of m_t , $E_t m_{t+1}$, p_t and $E_t p_{t+1}$. 2 pts

$$x_t = \tfrac{1}{2} p^*_{it} + \tfrac{1}{2} E_t p^*_{i\,t+1} = \tfrac{1}{2}(\phi m_t + (1-\phi) p_t) + \tfrac{1}{2}(\phi m^e_t + (1-\phi) p^t_{t+1})$$

b) Write down x_t as a function of m_t and $E_t m_{t+1}$ alone. 3 pts

From above, multiply both sides by 2,

$$2x_t = \phi m_t + (1-\phi) x_t + \phi m^e_{t+1} + (1-\phi) x_t$$

$$2x_t - 2(1-\phi) x_t = \phi m_t + \phi m^e_{t+1}$$

$$2\phi x_t =$$

$$x_t = \tfrac{1}{2}(m_t + m^e_{t+1})$$

c) Now suppose that m evolves as a random walk: $m_t = m_{t-1} + \epsilon_t$ where ϵ is mean-zero i.i.d. Using this fact and your answer to b), what are y_t , y_{t+1} and y_{t+3} ? 3 pts

$$m^e_{t+1} = m_t$$

so $x_t = m_t$, And $p_t = p_{t+1} = x_t$

$$Y_t = m_t - m_t = 0$$

$$Y_{t+1} = m_t - (m_t + \varepsilon_{t+1}) = \varepsilon_{t+1}$$

$$Y_{t+3} = \varepsilon_{t+3}$$

d) Suppose that you had time-series data on y from this economy. You run regressions. You regress y on y in the previous period. Would the estimated coefficient be positive, negative, or zero? 2

Zero.

e) Now suppose that you regress y on y two periods ago (y_t on y_{t-2}). Would the estimated coefficient be positive, negative, or zero? (2)

Zero.

f) Suppose you had time-series data on y from an economy described by the ordinary Taylor model (the one in the textbook and class). You run regressions of y_t on y_{t-2} (y_t on the left-hand side, y_{t-2} on the right-hand side). Would the estimated coefficient be positive, negative, or zero? 3

Positive.