ANSWERS

Economics 614, Advanced Macro, Spring 2021 Midterm examination

Look over the entire exam before you begin. Good luck!

1) Suppose that $y_t = m_t - p_t$ where *y* is the output gap (the difference between the log of output and the log of "natural rate" output), *p* is the price level and *m* is the money supply. *m* evolves as $m_t = m_{t-1} + \epsilon_t$ where ϵ is mean-zero i.i.d.

a) Suppose that the "aggregate supply" side of the model is as in the Lucas supply function model presented in class. You get a long run of time-series data from the economy and regress y_t on y_{t-2}. Will the estimated coefficient in this regression be positive, negative or zero? *Zero because* $y_t = (b/(1+b))\epsilon_t$ *and* $y_{t-2} = (b/(1+b))\epsilon_{t-2}$ *and there is no correlation between* ϵ_t , ϵ_{t-2} .

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b) Now suppose that the aggregate supply side of the model is the Fischer model presented in class. You get a long run of time-series data from the economy and regress y_t on y_{t-2}. Will the estimated coefficient in this regression be positive, negative or zero? *Zero because* $y_t = (1/(1+\phi))\epsilon_{t-1} + \epsilon_t$ *and* $y_{t-2} = (1/(1+\phi))\epsilon_{t-3} + \epsilon_2$ *and there is no correlation between....*

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c) Finally suppose that the aggregate supply side of the model is the Taylor model presented in class. Again you get a long run of time-series data from the economy and regress y_t on y_{t-2}. Will the estimated coefficient in this regression be positive, negative or zero? *Positive because* $y_t = \lambda y_{t-1} + (1/(1+\lambda))\epsilon_t = \lambda^2 y_{t-2} + \lambda(1/(1+\lambda))\epsilon_{t-1} + (1/(1+\lambda))\epsilon_t$

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2) In a simple real business cycle (RBC) model like the one in Romer's textbook, a surprise decrease in the productivity parameter *A* (a negative "technology shock") causes a reduction in employment and output. A surprise decrease in government purchases *G* also causes a reduction in employment and output. Why do we say that only a negative technology shock can cause a "recession" in the RBC model? That is, in what way does the model's response to a decrease in *G not* look like a real recession? *In the model, consumption would increase in a G-caused (in reality consumption is procyclical). Also, the real wage would rise a lot (in reality real wage is not strongly countercyclical).*

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3) Suppose demand for log real money balances is given by $(m-p)^d = y - bi$ where *y* is log output and *i* is the nominal interest rate. The money supply m^S is fixed. The natural rate of output has been growing at a low steady rate g_L. Unexpectedly, at time t_0, the natural rate of output begins growing at a higher rate g_H. The natural rate of interest does not change; it remains at the same stable value before and after t_0. There is one path for the log price level *p* that will allow output to remain equal to the natural rate at all points in time. Draw this path on a graph. Put time on the horizontal axis of the graph. Mark time t_0 on the horizontal axis. *Before* t_0, *the price level is falling at rate* g_L, *so that the supply of real money can rise at rate* g_L.

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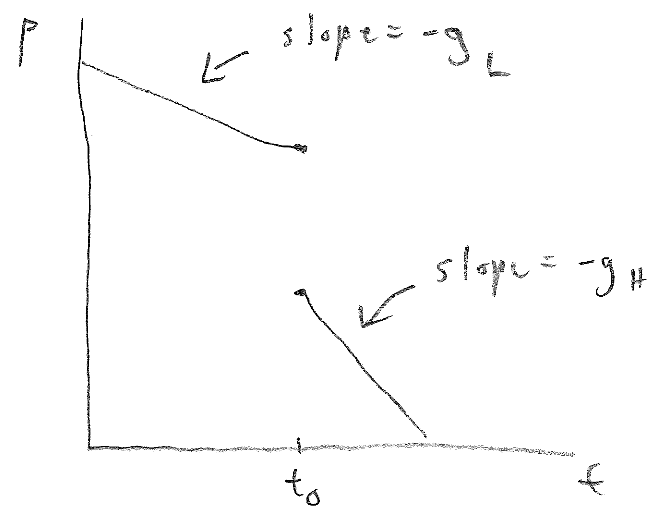
The nominal interest rate is $i = \bar{r} + \pi^e = \bar{r} - g_L$.
After t_0, *the price level is falling faster, at rate* g_H *and the nominal interest rate is lower:* $i = \bar{r} + \pi^e = \bar{r} - g_H$. *At* t_0, *expected inflation suddenly falls, the nominal interest rate suddenly falls, demand for real money balances suddenly rises; the price level suddenly falls - jumps down - so that the supply of real money can increase accordingly.*



4) Consider an economy where the central bank follows an interest-rate rule $r(\pi, y) + \epsilon$ where $r_\pi > 0$, $r_y > 0$. ϵ represents "shocks" to the central bank's behavior (times when the central bank sets an *r* different from the one specified by the usual interest-rate rule). "Aggregate supply" can be described by $\pi = \alpha + \beta y$. Demand for real money

balances is given by the usual function $(M/P)^D = L(i,y)$ where $L_i < 0$, $L_y > 0$. Assume expected future inflation is always equal to zero. Consider how the economy will respond to a positive realization of ϵ .

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a) Using graphs as appropriate, explain what will happen to output, the real interest rate and inflation.

b) What happens to the supply of real money balances $(M/P)^S$?

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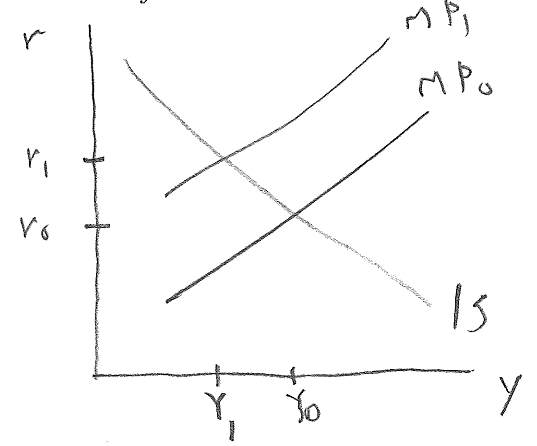
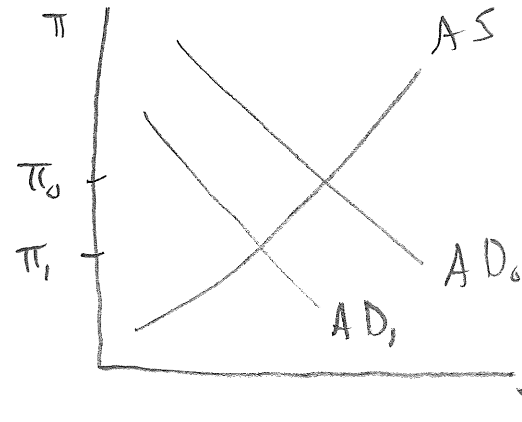
This is IS/MP with shocks to the interest-rate rule. The central bank adjusts the money supply to keep the interest rate at the desired value. A positive shock to the interest-rate rule looks like the graphs below. See that output falls, the real interest rate rises. Given that expected inflation is always equal to zero, the nominal interest rate is equal to the real interest rate, so the nominal interest rate rises too. With a decrease in y and an increase in i, demand for real money balances falls. So the central bank must be reducing the supply of real money balances accordingly.





5) Consider a model in which a representative-agent household maximizes

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + \frac{1}{1-v}\left(\Phi_t \frac{M_t}{P_t}\right)^{1-v} - \frac{1}{2}\theta L_t^2 \right] \text{ subject to } Z_{t+1} = \frac{P_t}{P_{t+1}}\left[\frac{M_t}{P_t} + (1+i_t)\left(Z_t - \frac{M_t}{P_t} - C_t + (W/P)_t L_t\right)\right]$$

In the utility function, C_t is the household's real consumption in period *t*. M_t is the household's nominal money balance held across period *t*. P_t is the price level. L_t is the quantity of labor supplied by the household. Φ_t is a parameter of the utility function that can change from period to period. β is close to, but a bit less than, one. Note that means $\ln(\beta) < 0$. And $0 < v < 1$. In the budget constraint, Z_{t+1} is a household's real wealth entering period *(t+1)*. i_t is the nominal interest rate paid on nonmonetary assets held across period *t*. The nominal wage is W_t .

Each period the household takes as given its wealth entering the period, the price level and the nominal wage. It chooses consumption, labor and its real money balance. Assume "certainty equivalence" holds, so that in the agent's optimization problem you take expected values of future variables to be equivalent to actual known values of future variables. In forming expected values, the household has rational expectations.

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a) Starting from the value function, derive the quantity of labor the household chooses to supply at time *t*, L_t^S in terms of the current real wage $(W/P)_t$ and current consumption C_t . What is the elasticity of labor supply here?

There are two ways to do this. One way is to take a first-order condition with respect to L and another first-order condition with respect to C, allowing Z_{t+1} to vary. In each of the equations there will be a term $\frac{\partial Z_{t+1}}{\partial X_t} E_t\left[\frac{\partial Vt+1}{\partial Zt+1}\right]$. You use the budget constraint to figure out $\partial V_{t+1}/\partial X_t$ Then you use the two equations together to get L_t in terms of C_t . To do this, you don't need to know what $E_t\left[\frac{\partial Vt+1}{\partial Zt+1}\right]$ is - that term disappears.

Another way is to take one first-order condition with respect to L, holding fixed Z_{t+1} . You use the budget constraint to figure out $\partial C_t/\partial L_t$.

To get an elasticity, take logs. Elasticity of LS is $\partial Ln[L_t^S]/\partial Ln[(W/P)_t]$

First way:

$$0 = \frac{\partial V}{\partial L} = -\theta L_t + \beta E_t V_Z (1+i_t)\frac{P_t}{P_{t+1}}(W/P)_t$$

this is $\frac{\partial Z_{t+1}}{\partial L_t}$

$$0 = \frac{\partial V}{\partial C} = \frac{1}{C_t} + \beta E_t V_Z (1+i_t)\frac{P_t}{P_{t+1}}(-1)$$

this is $\frac{\partial Z_{t+1}}{\partial L_t}$

From second eqn., $\beta E_t V_Z (1+i_t)\frac{P_t}{P_{t+1}} = \frac{1}{C_t}$

Substitute that into first equation, gives

$$L_t = \frac{1}{\theta}\frac{1}{C_t}\left(\frac{W}{P}\right)_t$$

Second way

$$0 = \left.\frac{\partial V}{\partial L}\right|_{\partial Z_{t+1}=0} = -\theta L_t + \frac{1}{C_t}(W/P_t)$$

this is $\left.\frac{\partial C_t}{\partial L_t}\right|_{\partial Z_{t+1}=0}$

which also gives $L_t = \frac{1}{\theta}\frac{1}{C_t}\left(\frac{W}{P}\right)_t$

Now log it:

$$l_t = -\ln\theta - c_t + (w-p)_t$$

$$\frac{\partial l_t}{\partial (w-p)_t} = 1 \sim \text{elasticity of LS}$$

b) Derive the agent's demand for real money balance $(M/P)_t$ in terms of consumption C_t, the nominal interest rate i_t and any other relevant variables. *Again, two ways to do this one.*

First way: use $\partial V/\partial C$ from a) along with

$$0 = \frac{\partial V}{\partial (M/P)} = \Xi_t^{1-\nu} (M/P)^{-\nu} + \beta E_t V_z \frac{P_t}{P_{t+1}} (1-(1+i))$$

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$$\text{gives} \quad \left(\frac{M}{P}\right)_t = \Xi_t^{\frac{1-\nu}{\nu}} \left(\frac{i_t}{1+i_t}\right)^{-1/\nu} C_t^{1/\nu}$$

Second way:

$$0 = \left.\frac{\partial V}{\partial (M/P)}\right|_{\partial Z_{t+1}=0} = \Xi^{1-\nu} (M/P)^{-\nu} - \underbrace{\frac{1}{C_t}}$$

this is $\left.\dfrac{\partial C_t}{\partial (M/P)_t}\right|_{\partial Z_t = 0}$

c) Derive current consumption C_t in terms of $E_t C_{t+1}$, i_t and $1 + E_t \pi_{t+1} = \dfrac{E_t P_{t+1}}{P_t}$.

This one is different from b) and c), because it stretches across two periods. Here you DO need to know what $E_t\left[\dfrac{\partial Vt+1}{\partial Zt+1}\right]$ *is! So you use the Benveniste-Scheinkman condition (envelope theorem). That means you use the fact that* $\partial V_{t+1}/\partial Z_{t+1} = \dfrac{\partial V_{t+1}}{\partial X_{t+1}} \dfrac{\partial Xt+1}{\partial Zt+1}$ *holding fixed all time-t+1 variables other than X. The "X" here will be C, because I asked for "in terms of* $E_t C_{t+1}$*."*

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$$\left.\frac{\partial C_{t+1}}{\partial C_t}\right|_{\partial Z_{t+1}=0} = (1+i) \frac{P_t}{P_{t+1}} (-1) \quad \text{and} \quad \left.\frac{\partial V_{t+1}}{\partial C_{t+1}}\right|_{\partial Z_{t+1}=0} = \frac{1}{C_{t+1}}$$

and use $\partial V/\partial C = 0$ from a)

$$\text{gives} \quad 0 = \frac{1}{C_t} + \beta \frac{1}{C^e_{t+1}} (1+i) \frac{P_t}{P_{t+1}} (-1)$$

rearrange to get

$$C_t = \beta^{-1} C^e_{t+1} (1+i)^{-1} (P^e_{t+1}/P_t)$$

$$= \beta^{-1} C^e_{t+1} (1+i)^{-1} (1+\pi^e) = \beta^{-1} C^e_{t+1} \frac{1+i_t}{1+\pi^e}$$

d) Suppose that real output in this economy is about equal to real consumption spending, and the price level is approximately fixed from one period to the next. Suppose also that there is a central bank that wants to stabilize real output (keep output as close as possible to LRSS output). In every period t the central bank's policy committee can set the money supply for the period M_t^S or the real interest rate for the period r_t. When it meets, the policy committee can observe the public's expected value for output in the following period. It cannot observe the current value of the variable Φ_t. Would it be better for the central bank's policy committee to set the money supply or or the real interest rate r? Explain!

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This relates to the Poole paper. Changes in Φ_t are movements in the LM curve, as you can see from the equation you derived in b). The IS curve corresponds to the equation from c). There is an unforeseeable disturbance term in the LM curve and there's no unpredictable disturbance term in the IS curve (because the central bank knows $E_t C_{t+1}$) so it's better to fix the interest rate.

6) Consider a model like Romer's textbook static (one-period) model of imperfect competition, in which each household i operates a monopoly firm and supplies labor to a perfectly competitive labor market. A household-firm does not use its own labor in production, but instead hires labor from the perfectly competitive labor market at a market-clearing nominal wage W per unit of labor. Each household acts to maximize:

$U_i = C_i - \frac{1}{\gamma} L_i^\gamma$ where C_i is a function of the household's consumption of individual goods. Each household-firm's production function is $Y_i = H_i$, where H is the number of labor units the household-firm hires from the labor market. Demand for the good produced by a household-firm is $Y_i^D = (P_i / P)^{-\eta} Y$ where P is the price level and Y is average real income or real GDP per household.

a) From profit-mazimization on the part of the firm, derive an equation that gives an individual firm's profit-maximizing price P_i^* as a function of the market nominal wage W. I'll do nominal profit.

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$$\pi = P_i Y_i - W L_i = P_i (P_i/P)^{-\eta} Y - W (P_i/P)^{-\eta} Y = P^\eta Y (P_i^{1-\eta} - W P_i^{-\eta})$$

$$0 = \frac{\partial \pi}{\partial P_i} = P^\eta \underbrace{Y\left((1-\eta) P_i^{-\eta} - W(-\eta) P_i^{-\eta-1}\right)}_{\text{must equal zero so...}}$$

$$(\eta - 1) P_i^{-\eta} = \eta W P_i^{-\eta-1}$$

$$\text{gives} \quad P_i = \frac{\eta}{\eta - 1} W = \frac{1}{1 - 1/\eta} W$$

b) From utility-maximization on the part of the household, derive an equation that gives labor supply per household L_i^S as a function of the real wage W/P.

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$$\text{Max } U_i = C_i - \frac{1}{\gamma} L_i^\gamma = \frac{W}{P} L_i - \frac{1}{\gamma} L_i^\gamma$$

$$0 = \frac{\partial U}{\partial L_i} = \frac{W}{P} - L_i^{\gamma - 1}$$

$$\text{gives} \quad L_i = (W/P)^{\frac{1}{\gamma - 1}}$$

c) From your answers to a) and b), derive what I called the "real rigidity equation," which gives the log of a firm's profit-maximizing price as a function of the log price level and the log of real GDP. Assume that aggregate output Y is equal to the output produced by the labor supplied by a representative household ($Y = L_i$).

$Y = L_i$ so $Y = (W/P)^{\frac{1}{\gamma-1}}$ so $W = PY^{\gamma-1}$

10 $P_i = \frac{\eta}{\eta-1} W = \frac{\eta}{\eta-1} P Y^{\gamma-1}$

Take logs

$p_i = \ln(\eta/\eta-1) + p + (\gamma-1)y$