Economics 614, Advanced Macro, Spring 2021 Final examination ANSWERS

Look over the entire exam before you begin. Good luck!

1) Consider a model in which a representative-agent household maximizes

$$E_t\sum_{t=0}^{\infty}\beta^t\left[\ln C_t + \frac{1}{1-\nu}\left(\frac{M_t}{P_t}\right)^{1-\nu} - \frac{1}{2}\theta L_t^2\right] \text{ subject to } Z_{t+1} = \frac{P_t}{P_{t+1}}\left[\frac{M_t}{P_t} + (1+i_t)\left(Z_t - \frac{M_t}{P_t} - C_t + (W/P)_t L_t\right)\right]$$

In the utility function, C_t is the household's real consumption in period t. M_t is the household's nominal money balance held across period t. P_t is the price level. L_t is the quantity of labor supplied by the household. β is close to, but a bit less than, one. And $0<\nu<1$.
In the budget constraint, Z_{t+1} is a household's real wealth entering period *(t+1)*. i_t is the nominal interest rate paid on nonmonetary assets held across period *t*. The nominal wage is W_t .

In each period, the household takes as given its wealth entering the period, the price level and the nominal wage. It chooses consumption, labor and its real money balance. Assume "certainty equivalence" holds, so that in the agent's optimization problem you take expected values of future variables to be equivalent to actual known values of future variables. In forming expected values, the household has rational expectations.

a) Using the value function, *derive* an equation that gives the log of current consumption c_t in terms of $E_t c_{t+1}$ and the real interest rate r_t using all the the usual approximations.

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$$V_t(Z_t) = \max_{C_t,\left(\frac{M}{P}\right)_t, L_t}\left[\ln C_t + \frac{1}{1-\nu}\left(\frac{M}{P}\right)^{1-\nu} - \frac{1}{2}\theta L_t^2 + \beta E_t[V_{t+1}(Z_{t+1})]\right]$$

Use Benveniste-Scheinkman condition to get Euler equation.

$$\frac{\partial V}{\partial C_t} = \frac{1}{C_t} + \beta E_t\frac{\partial V_{t+1}}{\partial Z_{t+1}}\frac{\partial Z_{t+1}}{\partial C_t}$$ At optimum $\frac{\partial V}{\partial C_t}$ must be zero.

From $Z_{t+1} = \ldots$, $\frac{\partial Z_{t+1}}{\partial C_t} = (1+i_t)\frac{P_t}{P_{t+1}}(-1)$

But what's $\frac{\partial V_{t+1}}{\partial Z_{t+1}}$? By envelope theorem it must be equal to $\frac{\partial V_{t+1}}{\partial Z_{t+1}}$ holding fixed C_{t+2}, L_{t+2} etc., that is spending all of ∂Z_{t+1} on C_{t+1}. So

$$\frac{\partial V_{t+1}}{\partial Z_{t+1}} = \left.\frac{\partial V_{t+1}}{\partial C_{t+1}}\right|_{\partial Z_{t+2}=0} = \frac{1}{C_{t+1}}$$

so

$$0 = \frac{1}{C_t} + \beta E_t \frac{\partial V_{t+1}}{\partial Z_{t+1}} \frac{\partial Z_{t+1}}{\partial C_t}$$

$$= \frac{1}{C_t} + \beta E_t \frac{1}{C_{t+1}} (1 + i_t) \frac{P_t}{P_{t+1}} (-1)$$

hence

$$C_t = \beta^{-1} C^e_{t+1} (1+i)^{-1} \frac{P^e_{t+1}}{P_t}$$ with usual approximations.

$$\frac{P^e_{t+1}}{P_t} = 1 + \pi^e \quad \text{so}$$

$$C_t = \beta^{-1} C^e_{t+1} \frac{1+\pi^e}{1+i} \approx \beta^{-1} C^e_{t+1} (1+r_t)$$

Take logs

$$c_t = -\ln \beta + c^e_{t+1} - \ln(1+r_t) = -\ln\beta + c^e_{t+1} - r_t$$

"small"

(3)

b) In this model, there is a relationship between the LRSS real interest rate and the LRSS rate of growth of consumption. Derive an expression that shows this relationship. Denote the LRSS rate of growth of consumption (a fraction, not a percent) as *g*.

In LRSS, $c_t = c_{t+1} - g$ *(consumption is in logs here). Using that gives* $r^{SS} = -Ln(\beta) + g$ *.*

$$c_{t+1} - g = -\ln\beta + c_{t+1} - r^{ss}$$

sorry

(5)

c) Assume the economy is closed, the capital stock is fixed, there is no investment. Output $Y_t = C_t + G_t$ where *G* is government purchases of goods and services. The share of *G* in total output is $\gamma_t = (G/Y)_t$. Assume γ is always a small number (in the same way that the real interest rate is a small number). Starting from your answer to b), derive an equation that gives the log of output y_t as a function of $E_t y_{t+1}$, the real interest rate r_t , γ and any other relevant variables.

(5) From notes,

From answer to a)

not log $\rightarrow$ $$C_t = \beta^{-1} C^e_{t+1} (1 + r_t)$$

Now say $Y_t = C_t + G_t$, which means

substitute in Euler equation

$$C_t = Y_t - G_t = Y_t\left(1 - \frac{G_t}{Y_t}\right) = Y_t(1 - \gamma_t)$$

and $$C^e_{t+1} = Y^e_{t+1}(1 - \gamma^e_{t+1})$$

$$Y_t(1-\gamma_t) = \beta^{-1} Y^e_{t+1}(1 - \gamma^e_{t+1})(1 + r_t)$$

Take logs...

$$\gamma_t + \ln(1-\gamma_t) = -\ln\beta + \gamma^e_{t+1} + \ln(1-\gamma^e_{t+1}) - r_t$$

here we used $\ln(1+r_t) \approx r_t$

I said γ is "small" like r,

so we can apply the same approximation

to $\ln(1-\gamma)$ that we do to $\ln(1+r)$.

$$\gamma_t - \gamma_t = -\ln\beta + y^e_{t+1} - \gamma^e_{t+1} - r_t$$

$$y_t = -\ln\beta + y^e_{t+1} - r_t + (\gamma_t - \gamma^e_{t+1})$$

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2) Clarida, Galiand Gertler (1999, "The Science of Monetary Policy: A New Keynesian Perspective") argue that in a New Keynesian model the dynamic inconsistency issue is, in one way, different from the issue in an old-fashioned Keynesian model with a Friedman-Phelps expectations-augmented Phillips curve. What is the difference? *See notes. For full credit, you had to mention supply shocks.*

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3) Consider an economy whose economy can be described by the "Lucas supply function" model. In the past, the country has been subject to many random, unpredictable money-supply shocks. At time t_0 the country unexpectedly adopts a new monetary regime under which there are fewer unpredictable money-supply shocks. The change in regime is immediately communicated to the public, and understood by them. Consider the expectations-augmented Phillips curve: $\pi_t = E_{t-1}\pi_t + \beta y_t$. At time t_0 , does the parameter β increase, decrease, or remain the same? Explain.

β decreases. In the LSF model, $y = b(p - E[p])$ which gives $\pi = E[\pi] + (1/b)y$. So (1/b) in this equation is equivalent to β in that expectations-augmented Phillips curve. b increases if the variance of unpredictable money-supply shocks decreases, holding fixed the variance of z. (See equation (6.93) in the textbook.) An increase in b means a decrease in β.

4) Consider an economy in which business investment is subject to the "asymmetric information" problem described by the Romer textbook's model of "financial-market imperfections." Suppose this economy is conquered by religious fanatics who ban the use of fixed-interest debt contracts (the type of debt contract described by the model). The only type of investment contract that can now take place is one in which an entrepreneur promises to pay an investor a *share* of the profit in the enterprise controlled by the entrepreneur. What will happen to the volume of business investment in this economy? Explain, perhaps using equations and/or graphs. But I am not asking for derivations here.

The expected value of the investor's return to investing in a business project is the expected value of the payment from the entrepreneur minus the "expected verification cost" denoted A. A project is undertaken if:

Expected return to project $\geq$ (1+r) + A

where r is the return on safe bonds. The "expected verification cost" is the verification cost times the probability that the investor must pay the verification cost. Under the fixed-interest debt contract, the probability that the investor must pay the verification cost is less than one. Under the share contract, that probability is one. Thus A is larger under the share contract. Fewer projects will be undertaken, decreasing the volume of business investment, if share contracts replace debt contracts.

5) Consider the following "three-equation" model:

(i) $y_t = E_t y_{t+1} - s r_t + u_t$ *where* $u_t = \rho u_{t-1} + \epsilon_t$ and ϵ is mean-zero i.i.d.

(ii) $\pi_t = E_t \pi_{t+1} + \kappa y_t$

(iii) $r_t = \phi \pi_t$

where y is the output gap and r is the gap between the real interest rate and the natural rate of interest.
Expectations are rational.
Assume there is a long-run steady state where $y = 0$, $\pi = 0$.

12 a) Derive equations that give y_t, π_t, r_t as functions of u_t . *See notes.*

b) In this model, what is the sign of the correlation between r_t and y_t (positive, negative or zero)?

2 *From your answer to a), you can see that the correlation is positive.*

c) What is the sign of the correlation between r_t and y_{t-1} (positive, negative or zero)?

2 *From your answer to a), you can see that the correlation is positive (since there is serial correlation in u).*

2 d) If you had long time-series of data from this economy and you ran a regression of y_t on y_{t-1} (that is, regress the output gap on one lag of the output gap), what would be the value of the estimated coefficient?

From your answer to a), you can see that the coefficient is ρ (serial correlation coefficient for u).

Answer to 5) from notes

Conjecture $y^e_{t+1} = \rho_y y_t$

Then we can apply "math trick" to

$\pi_t = \pi^e_{t+1} + \kappa y_t$ getting $\pi_t = \kappa \frac{1}{1-\rho_y} y_t$

putting that into $r_t = \phi \pi_t$ gives $r_t = \phi \kappa \frac{1}{1-\rho_y} y_t$

Substitute into r in IS equation, get

$$y_t = y^e_{t+1} - s\phi\kappa \frac{1}{1-\rho_y} y_t + u_t$$

Get y_t on LHS:

$$y_t = a y^e_{t+1} + a u_t \quad \text{where} \quad a = \frac{1}{1+s\phi\kappa\frac{1}{1-\rho_y}}$$

Apply math trick, get

$$y_t = a \frac{1}{1-a\rho_y} u_t$$

this tells us that serial correlation in y_t must be same as for u_t, so $\rho_y = \rho$, so:

$$y_t = a\frac{1}{1-a\rho} u_t = \frac{1}{\frac{1}{a}-\rho} u_t$$

$$\pi_t = \kappa\frac{1}{1-\rho} a\frac{1}{1-a\rho} u_t = \frac{\kappa}{1-\rho} \frac{1}{\frac{1}{a}-\rho} u_t$$

$$r_t = \phi\kappa\frac{1}{1-\rho} a\frac{1}{1-a\rho} u_t = \frac{\phi\kappa}{1-\rho} \frac{1}{\frac{1}{a}-\rho} u_t$$

Note they are all positive functions of u_t.

So y_t, π_t, r_t all + correlated

2 e) What is the sign of the correlation between π_t and y_t (positive, negative or zero)?
From your answer to a), positive.

2 f) If you had long time-series of data from this economy and you ran a regression of π_t on y_t, what would be the value of the estimated coefficient?

From your answer to a), you can see that $\Delta\pi / \Delta y = \kappa / (1-\rho)$.

5 g) How could you slightly modify the model so that the sign of correlation between output and inflation is the opposite of your answer to to e)?

Get rid of the IS shock, replace it with an "aggregate supply" (or "cost-push" or Phillips curve) shock.

6) Consider a model where:
(i) $y_t = E_t y_{t+1} - sr_t + u_t$ *where* $u_t = \rho u_{t-1} + \epsilon_t$ and ϵ is mean-zero i.i.d.
(ii) $\pi_t = E_t \pi_{t+1} + \kappa y_t$

and the central bank sets r_t to minimize a loss function: $E\left[\frac{1}{2} y_t^2 + \frac{1}{2}\pi_t^2\right]$

At the time the central bank sets r_t, it does *not* know what ϵ_t is. But it *does* know what π and y (and of course *r*) were in the past period *(t-1)* and earlier periods. Hint: in your answers to the questions below, I am not necessarily looking for much (if any) derivation.

5 a) In this model, what is the sign of the correlation between y_t and r_t (positive, negative or zero)?

From our discussions, you know that the central bank will fully counteract any known component of u_t . *The known component of* u_t *is* $\rho\, u_{t-1}$, *as* u_{t-1} *can be inferred from the observed values of* r_{t-1} *and* y_{t-1} . *Thus* $r_t = (1/s)\rho u_{t-1}$ *and* $y_t = \epsilon_t$. *Thus* y_t *and* r_t *are uncorrelated.*

5 b) What is the sign of the correlation between r_t and y_{t-1} (positive, negative or zero)?

$u_{t-1} = \rho u_{t-2} + \epsilon_{t-1} = \rho u_{t-2} + y_{t-1}$. *Thus* r_t *and* y_{t-1} *are positively correlated.*

5 c) If you had long time-series of data from this economy and you ran a regression of y_t on y_{t-1} (that is, regress the output gap on one lag of the output gap), what would be the value of the estimated coefficient?

Zero (because ϵ *is i.i.d.).*