Economics 614, Advanced Macro, Spring 2024 Midterm examination ANSWERS

1) Consider an expectations-augmented Phillips curve of the specific form $\pi_t = {}_{t-1}\pi^e{}_t + \alpha y_t$ where y denotes the output gap (the difference between the log of output and the log of the natural rate of output). If the expected-inflation term π^e has the characteristics of a *rational* expectation, it is not possible for there to be serial correlation in the output gap y.. Explain why, using equations. **10 pts.**

Rational expectations means ${}_{t-1}\pi^e_t = E_{t-1}[\pi_t]$

where $\pi_t = E_{t-1}[\pi_t] + \varepsilon_t$

ε_t is error in forecast. It must be uncorrelated with anything forecaster knows when he is making the forecast, That includes y_{t-1} & ε_{t-1}.

In $\pi_t = {}_{t-1}\pi^e_t + \alpha y_t$, ${}_{t-1}\pi^e_t = \pi_t - \varepsilon_t$

so $\pi_t = \pi_t - \varepsilon_t + \alpha y_t$

$\Rightarrow y_t = \frac{1}{\alpha}\varepsilon_t$

ε must be i.i.d. (so that it's uncorrelated with ε_{t-1}).

So no serial corr in y.

Also, ε_t must be uncorrelated with y_{t-1} (because forecaster knew y_{t-1} when he was making the forecast). So...

Note: you can just as well say "I define ε_{t-1} to be (−1) times the forecast error so that $E_{t-1}[\pi_t] = \pi_t + \varepsilon_t$." But that's not the convention.

2) Consider an expectations-augmented Phillips curve of the specific form $\pi_t = {}_t\pi^e_{t+1} + \alpha y_t$ where y again denotes the output gap (the difference between the log of output and the log of the natural rate of output). If the expected-inflation term π^e has the characteristics of a *rational* expectation, then the upcoming future change in inflation $(\pi_{t+1} - \pi_t)$ must be negatively related to the current output gap y_t, in the sense that a regression of the change in inflation on the output gap would give a negative coefficient. Explain why, using equations. **10 pts.**

Most of you missed (or at least failed to mention in your answer) the main point here. Maybe this was because you don't know what an OLS regression requires. Suppose you are regressing Y on X, like this: $Y_t = Constant + \beta X_t + \epsilon_t$. *Note that there is a "residual" term* ϵ *which is "things other than X that affect Y, which we would like to observe and add to the RHS of the regression, but sadly we can't do that because we can't observe those things, so they are in the residual." Will the regression coefficient be equal to* β*? It depends. If it so happens that the missing things in the regression behave so that* ϵ *is POSITIVELY correlated with X, then the regression coefficient will be BIGGER than* β*. If it so happens that the missing things in the regression behave so that* ϵ *is NEGATIVELY correlated with X, then the regression coefficient will be SMALLER than* β*. The regression coefficient will only be equal to* β *if* ϵ *is UNcorrelated with X.*

Now, the answer to the question is:

With rational expectations, ${}_t\pi^e_{t+1} = E_t[\pi_{t+1}]$

and $\pi_{t+1} = {}_t\pi^e_{t+1} + \varepsilon_{t+1}$ ← error in forecast, must be uncorrelated with...

so $\pi_t = \pi_{t+1} - \varepsilon_t + \alpha y_t$

$\pi_{t+1} - \pi_t = -\alpha y_t + \varepsilon_t$

ε_t must be uncorrelated with y_t, so estimated coefficient should be close to $(-\alpha)$, hence negative.

3) Consider an economy in which $\pi_t = {}_{t-1}\pi^e_t + (y_t - \bar{y})$ where y is output and $\bar{y}$ is the natural rate of output. Everyone in the economy has the same preferences with respect to inflation and output. The desired rate of inflation is π^*. The desired level of output y^* is greater than the natural rate of output, that is $y^* > \bar{y}$. The central bank policy committee knows the public's expected value for inflation, and also knows exactly what interest rate will deliver a given value of the output gap $(y_t - \bar{y})$. Each period it sets the interest rate to minimize a loss function:

$$L_t = \frac{1}{2}(y_t - y^*)^2 + \frac{1}{2}(\pi_t - \pi^*)^2$$

a) Derive the rate of inflation that will prevail in "rational expectations equilibrium." Show all steps in the derivation. **6 pts.**

$\pi = \pi^e + (y - \bar{y})$ so $y = \bar{y} + (\pi - \pi^e)$

$L = \frac{1}{2}(\bar{y} + (\pi - \pi^e) - y^*)^2 + \frac{1}{2}(\pi - \pi^*)^2$

$0 = \frac{\partial L}{\partial \pi} = (\bar{y} + \pi - \pi^e - y^*) + (\pi - \pi^*)$

$\pi = \frac{1}{2}(y^* - \bar{y}) + \frac{1}{2}(\pi^e + \pi^*)$

In R.E.E., $\pi^e = \pi$ so

$\pi = (y^* - \bar{y}) + \pi^*$

and $y^* - \bar{y} = \pi - \pi^*$

b) What is the value of loss that will be felt by a member of the public, in rational expectations equilibrum? 2 pts.

$L_t = \frac{1}{2}(\bar{y} - y^*)^2 + \frac{1}{2}(\bar{y} - y^*)^2 = (\bar{y} - y^*)^2$

$= (y^* - \bar{y})^2$

c) Now suppose monetary policy is delegated to a person with odd preferences: this person's preferred inflation rate is π^* but his preferred level of output y_i^* is equal to the natural rate of output $\bar{y}$. Thus he sets the interest rate to minimize:

$$L_t = \frac{1}{2}(y_t - \bar{y})^2 + \frac{1}{2}(\pi_t - \pi^*)^2$$

In that case, in rational expectations equilibrum,

- what is the rate of inflation that will prevail? (I don't necessarily need to see a complete derivation here.) **2 pts.**

To answer this, you can repeat the derivation in part a) with the new loss function, or you can just take your answer to a) and substitute in that $y^* = \bar{y}$. Either way, $\pi = \pi^*$.

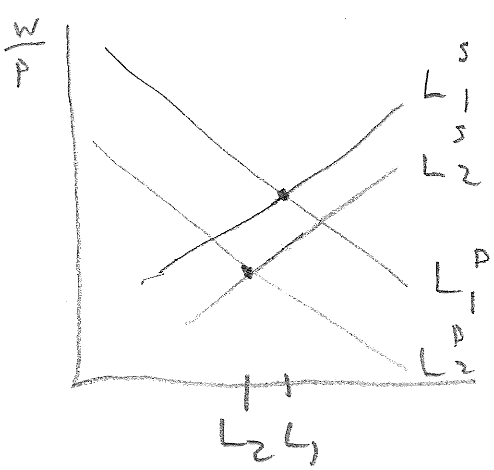
- what is the loss that will be felt by a member of the public, in "rational expectations equilibrum"? **4 pts.** *The key point here is to use the loss function of the public, not the odd person to whom monetary policy has been delegated. Hence the loss I am asking about is:*

$$L_t = \frac{1}{2}(\bar{y} - y^*)^2 \text{ which is smaller than the loss in part b). But not zero!}$$

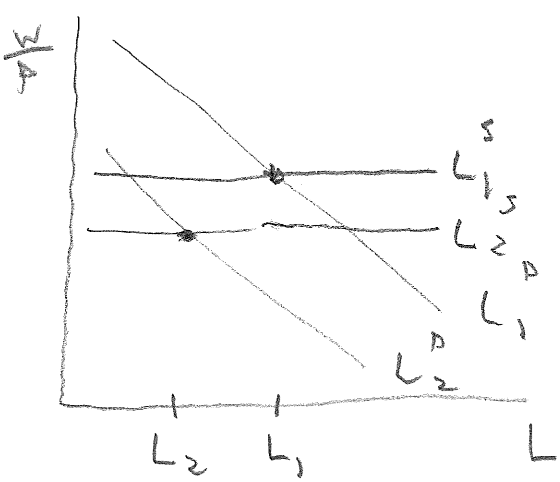
4) Recall Romer's "baseline" real business cycle model, in which output is affected by "productivity shocks" (shocks to the production function parameter A). Consider the *magnitude* of the output fluctuation that occurs in response to a productivity shock of given size. In Romer's baseline model, the representative household takes the real wage as given and chooses how much to work. Suppose you replaced this with "indivisible labor" plus "consumption insurance." Do you think this would increase, decrease, or have no effect on the magnitude of the output response to a productivity shock of given size? Explain. I am looking for nothing more than a verbal explanation and perhaps a graph or two. **10 pts.**

The trick here was to think about the use of our discussion of "indivisible labor" plus "consumption insurance" in the King and Rebelo paper. Why did they use it in that paper? To make labor supply more elastic. Why did they want to make labor supply more elastic? So that a given shift in the labor demand curve due to a productivity shock would have a bigger effect on output and employment. The "graph or two" I asked for could be these:

Baseline model



Model with indivisible labor plus consumption insurance



See: drop in L is bigger, so (for given drop in TFP) drop in Y is bigger.

To get full credit, you had to draw the shift in the labor supply curve, which exists in both cases, as well as the difference in slope. And the variable on the horizontal axis had to be L not Y. L=Y in some of our models, but not in Romer's RBC model - remember? it has a Cobb-Douglas production function!

5) Consider an economy where a firm's "desired" price for a period t is (in log terms) $p_{it}^* = p_t + \phi y_t$ and $y_t = m_t - p_t$. where y is the output gap. As in the Taylor model, a firm that can set its price in the current period first observes the realized value of m for the current period, then fixes its price at the same value for the current period and the upcoming period such that $x_{it} = p_{it} = p_{it+1}$. But there is *no staggering*. That is, all firms set their prices in periods *t, t+2, t+4, t+6*,....No firms set their prices in *t+1, t+3, t+5*,....

a) Derive an equation that gives x_{it} as a function of m_t and $E_t m_{t+1}$. **5 pts**

$x_{it} = \frac{1}{2} p_{it}^* + \frac{1}{2} E_t p_{it+1}^{*e}$ Want p_i^* in terms of m, so use $y_t = m_t - p_t$

to get $p_{it}^* = \phi m_t + (1-\phi) p_t$ so

$x_{it} = \frac{1}{2}(\phi m_t + (1-\phi) p_t) + \frac{1}{2}(\phi m_{t+1}^e + (1-\phi) p_{t+1}^e)$

All firms set same price as firm i, so $x_{it} = p_{it} = p_{t+1}$

so $x_{it} = \frac{1}{2}(\phi m_t + (1-\phi) x_{it}) + \frac{1}{2}(\phi m_{t+1}^e + (1-\phi) x_{it})$

Look! This defines x_{it} as fn. of m_t, m_{t+1}^e! Solve it—

$2x = \phi m_t + (1-\phi) x + \phi m^e + (1-\phi) x = \phi (m_t + m^e) + (2 - 2\phi) x$

$2\phi x_{it} = \phi (m_t + m_{t+1}^e)$ so $x_{it} = \frac{1}{2}(m_t + m_{t+1}^e)$.

And that means $p_t = p_{t+1} = \frac{1}{2}(m_t + m_{t+1}^e)$.

And $y_t = m_t - p_t$, $y_{t+1} = m_{t+1} - p_{t+1}$

b) Suppose m evolves as a random walk: $m_t = m_{t-1} + \epsilon_t$ where ϵ is mean-zero i.i.d. Using this and your answer to a), what are y_t , y_{t+1} , y_{t+2} and y_{t+3} ? **5 pts**

Random walk means $m_{t+i}^e = m_t$ for all t & i so

$y_t = m_t - \frac{1}{2}(m_t + m_t) = m_t - m_t = 0$

$y_{t+1} = m_{t+1} - \text{----} = m_{t+1} - m_t = \varepsilon_{t+1}$

$y_{t+2} = m_{t+2} - \frac{1}{2}(m_{t+2} + m_{t+2}) = \text{----} = 0$

$y_{t+3} = m_{t+3} - \text{----} = m_{t+3} - m_{t+2} = \varepsilon_{t+3}$

c) Would you say there is "persistence" (serial correlation) in the output gap? Explain why. **2 pts.**

No, because ε is i.i.d. And no AR(1) anyway.

6) In Romer's textbook static (one-period) model of imperfect competition, each household i operates a monopoly firm and supplies labor to a perfectly competitive labor market. A household-firm does not use its own labor in production, but instead hires labor from the perfectly competitive labor market at a market-clearing nominal wage W per unit of labor. Each household acts to maximize:

$U_i = C_i - \frac{1}{\gamma} L_i^{\gamma}$ where C_i is a function of the household's consumption of individual goods. The production function is $Y_i = L_i$, where L is the number of labor units hired from the labor market. Demand for the good produced by a household-firm is $Y_i^D = (P_i / P)^{-\eta} Y$ where P is the price level and Y is average real income or real GDP per household.

a) From profit-mazimization on the part of the firm, derive an equation that gives an individual firm's profit-maximizing price P_i^* as a function of the market nominal wage W. **4 pts**

b) From utility-maximization on the part of the household, derive an equation that gives labor supply per household L_i^S as a function of the real wage W/P. **4 pts**

c) From your answers to a) and b) , derive what I called the "real rigidity equation," which gives the log of a firm's profit-maximizing price as a function of the log price level and the log of output. **4 pts**

See notes, also 2021 midterm (6.