NEW KEYNESIAN PHILLIPS CURVE

"Romer's" derivation of Calvo model

x_t Price set by a firm that is allowed to reset in t, with probability α

$$p_t = \alpha x_t + (1-\alpha) p_{t-1} \quad (7.53)$$

price-changers chosen at random, so p_{t-1} is average price of non-choosers

$$\pi_t = p_t - p_{t-1} = \alpha x_t + (1-\alpha) p_{t-1} - p_{t-1}$$

$$= \alpha (x_t - p_{t-1}) \qquad (7.54)$$

Means $(x_t - p_{t-1}) = \frac{1}{\alpha}\pi_t$, $(x_{t+1} - p_t) = \frac{1}{\alpha}\pi_{t+1}$

1) Get x_t

Recall (7.15): $p_i = \sum_{t=0}^{\infty} \dfrac{\beta^t q_t}{\sum_{\tau=0}^{\infty} \beta^\tau q_\tau} E p_t^*$

Call $t \to j$ (so that we can use t for current period)

$p_i \to x_t$

$$q_j = (1-\alpha)^j$$

note $\beta(1-\alpha) < 1$ so...

$$\sum_{\tau=0}^{\infty} \beta^\tau q_\tau = \sum_{\tau=0}^{\infty} \beta^j (1-\alpha)^j = \sum_{\tau=0}^{\infty} (\beta(1-\alpha))^j = \frac{1}{1-\beta(1-\alpha)}$$

$$x_t = [1-\beta(1-\alpha)] \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j E_t p_{t+j}^* \qquad (7.56)$$

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Romer's derivation of Calvo

2) Get X_t in terms of $E_t X_{t+1}$

to collapse down that infinite term.

First, what is $E_t X_{t+1}$?

$$X_t = [1-\beta(1-\alpha)] \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j E_t p^*_{t+j} \qquad (7.56)$$

$$X_{t+1} = [\qquad] \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j E_{t+1} p^*_{t+1+j}$$

$$E_t X_{t+1} = [\qquad] \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j E_t E_{t+1} p^*_{t+1+j}$$

"Law of iterated projections" (expectations) says $E_t E_{t+1} X_{t+j} = E_t X_{t+j}$

so

$$E_t X_{t+1} = [\qquad] \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j E_t p^*_{t+1+j}$$

Where is $E_t X_{t+1}$ within (7.56)?

Take (7.56), pull out first period (i.e. $j=0$)

$$X_t = [1-\beta(1-\alpha)] \beta^0 (1-\alpha)^0 p^*_t + [1-\beta(1-\alpha)] \sum_{j=1}^{\infty} \beta^j (1-\alpha)^j E_t p^*_{t+j}$$

Now $\sum_{j=1}^{\infty} \beta^j (1-\alpha)^j E_t p^*_{t+j} = \beta(1-\alpha) \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j E_t p_{t+1+j}$ so

$$X_t = [1-\beta(1-\alpha)] p^*_t + \beta(1-\alpha) \underbrace{[1-\beta(1-\alpha)] \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j E_t p^*_{t+1+j}}_{\text{Look! } E_t X_{t+1}\text{!}}$$

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Romer's derivation of Calvo (cont.)

2) Get X_t in terms of $E_t X_{t+1}$ (cont.)

$$X_t = [1-\beta(1-\alpha)]\, p_t^* + \beta(1-\alpha) E_t X_{t+1} \qquad (7.57)$$

3) Get an expression with $(x_t - p_{t-1})$ & $(x_{t+1} - p_t)$ in it

Take (7.57), subtract p_t from both sides

$$x_t - p_t = [1-\beta(1-\alpha)]\, p_t^* + \beta(1-\alpha) E_t X_{t+1} - \overbrace{[1-\beta(1-\alpha) + \beta(1-\alpha)]}^{\text{equals one}} p_t$$

$$x_t - p_t = [1-\beta(1-\alpha)] \underbrace{(p_t^* - p_t)}_{\phi y_t \text{ (from } p_{it}^* - p_t = \phi y_t\text{)}} + \beta(1-\alpha)(E_t X_{t+1} - p_t)$$

same thing

$$x_t - p_{t-1} - (p_t - p_{t-1}) =$$

$$x_t - p_{t-1} - \pi_t = [1-\beta(1-\alpha)]\, \phi y_t + \beta(1-\alpha)(E_t X_{t+1} - p_t)$$

4) Using (7.54), substitute in for $(x_t - p_{t-1})$, $(x_{t+1} - p_t)$

Recall
$$\left.\begin{aligned} x_t - p_{t-1} &= \tfrac{1}{\alpha}\, \pi_t \\ E_t X_{t+1} - p_t &= \tfrac{1}{\alpha} E_t\, \pi_{t+1} \end{aligned}\right\} \text{(from 7.54)}$$

so...

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Romer's derivation of Calvo (cont.)

4) Using (7.54), substitute in ... (cont.)

$$\frac{1}{\alpha}\pi_t - \pi_t = [1-\beta(1-\alpha)]\phi y_t + \beta(1-\alpha)\frac{1}{\alpha}E_t\pi_{t+1}$$

Solve for π_t:

$$\pi_t = \underbrace{\frac{\alpha}{1-\alpha}[1-\beta(1-\alpha)]\phi}_{\text{call this } \kappa} y_t + \beta E_t \pi_{t+1} \qquad (7.60)$$