# NEW KEYNESIAN PHILLIPS CURVE
## "Indexation" (Romer 7.7, first part)

This is a component of Christiano, Eichenbaum, Evans (2005) which we will read later.

Like Calvo, except if you aren't allowed to adjust your $p_i$ this period, you don't hold $p_i$ fixed. Instead, you "index" $p_i$ to past inflation:

$$p_{it} - p_{it-1} = \pi_{t-1} = p_{t-1} - p_{t-2}$$

Result will be

$$\pi_t = \gamma \pi_{t-1} + (1-\gamma) E_t \pi_{t+1} + \varkappa y_t \qquad (7.77)$$

where $0 < \gamma < 1$

Sees: 1) built-in inertia

2) solves nonzero LRSS $\pi$ problem

Standard Calvo:

$\bar{\pi} = \beta \bar{\pi} + k y$. For $y = 0$, $\bar{\pi} = 0$

Indexation:

$\bar{\pi} = \gamma \bar{\pi} + (1-\gamma)\bar{\pi} + \varkappa y$. For $y = 0$, $\bar{\pi}$ can be anything

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## Indexation to lagged $\pi$

As in Calvo, prob. you can adjust is $\alpha$

so prob. price set today is still "in effect"

j periods in future is

$$q_j = (1-\alpha)^j$$

but here "in effect" means

$$p_{i_{t+j}} = \hat{x}_t = x_t + \sum_{j=0}^{j-1} \pi_{t+j}$$

not in Romer

In standard Calvo, our problem was

$$\min_{x_t} Z = E\left[\sum_{j=0}^{\infty} \beta^j q_{t+j} (x_{t+j} - p^*_{it+j})^2\right]$$

With indexation, it's

$$\min_{x_t} Z = E\left[\sum_{j=0}^{\infty} \beta^j q_{t+j} (\hat{x}_{t+j} - p^*_{it+j})^2\right]$$

Note this is different from model in 7.1,
so we can't use (7.15); must redo f.o.c. for
optimal $x_t$.

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"Indexation"

What is $\pi_t$ in terms of $x_t$?

Start by getting analogue to (7.53).

$$p_t = (1-\alpha)(p_{t-1} + \pi_{t-1}) + \alpha x_t \quad (7.68)$$

Subtract this from $x_t$:

$$x_t - p_t = x_t - (1-\alpha)(p_{t-1} + \pi_{t-1}) - \alpha x_t$$

$$= (1-\alpha)x_t - (1-\alpha)(p_{t-1} + \pi_{t-1})$$

$$= (1-\alpha)x_t - (1-\alpha)(p_{t-1} + \pi_{t-1}) + (1-\alpha)(p_t - p_t)$$

$$= (1-\alpha)(x_t - p_t) - (1-\alpha)(-p_t + p_{t-1} + \pi_{t-1})$$

$$= (1-\alpha)(x_t - p_t) - (1-\alpha)(-\pi_t + \pi_{t-1})$$

$$= (1-\alpha)(x_t - p_t) + (1-\alpha)(\pi_t - \pi_{t-1}) \quad (7.69)$$

Solve for $(x_t - p_t)$:

$$x_t - p_t = \frac{1-\alpha}{\alpha}(\pi_t - \pi_{t-1}) \quad (7.70)$$

So, if we can get $(x_t - p_t)$, ------

1) Redo f.o.c., get optimal $x_t$

$$\min_{x_t} Z = E\left[\sum_{j=0}^{\infty} \beta^j q_{t+j} (\hat{x}_{t+j} - p^*_{i\,t+j})^2\right]$$

What is $(\hat{x}_{t+j} - p^*_{i\,t+j})$?

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## "Indexation"

## 1) Redo F.O.C. (cont.)

$$\hat{x}_{t+j} = x_t + \sum_{\tau=0}^{j-1} \pi_{t+\tau}$$

$$p^*_{it+j} = p_{t+j} + \phi y_{t+j} = p_t + \sum_{\tau=1}^{j} \pi_{t+\tau} + \phi y_{t+1}$$

$$\hat{x}_{t+j} - p^*_{it+j} = x_t - p_t - \phi y_{t+1} + \sum_{\tau=0}^{j-1} \pi_{t+\tau} - \sum_{\tau=1}^{j} \pi_{t+\tau}$$

$$\sum_{\tau=0}^{j-1} \pi_{t+\tau} = \pi_t + \pi_{t+1} + \cdots \pi_{t+j-1}$$ (includes $\pi_{t}$, not $\pi_{t+j}$)

$$\sum_{\tau=1}^{j} \pi_{t+\tau} = \pi_{t+1} + \cdots \pi_{t+j-1} + \pi_{t+j}$$ (includes $\pi_{t+j}$, not $\pi_t$)

so $$\sum_{\tau=0}^{j-1} \pi_{t+\tau} - \sum_{\tau=1}^{j} \pi_{t+\tau} = \pi_t - \pi_{t+j}$$

so

$$\hat{x}_{t+j} - p^*_{it+j} = x_t - p_t - \phi y_{t+1} + \pi_t - \pi_{t+j}$$

Romer reverses it (doesn't matter):

$$e_{t,t+j} = p^*_{it+j} - \hat{x}_{t+j} = (p_t - x_t) + (\pi_{t+j} - \pi_t) + \phi y_{t+j} \quad (7.7))$$

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"Indexation"

1) Rel. to F.o.c. (cont.)

$$\min_{x_{it}} Z = \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j (x_{t+j} - p^*_{t+j})^2 = \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j e^2_{t,t+j}$$

(here, to simplify notation, I've left out E[ ]'s & variance)

$$= \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j (x_t - p_t - \pi_{t+j} + \pi_t - \phi y_{t+j})^2$$

$$0 = \frac{\partial Z}{\partial x} = \beta^0 (1-\alpha)^0 2 (x_t - p_t - \pi_t + \pi_t - \phi y_t)$$
$$+ \beta (1-\alpha) 2 (x_t - p_t - \pi_{t+1} + \pi_t - \phi y_{t+1})$$
$$+ \beta^2 (1-\alpha)^2 2 ( \ldots\ldots$$

Pull out $(x_t - p_t)$ from each row, put on LHS of an equation, divide both sides by 2, get:

$$(x_t - p_t)(\beta^0 (1-\alpha)^0 + \beta(1-\alpha) + \beta^2 (1-\alpha)^2 \ldots)$$
$$= \beta^0 (1-\alpha)^0 (\pi_t - \pi_t + \phi y_t) + \beta(1-\alpha)(\pi_{t+1} - \pi_t + \phi y_{t+1}) + \ldots$$

Note: $\beta^0 (1-\alpha)^0 + \beta(1-\alpha) + \beta^2 (1-\alpha)^2 + \ldots = \dfrac{1}{1-\beta(1-\alpha)}$ so

$$\frac{1}{1-\beta(1-\alpha)} (x_t - p_t) = \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j (\pi_{t+j} - \pi_t - \phi y_{t+j})$$

$$(x_t - p_t) = [1-\beta(1-\alpha)] \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j [E_t \pi_{t+j} - \pi_t + \phi E_t y_{t+j}] \quad (7.72)$$

Need to collapse down that infinity, as we did for (7.56). So ....

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2) Get $(x_t - p_t)$ in terms of $E_t(x_{t+1} - p_{t+1})$.

First, what is $E_t(x_{t+1} - p_{t+1})$? From 7.72),

$$x_{t+1} - p_{t+1} = [1-\beta(1-\alpha)] \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j [(E_{t+1}\pi_{t+1+j} - \pi_{t+1}) + \phi E_{t+1} y_{t+1+j}]$$

Replace $\pi_{t+1}$ with $\pi_t + (\pi_{t+1} - \pi_t)$

$$= [\quad] \sum \cdots [(E_{t+1}\pi_{t+1+j} - \pi_t - \pi_{t+1} + \pi_t + \phi \cdots\cdots]$$

$$= [1-\beta(1-\alpha)](-\pi_{t+1} + \pi_t) \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j$$

infinite series... $= \frac{1}{\beta(1-\alpha)}$

$$+ [1-\beta(1-\alpha)] \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j [(E_{t+1}\pi_{t+1+j} - \pi_t) + \phi E_{t+1} Y_{t+1+j}]$$

$$= -\pi_{t+1} + \pi_t + \cdots\cdots$$

$$E_t[x_{t+1} - p_{t+1}] = -E_t[\pi_{t+1} - \pi_t] + [1-\beta(1-\alpha)] \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j [E_t \cdots\cdots \tag{7.74}$$

Note that (7.74) assumes $E_t E_{t+1} x = E_t x$, which is "law of iterated projections" (expectations).

Note also that (7.74) means:

$$[1-\beta(1-\alpha)] \sum_{j=0}^{\infty} \cdots\cdots = E_t[x_{t+1} - p_{t+1}] + E_t[\pi_{t+1} - \pi_t]$$

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## "Indexation"

## 2) Get $(x_t - p_t)$ in terms of $E_t(x_{t+1} - p_{t+1})$ (cont.)

Now, where is $E_t(x_{t+1} - p_{t+1})$ within (7.72)?

Take (7.72), pull out first period:

$$x_t - p_t = [1-\beta(1-\alpha)]\left((\pi_t - \pi_t + \phi y_t) + \sum_{j=1}^{\infty} \beta^j (1-\alpha)^j [E_t \pi_{t+j} - \pi_t + \ldots]\right)$$

Now $\sum_{j=1}^{\infty} \beta^j (1-\alpha)^j [E_t \pi_{t+j} \ldots] = \beta(1-\alpha) \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j [E_t \pi_{t+1+j} \ldots]$

so
$$x_t - p_t = [1-\beta(1-\alpha)]\phi y_t + \beta(1-\alpha)\underbrace{[1-\beta(1-\alpha)]\sum_{j=0}^{\infty} \beta^j (1-\alpha)^j [E_t \pi_{t+j} - \pi_t + \phi E_t y_{t+j}]}_{\text{Look! This is bottom of previous page! Substitute that in!}}$$

$$x_t - p_t = [1-\beta(1-\alpha)]\phi y_t + \beta(1-\alpha)\left(E_t[x_{t+1} - p_{t+1}] + E_t[\pi_{t+1} - \pi_t]\right) \quad (7.75)$$

We're almost done! Look back at (7.70).

It says $x_t - p_t = \frac{1-\alpha}{\alpha}(\pi_t - \pi_{t-1})$

$$E_t[x_{t+1} - p_{t+1}] = \frac{1-\alpha}{\alpha}(E_t \pi_{t+1} - \pi_t)$$

Substitute that in, solve for $\pi_t$, gives...

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"Indexation"

3) Using (7.70), substitute in

$$\pi_t = \frac{1}{1+\beta}\pi_{t-1} + \frac{\beta}{1+\beta}E_t\pi_{t+1} + \frac{1}{1+\beta}\frac{\alpha}{1-\alpha}[1-\beta(1-\alpha)]\phi y_t \tag{7.76}$$

Recall $\beta$ is time-discounting term in utility function, must be close to one.

So $\beta \simeq 1$, $\frac{1}{1+\beta} \simeq \frac{1}{2}$, $\frac{\beta}{1+\beta} \simeq \frac{1}{2}$