IS/LM

William Poole (QJE 1970)

"Optimal Choice of Monetary Instruments in a Simple Stochastic Macro Model"

Example of a paper that uses IS/LM with fixed P to analyze issue in monetary policy (central banks).

To get LM in IS/LM, M^s is fixed (but controllable as a policy variable).

Early Keynesians thought of central banks as choosing M^s. When policy committee meets, choose a number for M^s to hold until next meeting.

An alternative way to do policy: when committee meets, choose i to hold until next meeting. (Staff then adjusts M^s as needed to keep i at desired level.)

Poole (1970) is about this.

We'll skip parts IV, THE COMBINATION POLICY
V. A DYNAMIC MODEL

What central banks do today: fix i.

IS/LM

Poole (1970): fix i or M?

Alternative ways to do monetary policy:

What Central Bank can do

Central Bank (policy committee) can set M_t <u>or</u> i_t based on information available at time t (or t-1)

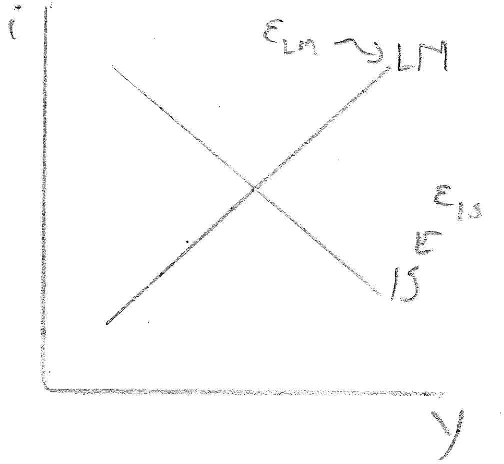
given IS stuff $Y = Y(r, G, \text{etc.}) + \varepsilon_{IS}$ ← "IS shock"

LM stuff $M/P = L(i, Y) + \varepsilon_{LM}$ ← "LM shock"

Fix M

Given $\hat{M}_t$,

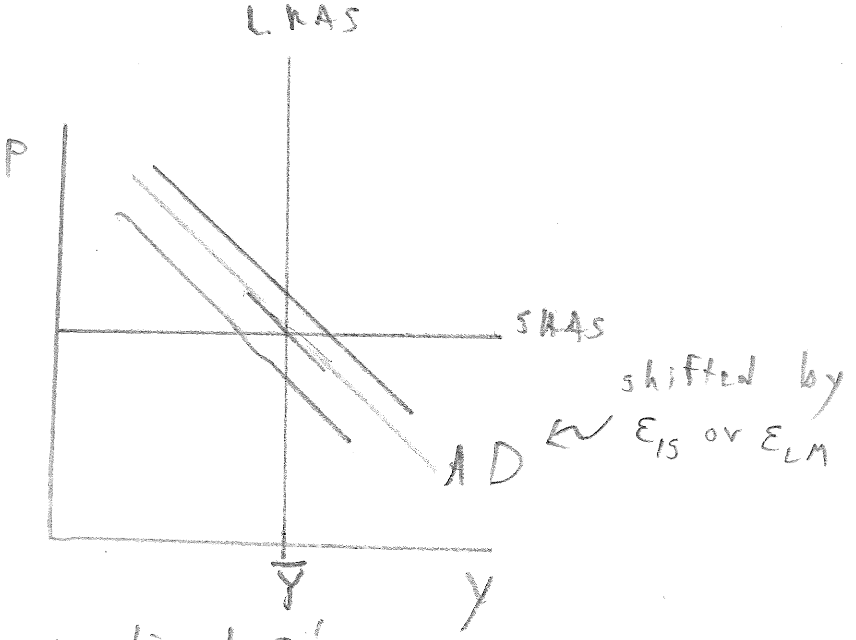
Fixed P



realized ε_{IS}

ε_{LM}

determine i, Y



realized ε's

determine i, Y, and P (given AS)

If Central Bank can't see ε_{IS}, ε_{LM} when setting M, realizations of ε's affect Y in SR

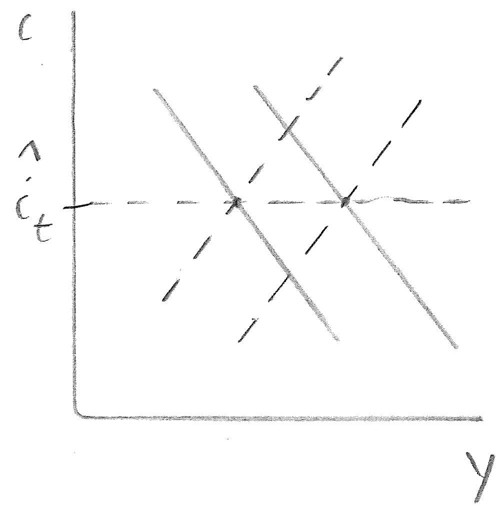
IS/LM

Poole [1970]

Alternative ways (cont.)

Fix i_t

Vary M_t to hold i_t at a fixed value $\hat{i}_t$



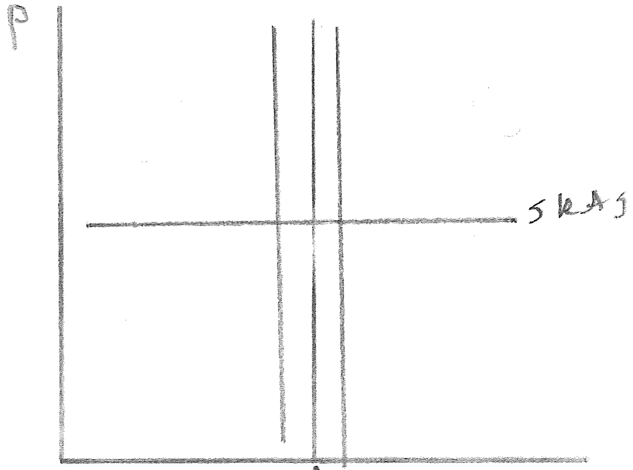
ε_{LM} is irrelevant

ε_{IS} affects Y

No relation between P & Y!

(Any effect of ΔP on $\frac{M}{P}$ is counteracted by ΔM)

How would we describe AD?



AD is vertical at $\hat{Y}$

$\hat{Y}$ shifted by π^e, G, ..., ε_{IS}

$$\hat{Y} = Y(\hat{i}_t + \pi^e, G, etc.) + \varepsilon_{IS}$$

? Should Central Bank fix i or M?

IS/LM
Poole (1970)

SOME THINGS ABOUT VARIANCES

Random variables u & v

$$u \quad \underset{\substack{\uparrow \\ \text{mean} \\ \boxed{E[u]}}}{\bar{u}} \quad \underset{\substack{\uparrow \\ \text{variance} \\ \boxed{E[(u-\bar{u})^2]}}}{\sigma_u^2} \qquad\qquad V \quad \bar{V} \quad \sigma_V^2$$

Say

$$Y = a + bu + cV$$

$$\bar{Y} = E[Y] = a + b\bar{u} + c\bar{V}$$

$$E[(Y-\bar{Y})^2] = \sigma_y^2$$

$$\sigma_y^2 = b^2\sigma_u^2 + c^2\sigma_V^2$$ if u & v independent, have no covariance

Covariance $\sigma_{uv} = E[(u-\bar{u})(v-\bar{v})]$

$$\sigma_y^2 = b^2\sigma_u^2 + c^2\sigma_V^2 + 2bc\,\sigma_{uv}$$

IS/LM
POOLE (1970): FIX i OR M?

ASSUMPTIONS ABOUT ECONOMY

AS: FIXED PRICE LEVEL $P = \bar{P}$ SO $\underset{\text{NOMINAL RATE}}{\nwarrow i} = \underset{\text{REAL RATE}}{r \nwarrow}$

AD:

IS CURVE $Y = a_0 + \overset{\swarrow\text{NEGATIVE}}{a_1} r + u \leftarrow \boxed{\text{IS shock}}$

LM CURVE $M = b_0 + b_1 Y + \underset{\nwarrow\text{NEGATIVE}}{b_2} r + v \leftarrow \boxed{\text{LM shock}}$

VARIANCES σ_u^2, σ_v^2

WE'LL SIMPLIFY POOLE, ASSUME u & v INDEPENDENT (NO COVARIANCE)

WHAT CENTRAL BANK CAN DO

SET M OR SET r OR SET $M(r)$

CENTRAL BANK'S PREFERENCES

MINIMIZE $L = E\left[(Y - Y_F)^2\right]$ "Loss Function"

Note this assumes

$Y < Y_F$ is bad (recessions are bad)

$Y > Y_F$ is bad (booms are bad) Huh?

IS/LM

POOLE (1970): FIX r OR M?

CENTRAL BANK'S PROBLEM WITHOUT UNCERTAINTY $(u = v = 0)$

FIX r

$$\min_{r} \; L = E\left[(Y - Y_f)^2\right] \text{ s.t. } Y = a_0 + a_1 r$$

$$L = (a_0 + a_1 r - Y_f)^2$$

$$\frac{\partial L}{\partial r} = 0 = 2(a_0 + a_1 r^* - Y_f)\, a_1$$

$$r^* = \frac{Y_F - a_0}{a_1}$$

OUTCOME: $Y = a_0 + a_1\left(\frac{Y_f - a_0}{a_1}\right) = Y_F$

$L = 0$

$$M = b_0 + b_1 Y_F + b_2\left(\frac{Y_f - a_0}{a_1}\right) = \frac{(a_1 b_1 + b_2) Y_F - a_0 b_2 + a_1 b_0}{a_1}$$

FIX M

$$\min_{M} \; L = E\left[(Y - Y_f)^2\right] \text{ s.t. } Y = \frac{a_0 b_2 + a_1(M^s - b_0)}{a_1 b_1 + b_2}$$

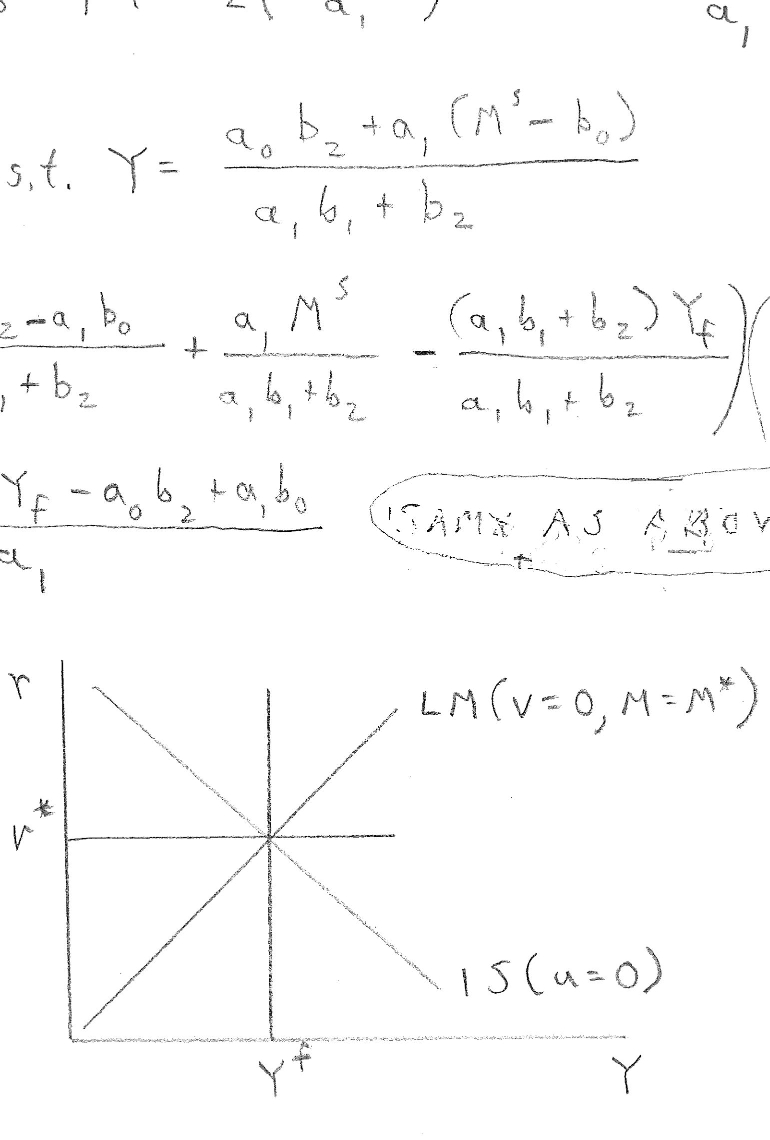
$$\frac{\partial L}{\partial M^s} = 0 = 2\left(\frac{a_0 b_2 - a_1 b_0}{a_1 b_1 + b_2} + \frac{a_1 M^s}{a_1 b_1 + b_2} - \frac{(a_1 b_1 + b_2) Y_f}{a_1 b_1 + b_2}\right)\left(\frac{a_0 b_2 - a_1 b_0}{a_1 b_1 + b_2}\right)$$

$$M^* = \frac{(a_1 b_1 + b_2) Y_F - a_0 b_2 + a_1 b_0}{a_1}$$ SAME AS ABOVE!

$Y = \ldots = Y_F$

$L = 0$

$$r = \frac{Y_f - a_0}{a_1}$$



IS/LM

POOLE (1970): FIX r OR M?

Central Bank's PROBLEM WITH UNCERTAINTY

FIX r

$$\min_{r} E[(Y-Y_f)^2] \text{ s.t. } Y = a_0 + a_1 r + u \quad \text{and } E[u] = 0$$

$$E[(Y-Y^f)^2] = E[(a_0 + a_1 r + u - Y^f)^2]$$

$$= E[\text{stuff} + a_1 r(a_0 + a_1 r + u - Y^f) + a_1 r(a_0 + u - Y^f)]$$

$$= E[\text{stuff} + a_1 r(2a_0 + 2u - 2Y^f) + a_1^2 r^2]$$

F.O.C.:

$$\frac{\partial E[\ ]}{\partial r} = E[a_1(2a_0 + 2u - 2Y^f) + 2a_1^2 r^*] = 0$$

$$\text{hence} \quad r^* = \frac{Y^f - a_0 - E[u]}{a_1} = \frac{Y^f - a_0}{a_1} \leftarrow$$

Same as r^* without uncertainty!

This is an example of "certainty equivalence": it holds because loss function is quadratic and model is linear.

(otherwise, r^* with uncertainty might be different from r^*

IS/LM

POOLE (1970): FIX r OR M?

CENTRAL BANK'S PROBLEM WITH UNCERTAINTY

FIX r

WHAT IS OUTCOME IF YOU CHOOSE

$$r = r^* = \frac{Y_f - a_0}{a_1}$$

OUTCOME: $Y = a_0 + a_1\left(\frac{Y_f - a_0}{a_1}\right) + u = Y_f + u$

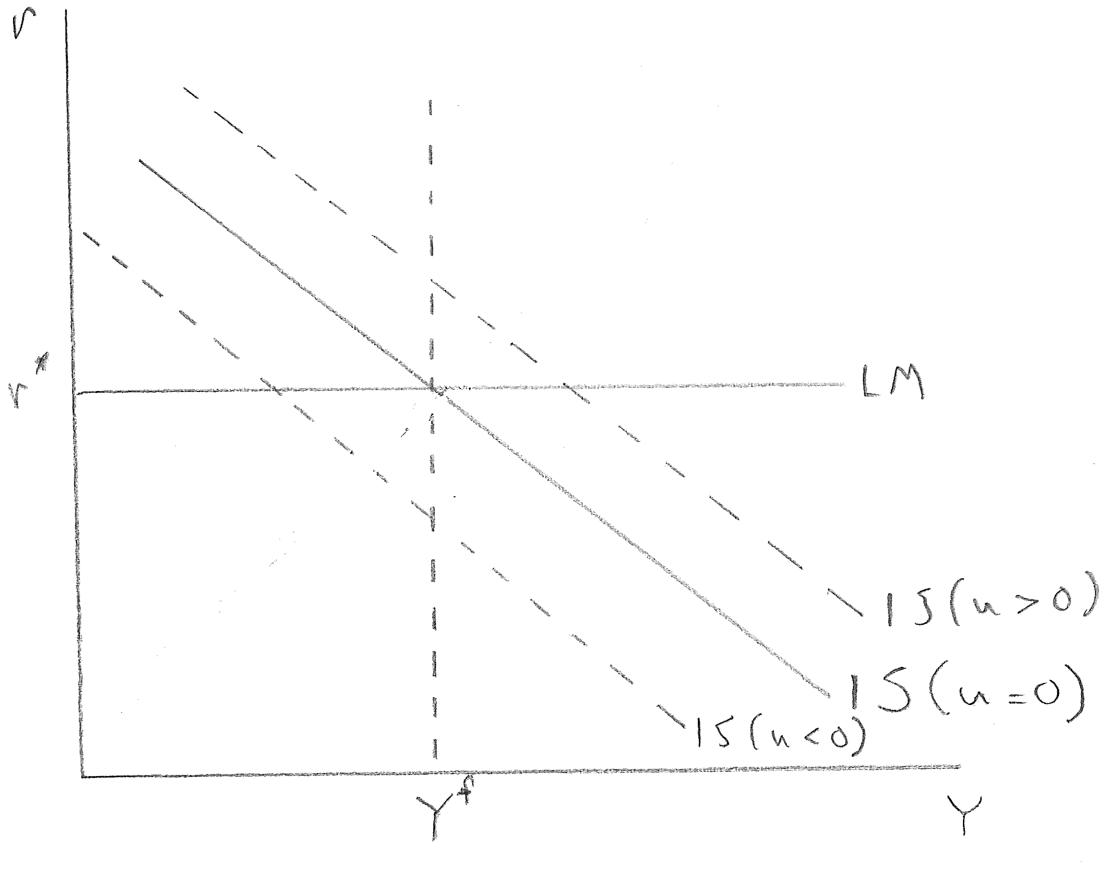
$$Y - Y_f = u$$

$$(Y - Y_f)^2 = u^2$$

$$E[Y] = Y_f$$

$$L = E\left[(Y - Y_f)^2\right] = E\left[u^2\right] = \sigma_u^2$$

$\boxed{\text{Note: LM shocks have no effect on } Y}$



IS/LM

POOLE (1970): FIX r OR M?

CENTRAL BANK'S PROBLEM WITH UNCERTAINTY

FIX M

$$\min_{M^s} L = E\left[(Y-Y_f)^2\right] \text{ s.t. } Y = \frac{a_0 b_2 + a_1(M^s - b_0) + b_2 u - a_1 v}{a_1 b_1 + b_2}$$

$$\text{SOLUTION: } M^* = \frac{(a_1 b_1 + b_2) Y_f - a_0 b_2 + a_1 b_0}{a_1} \quad \boxed{\text{Certainty equivalence}}$$

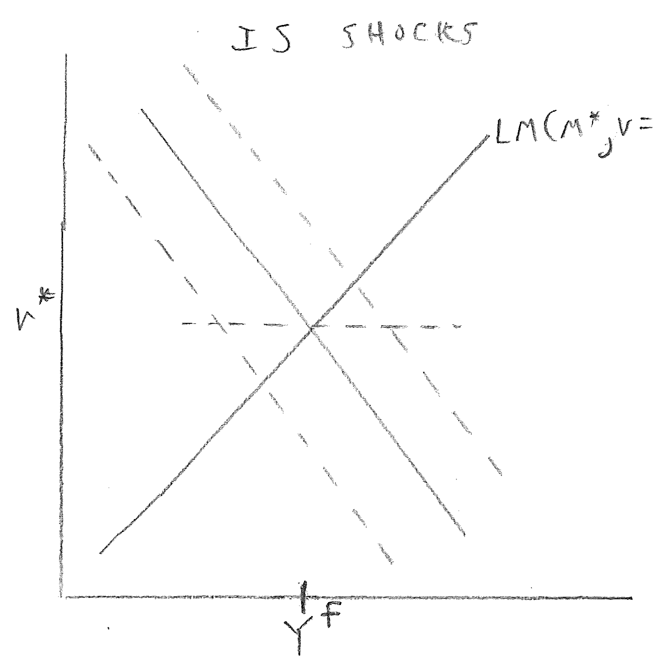
$$\text{OUTCOME: } Y = \ldots\ldots = Y_f + \frac{b_2}{a_1 b_1 + b_2} u - \frac{a_1}{a_1 b_1 + b_2} v$$

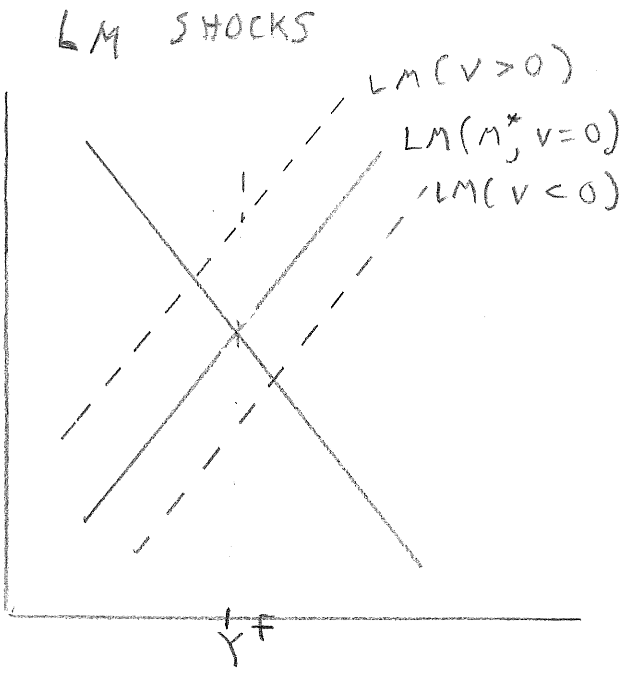
$$Y - Y^f = \frac{b_2}{a_1 b_1 + b_2} u - \frac{a_1}{a_1 b_1 + b_2} v$$

$$(Y - Y^f)^2 = \frac{b_2^2}{(a_1 b_1 + b_2)^2} u^2 - \frac{2 a_1 b_2}{(a_1 b_1 + b_2)^2} u v + \frac{a_1^2}{(a_1 b_1 + b_2)^2} v^2$$

$$E[Y] = Y^f$$

$$L = E\left[(Y - Y^f)^2\right] = \frac{b_2^2}{(a_1 b_1 + b_2)^2} \sigma_u^2 + \frac{a_1^2}{(a_1 b_1 + b_2)^2} \sigma_v^2$$





IS/LM

POOLE (1970): FIX r OR M?

WITH UNCERTAINTY

COMPARE TWO POLICIES

FIX r

$$L = \sigma_u^2$$

LM shocks have no effect on Y if you fix r, but IS shocks have a big effect.

. . .

FIX M

$$L = \underbrace{\frac{b_2^2}{(a_1 b_1 + b_2^2)}}\sigma_u^2 + \frac{a_1^2}{(a_1 b_1 + b_2)^2}\sigma_v^2$$

Coefficient on σ_u^2 is less than one, so IS shocks have smaller effect on Y if you fix M. Why? An omniscient central banker would fix r and raise r^* in response to positive IS shock (u↑). These central bankers can't see u, so they can't do that. But fixing M causes automatic r↑ in response to u↑.

Lesson for policymakers

Better to

- Fix r if you have lots of confidence in your forecasts of GDP at a given r, but know little about money demand.
- Fix M if you know lots about money demand, but find it hard to forecast spending.

[Poole says we probably know more about LM, so better to fix M.