

1) Consider the following model:

$$y_t = E_t y_{t+1} - r_t$$

$$\pi_t = E_t \pi_{t+1} + y_t + u_t \text{ where } u_t = \rho u_{t-1} + \epsilon_t \text{ and } \epsilon_t \text{ is "white noise" (mean-zero i.i.d.).}$$

$y$  is the output gap.  $r$  is the gap between the real interest rate and the natural rate of interest. Expectations are rational. Assume there is a long-run steady state where  $y = 0$ ,  $\pi = 0$ .

Every period a central bank chooses  $r_t$  to minimize a loss function  $L_t = E_t \sum_{\tau=0}^{\infty} \beta^\tau \frac{1}{2} (y_{t+\tau}^2 + \pi_{t+\tau}^2)$

The central bank has "discretion," that is, its choice of  $r_t$  in no way constrains its future behavior.

At the time the central bank chooses  $r_t$ , it knows  $u_t$  and the public's  $E_t \pi_{t+1}$ .

a) Solve this model for  $\pi_t$ ,  $y_t$  and  $r_t$ . Hint one: this model is similar to the one in Clarida, Gali and Gertler, "The Science of Monetary Policy: a New Keynesian Perspective." Solve it in the same way.

b) Draw a graph with time on the horizontal axis and  $\pi$  on the vertical axis. On the graph draw a line that shows how  $\pi_t$  evolves over time in response to a positive realization of  $\epsilon$  at time  $t_0$ , assuming that just before  $t_0$ , the variables  $y$ ,  $\pi$  and  $u$  were all zero.

c) Draw a graph with time on the horizontal axis and  $r$  on the vertical axis. On the graph draw a line that shows how  $r_t$  evolves over time in response to a positive realization of  $\epsilon$  at time  $t_0$ , assuming that just before  $t_0$ , the variables  $y$ ,  $\pi$  and  $u$  were all zero.

d) Would it be possible to reproduce the behavior of this central bank with an "interest rate rule" of the form  $r_t = \phi \pi_t$ ? If your answer is yes, what would be the coefficient  $\phi$ ?

2) Consider the following model:

$$y_t = E_t y_{t+1} - r_t + g_t \text{ where } g_t = \rho g_{t-1} + \epsilon_{gt} \text{ and } \epsilon_{gt} \text{ is "white noise" (mean-zero i.i.d.).}$$

$$\pi_t = E_t \pi_{t+1} + y_t$$

$y$  is the output gap.  $r$  is the gap between the real interest rate and the natural rate of interest. Expectations are rational. Assume there is a long-run steady state where  $y = 0$ ,  $\pi = 0$ .

Every period a central bank chooses  $r_t$  to minimize a loss function  $L_t = E_t \sum_{\tau=0}^{\infty} \beta^\tau \frac{1}{2} (y_{t+\tau}^2 + \pi_{t+\tau}^2)$

The central bank has "discretion," that is, its choice of  $r_t$  in no way constrains its future behavior.

At the time the central bank chooses  $r_t$ , it knows  $g_t$  and the public's  $E_t \pi_{t+1}$ .

a) Solve this model for  $\pi_t$ ,  $y_t$  and  $r_t$ . Hint one: this model is similar to the one in Clarida, Gali and Gertler, "The Science of Monetary Policy: a New Keynesian Perspective." Solve it in the same way.

b) Draw a graph with time on the horizontal axis and  $y$  on the vertical axis. On the graph draw a line that shows how  $y_t$  evolves over time in response to a positive realization of  $\epsilon$  at time  $t_0$ , assuming that just before  $t_0$ , the variables  $y$ ,  $\pi$  and  $g$  were all zero.

c) Draw a graph with time on the horizontal axis and  $r$  on the vertical axis. On the graph draw a line that shows how  $\pi_t$  evolves over time in response to a positive realization of  $\epsilon$  at time  $t_0$ , assuming that just before  $t_0$ , the variables  $y$ ,  $\pi$  and  $g$  were all zero.

d) Would it be possible to reproduce the behavior of this central bank with an "interest rate rule" of the form  $r_t = \phi y_t$ ? If your answer is yes, what would be the coefficient  $\phi$ ?