Economics 614, Advanced Macro. Problem set on capacity utilization

Consider a model in which a representative-agent household maximizes $E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\theta} C_t^{1-\theta} - \frac{1}{1+\lambda} L_t^{1+\lambda} \right]$

The agent can hold capital *K* and choose the capital utilization rate u_t. The agent earns a nominal rental rate R for each unit of "effective capital" $(uK)_t$. The agent's nominal wealth is *A*. At time *t*, the agent takes as given A_t, the wage W_t, the nominal interest rate i_t and the price level P_t. She chooses consumption, labor, investment I_t (purchases of more capital, or sales in which case *I* is negative) and u_t, among other things. The price of a unit of capital is always equal to the price of a unit of consumption. The effective-capital rental rate is determined in a competitive market. For a firm, capital is a "variable factor" like labor.

1) Suppose that the agent's nominal wealth evolves as:

$$A_{t+1} = (1+i_t)[A_t + W_t L_t + R_t u_t K_t - P_t(C_t + I_t + u_t^2 K_t)] + P_{t+1}[I_t + (1-\delta)K_t]$$

The term u^2 represents a current cost of running a unit of capital harder. δ is the depreciation rate (a fixed parameter). Starting from the Bellman equation, derive the value of u_t that the agent will set for period *t*.

2) Now suppose that the agent's nominal wealth evolves as:

$$A_{t+1} = (1+i_t)[A_t + W_t L_t + R_t u_t K_t - P_t(C_t + I_t)] + P_{t+1}[I_t + (1-\theta u_t^2)K_t]$$

Here there is no current cost of running capital harder, but the depreciation rate increases with u_t.

Starting from the Bellman equation, derive the value of u_t that the agent will set for period *t* in terms of the real interest rate *r*, using the usual approximation that $(1+r)_t \approx (1+i_t)/(1+E_t\pi_{t+1})$ where π is the inflation rate.