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4.10) For LRSS without shocks,

a) Get expressions defining $y^*, k^*, c^*, w^*, l^*, r^*$

recall c/AL vs. c in utility function

b) Assuming $\alpha = \frac{1}{3}$, $g = 0.5\% = 0.005$, $n = 0.25\% = 0.0025$,
$\delta = 2.5\% = 0.025$, $(G^*/Y) = 0.2$, ← share of G in Y
$r^* = 1.5\% = 0.015$, $l^* = \frac{1}{3}$,
what is $\frac{C}{Y}$, $\frac{I}{Y}$, $\frac{K}{\text{Annual } Y} = \frac{K}{4 \cdot Y}$ ← quarterly Y

Note: in class I derived LRSS values for $G = 0$.

a) From $\frac{1}{c_t} = e^{-\theta} \frac{1}{c_{t+1}}(1+r^*)$ and $\frac{c_{t+1}}{c_t} = e^g$,

we get $(1+r^*) = e^{g+\theta}$

$$r^* = e^{g+\theta} - 1$$

From $y^* = k^{*\alpha}$ we get $r^* = f'(k) - \delta = \alpha k^{*-(1-\alpha)} - \delta$

solve for k^*,

$$k^* = \left(\frac{\alpha}{e^{g+\theta} \cdot \delta - 1}\right)^{\frac{1}{1-\alpha}} = \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{1}{1-\alpha}}$$

$$y^* = \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{\alpha}{1-\alpha}}$$

4.10) (cont.)

Recall $w = MPL$ where $Y = K^{\alpha}(AL)^{1-\alpha}$

hence $w = K^{\alpha}(1-\alpha)(AL)^{-\alpha} A = A(1-\alpha)\left(\frac{K}{AL}\right)^{\alpha} = A(1-\alpha)k^{\alpha}$

$w^* = A(1-\alpha)k^{*\alpha}$ ← (defined above)

what about $\frac{C}{L}$ and $\frac{C}{AL}$?

$c^* = \left(\frac{C}{AL}\right)^* = f(k^*) - (n+g+\delta)k^* - G^*$ ← $\left(\frac{G}{AL}\right)^*$

We're not told G^*; we're told $\frac{G^*}{Y}$ ← $\frac{\text{Govt./AL}}{Y/AL} = \frac{\text{Govt.}}{Y} = 0.2$

so put it in terms of G^*/Y:

$c^* = y^* - y\frac{G^*}{Y} - (n+g+\delta)k^*$

$= \left(1 - \frac{G^*}{Y}\right)y^* - (n+g+\delta)k^*$

$= \left(1 - \frac{G^*}{Y}\right)k^{*\alpha} - (n+g+\delta)k^*$

and

$\frac{C^*}{L} = \frac{C}{AL} \cdot A = Ac^* = A\left(\left(1 - \frac{G^*}{Y}\right)k^{*\alpha} - (n+g+\delta)k^*\right)$

Finally, what's ℓ^*?

From $U'_{1-\ell}(1-\ell) = wU'_c(C/L)$, here $b\frac{1}{1-\ell} = w\frac{1}{C/L}$,

we have $\ell^* = 1 - b\frac{(C/L)^*}{w^*}$

$= 1 - b\frac{\left(\left(1 - G/Y^*\right)k^{*\alpha} - (n+g+\delta)k^*\right)}{(1-\alpha)k^{*\alpha}}$

4.10) (cont.)

b) Put in parameter values

$$k^* = \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{1}{1-\alpha}} = \left(\frac{1/3}{0.015 + 0.025}\right)^{\frac{1}{1-1/3}} \approx 24.1$$

$$y^* = k^{\alpha} = 24.1^{1/3} \approx 2.9$$

$$c^* = \frac{C}{AL} = (1 - G^*)y^* - (n + g + \delta)k^*$$

$$= (1 - 0.2)2.9 - (0.0025 + 0.0050 + .025)24.1$$

$$\approx 1.53$$

What's $\frac{C}{Y}$, $\frac{I}{Y}$, $\frac{K}{4Y}$?

$$\frac{C}{Y} = \frac{C/AL}{Y/AL} = \frac{1.53}{2.9} \approx 0.53 \text{ or } 53\%$$

$$\frac{I}{Y} = 1 - \frac{G}{Y} - \frac{C}{Y} = 1 - 0.2 - 0.53 = .27 \text{ or } 27\%$$

$$\frac{K}{4Y} = \frac{1}{4}\frac{K/AL}{Y/AL} = \frac{1}{4}\frac{24.1}{2.9} \approx 2.07$$