RBC Thy Problems

5.8) $E[U] = E\left[\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t u(C_t)\right]$

where $u(C_t) = C_t - \Theta C_t^2$ hence $u'(C_t) = 1 - 2\Theta C_t$

$Y_t = AK_t + e_t$ means $\frac{\partial Y}{\partial K} = A$ ← ("interest rate")

$K_{t+1} = K_t + Y_t - C_t$ hence $\partial K_{t+1}/\partial C_t = -1$, $C_t = K_{t+1} + K_t + Y_t$

$e_t = \phi e_{t-1} + \varepsilon_t$ where $E[\varepsilon] = 0$ hence $E[e_t] = \phi e_{t-1}$

and $A = \rho$

("productivity shock")

a) Find Euler equation

Variables that define current state: K_t, e_t

$$V(K_t, e_t) = \max_{C_t}\left\{u_c(C_t) + \frac{1}{1+\rho}E_t\left[V(K_{t+1}, e_{t+1})\right]\right\}$$

$$\frac{\partial V}{\partial C_t} = 0 = 1 - 2\Theta C_t + \frac{1}{1+\rho}E_t\left[V_K(\ )\underbrace{\frac{\partial K_{t+1}}{\partial C_t}}_{-1}\right]$$

What's V_K? Envelope theorem, Benveniste-Scheinkman

$$V_K(K_t, e_t) = \frac{\partial u}{\partial C_t}\frac{\partial C_t}{\partial K_t} = (1 - 2\Theta C_t)\frac{\partial C_t}{\partial K_t}$$

and $$\frac{\partial C_t}{\partial K_t} = 1 + \frac{\partial Y_t}{\partial K_t} = 1 + A = 1 + \rho$$

so $V_K = (1 - 2\Theta C_t)(1+\rho)$

$$E_t[V_K(\ _{+1}\ )] = E[(1 - 2\Theta C_{t+1})(1+\rho)]$$
$$= (1+\rho)(1 - 2\Theta C^e_{t+1})$$

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5,8) a) (cont.)

$$0 = 1 - 2\Theta C_t + \frac{1}{1+\rho}(1+\rho)(1 - 2\Theta C^e_{t+1})$$

$$C_t = C^e_{t+1}$$ "Consumption is a random walk"

b) Conjecture $C_t = \alpha + \beta K_t + \gamma e_t$

then $K_{t+1} = K_t + \underbrace{Y_t}_{A K_t + e_t} - C_t = (1+\rho-\beta)K_t + (1-\gamma)e_t - \alpha$

c) What values of α, β, γ could work (satisfy Euler equation)?

$$C^e_{t+1} = E_t[\alpha + \beta K_{t+1} + \gamma e_{t+1}] = \alpha + \beta E[K_{t+1}] + \gamma \overbrace{E[e_{t+1}]}^{\phi e_{t-1}}$$

$$= \alpha + \beta((1+\rho-\beta)K_t + (1-\gamma)e_t - \alpha) + \gamma\phi e_{t-1}$$

$$= (1-\beta)\alpha + (1+\rho-\beta)\beta K_t + (\beta + \gamma(\phi-\beta))e_t$$

and $C^e_{t+1} = C_t = \alpha + \beta K_t + \gamma e_t$

so question is, what values of α, β, γ satisfy

$$\overbrace{\alpha + \beta K_t + \gamma e_t}^{C_t} = \ldots\ldots$$

means
$$\alpha = (1-\beta)\alpha$$
$$\beta = (1+\rho-\beta)\beta$$
$$\gamma = \beta + \gamma(\phi-\beta)$$

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c) (cont.)

$$\alpha = (1-\beta)\alpha \qquad \beta = (1+\rho-\beta)\beta \qquad \gamma = \beta + \gamma(\phi - \beta)$$

One solution:

$$1 = (1-\beta)$$

hence $\beta = 0 \Rightarrow 0 = (1+\rho-0)0 \Rightarrow \gamma = 0 + \gamma(\phi - 0)$

$$\gamma = \gamma\phi$$

$\gamma = 0$ and α can be anything.

Another solution:

$\alpha = 0$

$$\beta = (1+\rho-\beta)\beta$$

$$1 = 1+\rho-\beta$$

$\beta = \rho \ (=A) \Rightarrow \gamma = \rho + \gamma(\phi - \rho)$

$$= \rho + \gamma\phi - \gamma\rho$$

$$\gamma + \gamma\rho - \gamma\phi = \rho$$

$$(1+\rho-\phi)\gamma = \rho$$

$$\gamma = \frac{\rho}{1+\rho-\phi}$$

substituting these into conjectured equation for C_t gives

$$C_t = \rho K_t + \left(\frac{\rho}{1+\rho-\phi}\right) e_t$$

and

$$K_{t+1} = (1+\rho-\rho)K_t + \left(1 - \frac{\rho}{1+\rho-\phi}\right) e_t$$

$$= K_t + \frac{1-\phi}{1-\phi+\rho} e_t$$

With these two equations, you can trace out path of C & K over time in response to ε_t

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5.8) cont.

d) Effects of one-time ε shock on paths of Y, K, C

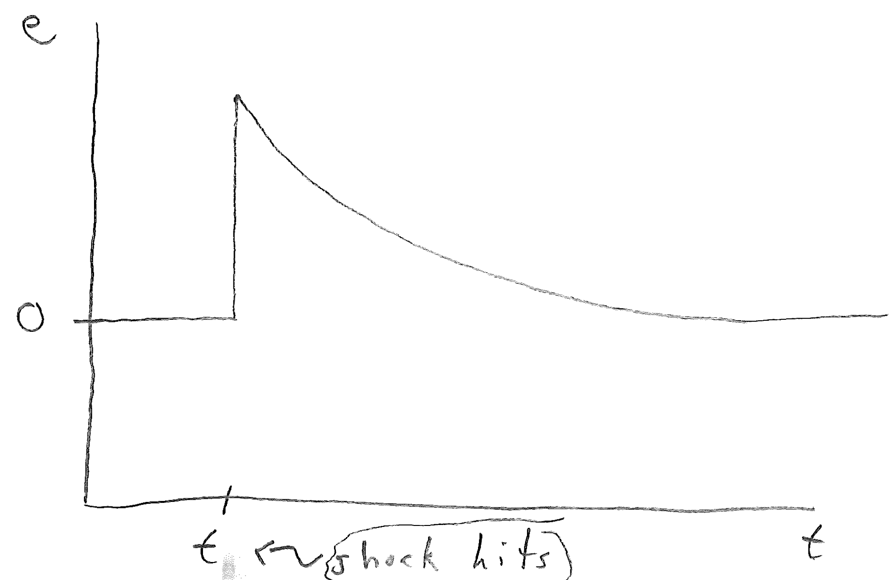
$$e = \phi e_{t-1} + \varepsilon_t$$

$$C_t = \rho K_t + \left(\frac{\rho}{1+\rho-\phi}\right) e_t$$

$$K_{t+1} = K_t + \frac{1-\phi}{1-\phi+\rho} e_t$$

$$Y_t = A K_t + e_t$$

Path for e:



Note: e is zero before & eventually after.

With zero e,

$$K_{t+1} = K_t \quad \text{hence}$$

before shock $K_t = K_{t-1} = K_{t-2} = K_{t-3} \cdots$

eventually, after shock $K_{t+j} = K_{t+j+1} = K_{t+j+2} \cdots$

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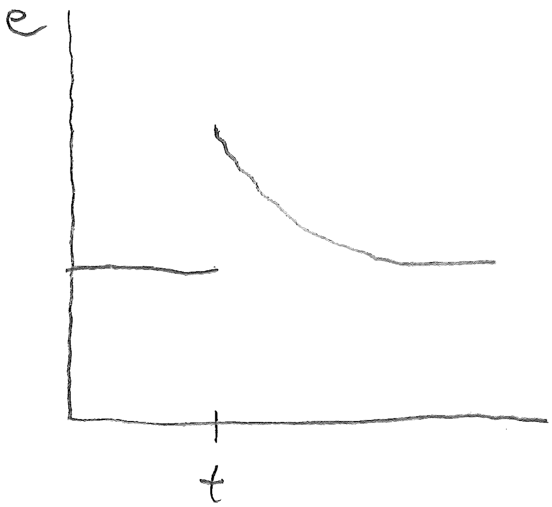
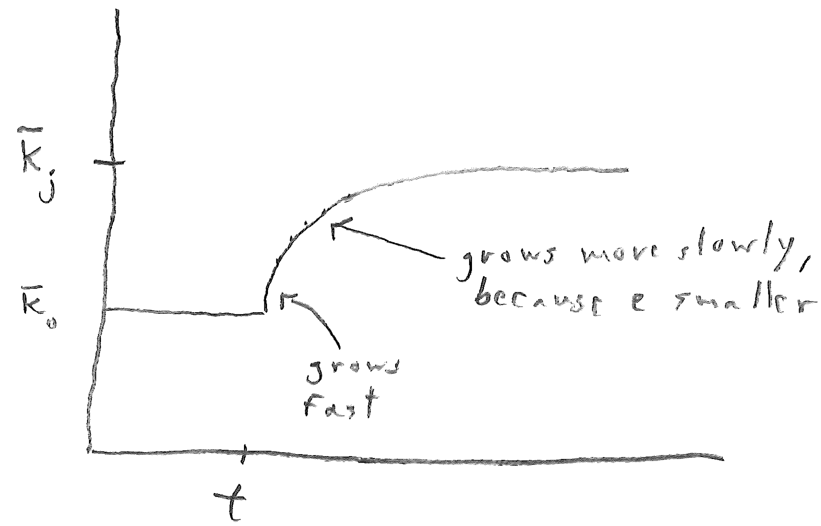
5.8) cont.

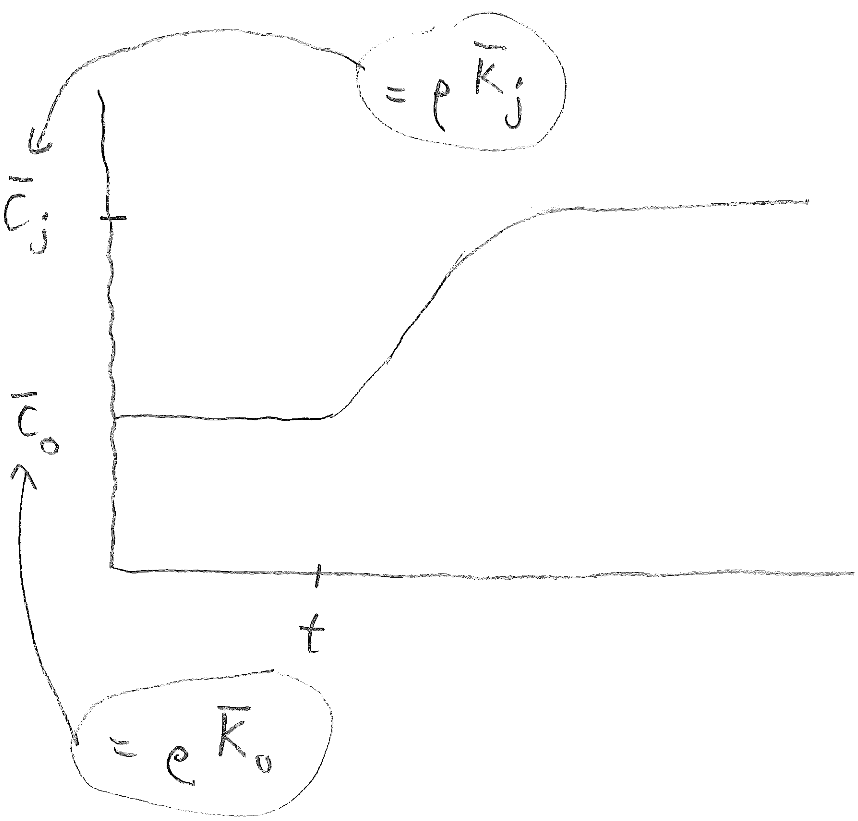
In the meantime, before e has settled down, K grows

$$K_{t+1} = K_t + \frac{1-\phi}{1-\phi+\rho}\, e_t$$





$$C_t = \rho K_t + (\quad) e_t$$



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5.8) cont.

Finally, $Y_t = A K_t + e_t$