ANSWER TO 6.12 (menu costs)

$\pi(y, r)$ Profit of firm

y Aggregate output

r Relative price

$r^*(y)$ Profit-maximizing price

Note: profit-maximization

$\max_r \pi(y,r)$ means $\frac{\partial \pi(y,r)}{\partial r} = 0$ at r^*.

Funny notation:

$$\pi_1(y, r) = \frac{\partial \pi_1}{\partial y}$$

$$\pi_2(y, r) = \frac{\partial \pi}{\partial r}$$

$$\pi_{11} = \frac{\partial^2 \pi}{\partial y^2} \qquad \pi_{22} = \frac{\partial^2 \pi}{\partial r^2}$$

$$\pi_{12} = \frac{\partial^2 \pi}{\partial y \, \partial r} \qquad \pi_{21} = \frac{\partial^2 \pi}{\partial r \, \partial y}$$

A math thing: $\pi_{12} = \pi_{21}$

a) Incentive to adjust price when y goes from y_0 to y_1

$\pi(y_1, r^*(y_0))$ Profit if $y = y_1$ but price is set at profit-maximizing level for y_0

$\pi(y_1, r^*(y_1))$ " " " and I adjust price to profit-maximizing level for y_1

Gain in profit from adjusting price:

$$G = \pi(y_1, r^*(y_1)) - \pi(y_0, r^*(y_0))$$

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b) Second-order Taylor approximation

For y_1 close to y_0,

$$G \approx G\Big|_{y_1=y_0} + \frac{\partial G}{\partial y_1}\Big|_{y_1=y_0} (y_1-y_0) + \frac{1}{2}\frac{\partial^2 G}{\partial y_1^2}\Big|_{y_1=y_0} (y_1-y_0)^2$$

$$G = \pi(y_1, r^*(y_1)) - \pi(y_1, r^*(y_0)) = 0 \text{ for } y_1 = y_0$$

$$\frac{\partial G}{\partial y} = \frac{\partial \pi(y_1, r^*(y_1))}{\partial y_1} + \frac{\partial \pi(\ )}{\partial r}\frac{\partial r^*(y_1)}{\partial y_1} - \frac{\partial \pi(y_1, r^*(y_0))}{\partial y_1}$$

For $y_0 = y_1$, $\dfrac{\partial \pi(y_1, r^*(y_1))}{\partial y_1} = \dfrac{\partial \pi(y_1, r^*(y_0))}{\partial y_1}$

and profit-max means $\dfrac{\partial \pi(\ )}{\partial r} = 0$ at r^*

hence

$$\frac{\partial G}{\partial y} = 0 \text{ for } y_1 = y_0$$

What about $\dfrac{\partial^2 G}{\partial y_1^2}$?

$$= \frac{\partial^2 \pi(y_1, r^*(y_1))}{\partial y_1 \partial y_1} + \frac{\partial^2 \pi(\ )}{\partial y\, \partial r}\frac{\partial r}{\partial y}$$

$$+ \frac{\partial^2 \pi(\ )}{\partial r\, \partial y}\frac{\partial r}{\partial y} + \frac{\partial^2 \pi(\ )}{\partial r\, \partial r}\frac{\partial r}{\partial y}\frac{\partial r}{\partial y} + \frac{\partial \pi(\ )}{\partial r}\frac{\partial^2 r(\ )}{\partial y\, \partial y}$$

$$- \frac{\partial^2 \pi(y_1, r^*(y_0))}{\partial y\, \partial y}$$

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b) Cont.

Using $\frac{\partial \pi}{\partial r} = 0$ at r^* and $\frac{\partial^2 \pi}{\partial r \partial y} = \frac{\partial^2 \pi}{\partial y \partial r}$, above becomes:

$$\frac{\partial^2 G}{\partial y_1^2} = \frac{\partial^2 \pi(y_1, \ldots)}{\partial y^2} + 2\frac{\partial^2 \pi}{\partial y \partial r}\frac{\partial r}{\partial y} + \frac{\partial^2 \pi}{\partial r^2}\left(\frac{\partial r}{\partial y}\right)^2 - \frac{\partial^2 \pi(y_0, \ldots)}{\partial y^2}$$

For $y_1 = y_0$, $\frac{\partial^2 \pi(y_1, \ldots)}{\partial y^2} = \frac{\partial^2 \pi(y_0, \ldots)}{\partial y^2}$

so above becomes:

$$\frac{\partial^2 G}{\partial y_1^2} = 2\frac{\partial^2 \pi}{\partial y \partial r}\frac{\partial r}{\partial y} + \frac{\partial^2 \pi}{\partial r^2}\left(\frac{\partial r}{\partial y}\right)^2$$

Something else we know:

since $\frac{\partial \pi}{\partial r} = 0$ at r^* (profit-max),

$$\frac{\partial(\partial \pi / \partial r)}{\partial y} = 0 \text{ at } r^* \text{ hence}$$

$$\frac{\partial^2 \pi}{\partial r \partial y} + \frac{\partial^2 \pi}{\partial r^2}\frac{\partial r^*}{\partial y} = 0$$

$$\Rightarrow \frac{\partial^2 \pi}{\partial r \partial y} = \left(-\frac{\partial^2 \pi}{\partial r^2}\frac{\partial r^*}{\partial y}\right)$$

substitute this into above

$$\Rightarrow \frac{\partial^2 G}{\partial y_1^2} = -\frac{\partial^2 \pi}{\partial r^2}\left(\frac{\partial r^*}{\partial y}\right)^2$$

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b) cont.

hence

$$G \simeq -\frac{1}{2}\frac{\partial^2 G}{\partial y_1^2} = -\frac{1}{2}\frac{\partial^2 \pi}{\partial r^2}\left(\frac{\partial r^*}{\partial y}\right)^2 (y_1 - y_0)$$

where $\frac{\partial^2 \pi}{\partial r^2}$ equivalent to π_{22}

$\frac{\partial r^*}{\partial y}$ equivalent to $r^{*\prime}(y)$ } book's notation

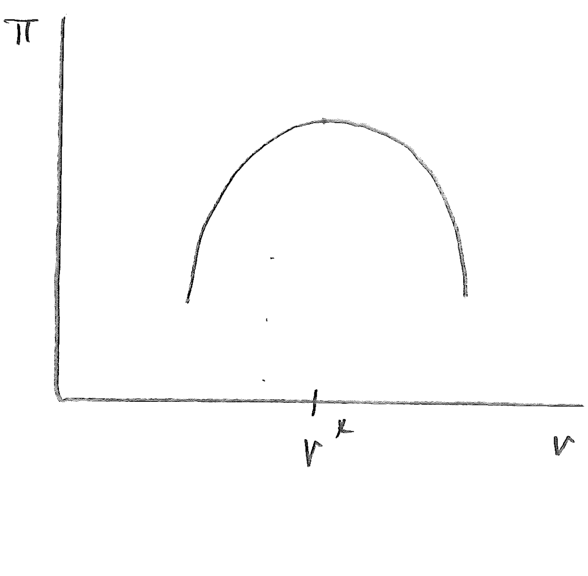
c)

What's "real rigidity"? $\rightarrow \frac{\partial r^*}{\partial y}$ ← "how much does Δy affect $r^* = p^* - p$?"

What's "insensitivity of profit function"? $\frac{\partial^2 \pi}{\partial r^2}$

Sensitive profit function

$\left|\frac{\partial^2 \pi}{\partial r^2}\right|$ big



Insensitive profit function

$\left|\frac{\partial^2 \pi}{\partial r^2}\right|$ small