Does Certainty Equivalence hold for labor supply in LSF model?

Answer to 6.14

LS under C.E.: $\ell_i = \frac{1}{\gamma-1} E[p_i - p/p_i] = \frac{1}{\gamma-1} E\left[\ln\left(\frac{p_i}{p}\right)\right]$ (6.87)

or y_i

(a) LS under EU:

$$U_i = \frac{P_i Y_i}{P} - \frac{1}{\gamma} Y_i^{\gamma} \qquad (6.72)$$

Behavior under uncertainty & "Expected Utility"

$$\max_{Y_i} E[U_i] = E\left[\frac{P_i Y_i}{P} - \frac{1}{\gamma} Y_i^{\gamma}\right]$$

$$= Y_i E\left[\frac{P_i}{P}\right] - \frac{1}{\gamma} Y_i^{\gamma}$$

First-order condition:

$$\frac{\partial E[U]}{\partial Y_i} = 0 = E\left[\frac{P_i}{P}\right] - Y_i^{*\,\gamma-1}$$

$$\Rightarrow Y_i^* = \left(E\left[\frac{P_i}{P}\right]\right)^{\frac{1}{\gamma-1}} \quad \text{in logs } y_i^* = \frac{1}{\gamma-1} \ln\left(E\left[\frac{P_i}{P}\right]\right)$$

or ℓ_i^*

(b) Is

$$\underbrace{\left(\frac{1}{\gamma-1}\right) \ln\left(E\left[\frac{P_i}{P}\right]\right)}_{\ell^* \text{ under EU}} \stackrel{?}{=} \underbrace{\left(\frac{1}{\gamma-1}\right) E\left[\ln\left(\frac{P_i}{P}\right)\right]}_{\ell^* \text{ under CE}} \quad ?$$

$\boxed{\text{No!}}$ $\ln\left(E\left[\frac{P_i}{P}\right]\right) > E\left[\ln\left(\frac{P_i}{P}\right)\right]$

so ℓ^* under EU $> \ell^*$ under CE

How do we know this?

"Jensen's Inequality" says:

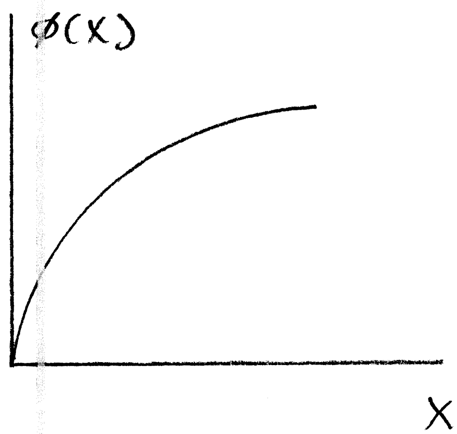
— If $\phi(X)$ is a concave function of random variable X,

$$E[\phi(X)] < \phi(E[X])$$

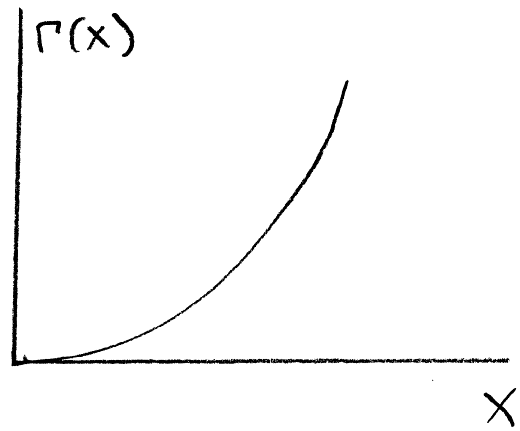
— If $\Gamma(x)$ is a convex function of random variable X,

$$E[\Gamma(x)] > \Gamma(E[X])$$

Concave function



Convex function



Ln is a concave function so

$$\mathrm{Ln}(E[X]) > E[\mathrm{Ln}(X)]$$

Does Certainty Equivalence hold? (cont.)

(c) Suppose $p_i - p = \underbrace{E[p_i - p]}_{\text{not a random variable}} + u_i \leftarrow$ random variable

and u_i is mean zero and independent of $(p_i - p)^e$

What is $\ln(E[p_i/p])$?

$$\frac{p_i}{p} = e^{E[p_i - p]} e^{u_i}$$

$$E\left[\frac{p_i}{p}\right] = E\left[e^{E[p_i-p]} e^{u_i}\right] = e^{E[p_i-p]} \underbrace{E\left[e^{u_i}\right]}_{\text{a constant}}$$

$$\ln(E[p_i/p]) = \ln\left(e^{E[p_i-p]}\right) + \ln\left(E\left[e^{u_i}\right]\right)$$

$$= E[p_i - p] + \text{constant}$$

Hence if

$$l_i^* = \frac{1}{\gamma - 1} \ln(E[p_i/p])$$

then

$$l_i^* = \frac{1}{\gamma-1} E[p_i - p] + \underbrace{\frac{1}{\gamma-1}\text{ constant}}_{\text{another constant}}$$