ANSWER TO 7.2

To assumptions for problem 7.1, add:

$$m_t = m_{t-1} + u_t \leftarrow \text{i.i.d.} \quad (m \text{ is random walk})$$

$f < \frac{1}{2}$ (more folks set prices in even periods)

Pricesetter's Loss over two periods:

$$L = (p_{it} - p_{it}^{*})^2 + (p_{it+1} - p_{it+1}^{*})^2$$

'S $E[L^1] \geq E[L^2]$

Loss to odd pricesetters

Loss to even pricesetters (guys in larger group)

First find for firms setting price in odd periods (larger group)

loss & expected loss for period t

loss & expected loss for period $t+1$

total expected loss

Second do same for firms setting price in even periods (when more firms set)

Notation t is an even period

Keep in mind since m is a random walk

$$E_{t-1}[m_{t+j}] = m_{t-1}$$

$$E_{t-1}[m_{t-j} - m_{t-1}] = 0$$

ANSWER TO 7.2

First find for firms setting price in odd periods (smaller group) loss & expected loss in period t (even period)

from answer to 7.1, expression (4):

$$p_{it} = E_{t-2}[m_t] + \frac{\phi}{1-(1-\phi)f}\left(E_{t-1}[m_t] - E_{t-2}[m_t]\right)$$

with m a random walk

$$p_{it} = m_{t-2} + \frac{\phi}{1-(1-\phi)f}\left(m_{t-1} - m_{t-2}\right)$$

while

$$p_{it}^* = \phi m_t + (1-\phi)p_t$$ get from answer to 7.1, expression (5)

$$p_t = E_{t-2}[m_t] + \frac{\phi f}{1-(1-\phi)f}\left(E_{t-1}[m_t] - E_{t-2}[m_t]\right)$$

with m a random walk:

$$p_t = m_{t-2} + \frac{\phi f}{1-(1-\phi)f}\left(m_{t-1} - m_{t-2}\right)$$

substitute into expression for p_{it}^* gives

$$p_{it}^* = \phi m_t + \frac{(1-\phi)\phi f}{1-(1-\phi)f}\left(m_{t-1} - m_{t-2}\right) + (1-\phi) m_{t-2}$$

hence $$p_t - p_{it}^* = m_{t-2} + \frac{\phi f}{1-(1-\phi)f}\left(m_{t-1} - m_{t-2}\right) - \phi m_t + \frac{(1-\phi)\phi f}{1-(1-\phi)F}\left(m_{t-1} - m_{t-2}\right) + (1-\phi) m_{t-2}$$

$$= -\phi m_t + \phi m_{t-2} + \frac{\phi[1-(1-\phi)f]}{1-(1-\phi)f}\left(m_{t-1} - m_{t-2}\right)$$

$$= -\phi m_t + \phi m_{t-1} - \phi m_{t-2} + \phi m_{t-2}$$

$$= \phi\left(m_{t-1} - m_t\right)$$

hence $$E\left[\left(p_{it} - p_{it}^*\right)^2\right] = \phi^2 E\left[\left(m_{t-1} - m_t\right)^2\right] = \phi^2 E\left[\left(-u_t\right)^2\right] = \phi^2 \sigma_u^2$$

variance of u

ANSWER to 7.2

First find for firms setting price in odd periods (smaller group) loss & expected loss for period t+1 (odd period)

From answer to 7.1, expression (3):

$$p_{it+1} = E_{t-1}[m_{t+1}]$$

with m a random walk

$$p_{it+1} = m_{t-1}$$

while

$$p_{it+1}^* = \phi m_{t+1} + (1-\phi) p_{t+1}$$

$$= \phi m_{t+1} + (1-\phi)\underbrace{\left[E_{t-1}[m_{t+1}] + \frac{\phi(1-f)}{1-(1-\phi)(1-f)}\left(E_t[m_{t+1}] - E_{t-1}[m_{t+1}]\right)\right]}_{\text{from answer to 6.7, expression (5), modified for odd period}}$$

with m a random walk

$$p_{it+1}^* = \phi m_{t+1} + (1-\phi)\left[m_{t-1} + \frac{\phi(1-f)}{1-(1-\phi)(1-f)}(m_t - m_{t-1})\right]$$

hence $p_{it+1} - p_{it+1}^* = m_{t-1} - \phi m_{t+1} - m_{t-1} + \phi m_{t-1} + \frac{(1-\phi)(1-f)\phi}{1-(1-\phi)(1-f)}(m_t - m_{t-1})$

$$= \phi(m_{t-1} - m_{t+1}) + \frac{(1-\phi)(1-f)\phi}{1-(1-\phi)(1-f)}(m_t - m_{t-1})$$

$$(p_{it+1} - p_{it+1}^*)^2 = \phi^2(m_{t-1} - m_{t+1})^2 + 2\frac{(1-\phi)(1-f)\phi^2}{1-(1-\phi)(1-f)}(m_{t-1} - m_{t+1})(m_{t-1} - m_t) + \left[\frac{(1-\phi)(1-f)\phi}{1-(1-\phi)(1-f)}\right]^2(m_{t-1} - m_t)^2$$

$$= \phi^2(-u_t - u_{t+1})^2 + 2\frac{(1-\phi)(1-f)\phi^2}{1-(1-\phi)(1-f)}(-u_t - u_{t+1})(-u_t) + \left[\frac{(1-\phi)(1-f)\phi}{1-(1-\phi)(1-f)}\right]^2(-u_t)^2$$

$$= \phi(u_t^2 + u_t u_{t+1} + u_{t+1}^2) + 2\frac{(1-\phi)(1-f)\phi^2}{1-(1-\phi)(1-f)}(u_t^2 + u_t u_{t+1}) + \left[\frac{(1-\phi)(1-f)\phi}{1-(1-\phi)(1-f)}\right]^2 u_t^2$$

ANSWER TO 7.2

First find for firms setting price in odd periods (smaller group)

loss & expected loss for period $t+1$ (odd period) cont.

$$E\left[(p_{it+1} - p^*_{it+1})^2\right] = \phi\left(\overbrace{E[u_t^2]}^{\sigma_u^2} + \overbrace{E[u_t u_{t+1}]}^{E[u_t]\cdot E[u_{t+1}]=0} + \overbrace{E[u_{t+1}^2]}^{\sigma_u^2}\right)$$

$$+2\,\frac{(1-\phi)(1-f)\phi^2}{1-(1-\phi)(1-f)}\left(\underbrace{E[u_t^2]}_{\sigma_u^2} + \underbrace{E[u_t u_{t+1}]}_{0}\right) + \left(\frac{(1-\phi)(1-f)\phi}{1-(1-\phi)(1-f)}\right)^2 \underbrace{E[u_t^2]}_{\sigma_u^2}$$

$$= \sigma_u^2\left[2\phi + \frac{2(1-\phi)(1-f)\phi^2}{1-(1-\phi)(1-f)} + \left(\frac{(1-\phi)(1-f)\phi}{1-(1-\phi)(1-f)}\right)^2\right]$$

total expected loss for smaller group

$$= \text{above} + \phi^2\sigma_u^2$$

$$= \phi^2\sigma_u^2 + \sigma_u^2\left[2\phi + \frac{2(1-\phi)(1-f)\phi^2}{1-(1-\phi)(1-f)} + \left(\frac{(1-\phi)(1-f)\phi}{1-(1-\phi)(1-f)}\right)^2\right]$$

recall $f < 1/2$

ANSWER TO 7.2

Second find for firms setting price in even periods (larger group)

loss & expected loss in period t (odd period)

Equivalent to an odd firm in even period so

$$E(p_{it} - p^*_{it})^2 = \phi^2 \sigma_u^2$$

loss & expected loss in period $t+1$ (even period)

Equivalent to odd Firm in even period except $(1-f) \rightarrow f$

$$E(p_{it+1} - p^*_{it+1})^2 = \sigma_u^2 \left[2\phi + \frac{2(1-\phi) f \phi^2}{1-(1-\phi) f} + \left(\frac{(1-\phi) f \phi}{1-(1-\phi) f} \right)^2 \right]$$

total expected loss for larger group

$$= \phi^2 \sigma_u^2 + \sigma_u^2 \left[2\phi + \frac{2(1-\phi) f \phi^2}{1-(1-\phi) f} + \left(\frac{(1-\phi) f \phi}{1-(1-\phi) f} \right)^2 \right]$$

compare with smaller group's loss (recall $f < \frac{1}{2}$)

$$= \phi^2 \sigma_u^2 + \sigma_u^2 \left[2\phi + \frac{2(1-\phi)(1-f)\phi^2}{1-(1-\phi)(1-f)} + \left(\frac{(1-\phi)(1-f)\phi}{1-(1-\phi)(1-f)} \right)^2 \right]$$

If $\phi < 1$, small group's loss is greater