Econ 614 Problem on the "dynamic inconsistency" of low-inflation monetary policy

Consider an economy where aggregate supply has the same form as the Lucas supply function:

$$y - \bar{y} = b(p - p^{e})$$

In this expression, $\bar{y}$ is the natural rate of output and p^{e} is the expected price level, that is the public's expected value for this period's price level based on information from last period. This is equivalent to: $y - \bar{y} = b(\pi - \pi^{e})$ (Do you see how?)

Nearly everyone living in the economy likes high output, with a desired level of output y^* greater than the natural rate of output: $y^* > \bar{y}$. Their desired rate of inflation is π^*. Their preferences across output and inflation can be described by a loss function:

$$L = \frac{1}{2}(y - y^*)^2 + \frac{1}{2}a(\pi - \pi^*)^2$$

Call this "society's loss function." The central bank observes the public's inflation expectation π^e and chooses the real interest rate to control the level of output: $y - \bar{y} = -d(r - \bar{r})$ where $\bar{r}$ is the natural rate of interest. Given π^e, each level of output corresponds to a certain level of inflation. Note that there is no uncertainty in the relationship between the real interest rate and output or in the relationship between inflation and output. So, to keep notation simpler, you can describe the central bank as controlling inflation *directly* - that is, choosing a level of inflation (which implies a corresponding level of output) given π^e. If the central bank chooses inflation $\hat{\pi}$, output is $y = \bar{y} + b(\hat{\pi} - \pi^e)$.

1) Suppose the central bank's policy committee has exactly the same preferences as the public.
a) Taking π^e as given, what value of π will the central bank choose?

$$L = \frac{1}{2}\left(\bar{y} + b(\pi - \pi^e) - y^*\right)^2 + \frac{1}{2}a(\pi - \pi^*)^2$$

$\frac{\partial L}{\partial \pi} = 0$ solve for π, gives:

$$\pi = \frac{b^2}{a + b^2}\pi^e + \frac{a}{a+b^2}\pi^* + \frac{b}{a+b^2}(y^* - \bar{y})$$

b) Suppose the public knows that the central bank will behave this way. That means the public will use the equation you just derived in 1) to form their inflation expectation π^e. The level of *realized* inflation in your answer to 1) must be equal to π^e. What does this mean for the realized level of inflation? That is, what will be the level of inflation in the economy if the public forms its inflation expectation this way?

$$\pi = \frac{b^2}{a+b^2}\pi + \frac{a}{a+b^2}\pi^* + \ldots$$

Solve for π, gives:

$$\pi = \pi^* + \frac{b}{a}(y^* - \bar{y})$$

c) What is the resulting value of loss?
With $\pi=\pi^e$, $y=\bar{y}$.

$$L = \frac{1}{2}(y^*-\bar{y})^2 + \frac{1}{2}a\left(\pi^* + \frac{b}{a}(y^*-\bar{y}) - \pi^*\right)^2$$
$$= \frac{1}{2}(y^*-\bar{y})^2 + \frac{1}{2}a\left(\frac{b}{a}(y^*-\bar{y})\right)^2$$

2) Suppose society can delegate monetary policy choices to an individual person. An individual's loss function may have a value of a, call it a_i, that is different from the value of a in society's loss function. If monetary policy is delegated to an individual, he or she will use his or her own loss function, not society's loss function, in choosing π. Monetary policy can be delegated to a person with $a_i = a$, or a person with $a_i<a$, or a person with $a_i>a$. What sort of person will end up creating the smallest loss for society - that is, the best outcome based on *society's* loss function?

From your answer to b), inflation will be $\pi = \pi^* + \frac{b}{a_i}(y^*-\bar{y})$

so loss is: $\frac{1}{2}(y^*-\bar{y}) + \frac{1}{2}a\left(\frac{b}{a_i}(y^*-\bar{y})\right)^2$

When $a_i>a$*, loss is smaller.*

3) Again suppose society can delegate monetary policy choices to an individual person. Now suppose each individual's loss function has the same value of *a* as society's loss function. But an individual's loss function may have a value of y^*, call it y_i^*, that is different from the value of y^* in society's loss function. Again, if monetary policy is delegated to an individual, he or she will use his or her own loss function in choosing π. Monetary policy can be delegated to a person with $y_i^* = y^* > \bar{y}$, or a person with $y_i^* = \bar{y}$, or a person with $y_i^* < \bar{y}$. What sort of person will end up creating the smallest loss for society - that is, the best outcome based on *society's* loss function?

From your asnwer to b), inflation will be: $\pi = \pi^* + \frac{b}{a}(y_i^*-\bar{y})$

so loss is: $\frac{1}{2}(y^*-\bar{y})^2 + \frac{1}{2}a\left(\frac{b}{a}(y_i^*-\bar{y})\right)^2$

When $y_i^* = \bar{y}$*, loss is minimized.*

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