

# ANSWER TO INDIVISIBLE LABOR PROBLEM

$N$  Members of group

$W$  Wage earned by working member

$C_1$  Consumption of worker

$C_2$  Consumption of nonworker

$\ln(C_1)$  Utility of worker

$\ln(C_2) + Z$  Utility of nonworker

$p$  Fraction assigned to work

Constraint: group income = total consumption

$$pNW = pNC_1 + (1-p)NC_2$$

$$pW = pC_1 + (1-p)C_2$$

means  $C_2 = \frac{p}{1-p}W - \frac{p}{1-p}C_1$

ANSWER...

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Social planner's problem:

$$\text{Max}_{p, c_1, c_2} p \ln(c_1) + (1-p) [\ln(c_2) + Z]$$

$$\text{s.t. } pW = p c_1 + (1-p) c_2$$

can be expressed as

$$\text{Max}_{p, c_1} p \ln(c_1) + (1-p) \left[ \ln \left( \frac{p}{1-p} W - \frac{p}{1-p} c_1 \right) + Z \right]$$

$$\frac{\partial [\ ]}{\partial c_1} = p \frac{1}{c_1} + (1-p) \frac{1}{\frac{p}{1-p} W - \frac{p}{1-p} c_1} \left( -\frac{p}{1-p} \right) = 0$$

$$0 = \frac{p}{c_1} - \frac{p}{\frac{p}{1-p} W - \frac{p}{1-p} c_1}$$

$$\Rightarrow c_1 = \frac{p}{1-p} W - \frac{p}{1-p} c_1$$

$$c_1 + \frac{p}{1-p} c_1 = \frac{p}{1-p} W$$

$$\frac{1-p}{1-p} c_1 + \frac{p}{1-p} c_1 = \frac{p}{1-p} W$$

$$\frac{1}{1-p} c_1 = \frac{1}{1-p} p W$$

$$\Rightarrow c_1 = p W, \quad c_2 = \frac{p}{1-p} W - \frac{p}{1-p} p W = \frac{1}{1-p} p W - \frac{p}{1-p} p W = \frac{1-p}{1-p} p W = p W$$

ANSWER...

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Now substitute  $c_1 = c_2 = pw$  into EU

$$\max_p p \ln(pw) + (1-p)[\ln(pw) + Z]$$

$$= p \ln(pw) + \ln(pw) + Z - p \ln(pw) - pZ$$

$$= \ln(pw) + Z - pZ$$

$$\frac{\partial [\ ]}{\partial p} = \frac{1}{pw} w - Z = 0$$

$$\frac{1}{p} - Z = 0$$

$$\frac{1}{p} = Z$$

$$p = \frac{1}{Z} \leftarrow \text{inelastic labor supply}$$

