KEYNESIAN DSGE

Persistent shocks + interest-rate rule with ϕ_π and ϕ_y

$$y_t = {}_t y^e_{t+1} - s r_t + u^{IS}_t$$

$$\pi_t = {}_t \pi^e_{t+1} + k y_t + u^{AS}_t$$

$$r_t = \phi_\pi \pi_t + \phi_y y_t$$

1) Effect of u^{IS}

Conjecture $\pi = \frac{k}{1-\rho_y} y$

hence $r_t = \left(\frac{\phi_\pi k}{1-\rho_y} + \phi_y \right) y_t$

$$y_t = {}_t y^e_{t+1} - s\left(\qquad \right) y_t + u^{IS}_t$$

$$\left[1 + s\left(\quad \right) \right] y_t = \ldots\ldots + u_t \qquad \text{apply math trick}$$

$$\Rightarrow y_t = \frac{1}{1-\rho_{IS} + \frac{s\phi_\pi k}{1-\rho_{IS}} + \phi_y} u^{IS}_t \qquad \left(\text{note } \rho_y = \rho_{IS} \right)$$

$$\pi_t = \frac{k}{(1-\rho_{IS})^2 + s\phi_\pi k + \phi_y} u^{IS}_t$$

$$r_t = \frac{\frac{\phi_\pi k}{1-\rho_{IS}} + \phi_y}{1-\rho_{IS} + \frac{s\phi_\pi k}{1-\rho_{IS}} + \phi_y} u^{IS}_t$$

...with ϕ_π & ϕ_y

2) Effect of u^{AS}

$$y_t = y^e_{t+1} - s r_t = y^e - s\phi_\pi \pi_t - s\phi_y y_t$$

hence

$$y_t + s\phi_y y_t = y^e_t - s\phi_\pi \pi_t$$

$$\Rightarrow y_t = \frac{1}{1+s\phi_y} y^e - \frac{s\phi_\pi}{1+s\phi_y} \pi_t$$

apply math trick, conjecture $\pi^e_{t+1} = \rho_\pi \pi_t$

$$y_t = -\frac{s\phi_\pi}{1+s\phi_y} \frac{1}{1-\frac{1}{1+s\phi_y}\rho_\pi} \pi_t$$

$$= \frac{-s\phi_\pi}{1+s\phi_y - \rho_\pi} \pi_t$$

substitute into $\pi_t = \pi^e_{t+1} + k y_t + \varepsilon^{AS}_t$

$$\Rightarrow \pi_t = \pi^e_{t+1} - \frac{k s\phi_\pi}{1+s\phi_y - \rho_\pi} \pi_t + \varepsilon^{AS}_t$$

$$\Rightarrow \pi_t = \frac{1}{1+\frac{k s\phi_\pi}{1+s\phi_y-\rho_\pi}} \pi^e_{t+1} + \text{something } u^{AS}_t$$

apply math trick...

with ϕ_π & ϕ_y

2) Effect of u^{AS} (cont.)

$$\pi_t = \frac{1}{1 + \frac{\kappa s \phi_\pi}{1 + s\phi_y - \rho_{AS}}} \; \frac{1}{1 - \frac{1}{1 + \frac{\kappa s \phi_\pi}{1 - \rho_\pi + s\phi_y}} \rho_{AS}} \; u_t^{AS}$$

$$\pi_t = \frac{1}{1 + \frac{\kappa s \phi_\pi}{1 + s\phi_y - \rho_{AS}} - \rho_{AS}} \; u_t^{AS}$$

$$y_t = \frac{-s\phi_\pi}{\kappa s \phi_\pi + (1 - \rho_{AS})(1 - \rho_{AS} + s\phi_y)} \; u_t^{AS}$$

$$= -\frac{1}{\kappa + \frac{(1-\rho_{AS})(1-\rho_{AS} + s\phi_y)}{s\phi_\pi}} \; u_t^{AS}$$

What about r?

If you stick π & y above into $r = \phi_\pi \pi + \phi_y y$,

it looks like effect on r is ambiguous.

If you did enough algebra you'd eventually

see it's not ambiguous.

But there's a shortcut.

with ϕ_π & ϕ_y

3) Effect of u^{AS} (cont'd)

(because serial corr. in y is ρ_{AS})

From $y_t = y^e_{t+1} - s\, r_t$,

$$r_t = \frac{1}{s}\left(y^e_{t+1} - y_t\right) = \frac{1}{s}\left(\rho_{AS} y_t - y_t\right);$$

$$= -\frac{1}{s}\left(1-\rho_{AS}\right) y_t$$

hence, from above equation giving y_t as function of u^{AS}, you get

$$r_t = \frac{\phi_\pi}{\dfrac{\kappa\, s\phi_\pi}{1-\rho_{AS}} + (1-\rho_{AS}) + s\phi_y}\, u^{AS}_t.$$