REAL RIGIDITY

Problem

Change prodn. fn. from $Q_i = H_i$
to $Q_i = H_i^{1-\alpha}$ $\left(Q_i = K_i^{\alpha} H_i^{1-\alpha}\right.$
and $\left.K_i = 1\right)$

How does this affect real rigidity?

$p_i - p = c + \phi y$

Note: $\alpha = 0$ is equivalent to $Q_i = H_i$
baseline model

1) Derive $P_i^*(W, P, Y)$

2) Derive $P_i^*(\frac{W}{P}, P, Y)$

3) Use $\frac{W}{P}(L_i)$ & $L_i = H_i$ to derive $P_i^*(P, Y)$

4) Take logs to get $p_i - p = \ldots$

1) Derive $P_i^*(W, P, Y)$

nominal

$\pi = Q_i^D P_i - W H_i$

recall $\eta > 1$

where $Q_i^D = Y (P_i / P)^{-\eta}$

$H_i = Q_i^{\frac{1}{1-\alpha}}$ (From $Q_i = H_i^{1-\alpha}$)

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Problem (cont.)

Get $\pi(P_i, P, Y)$ & take f.o.c.

$$\pi = \underbrace{Y(P_i/P)^{-\eta}}_{Q_i} P_i - w \underbrace{\Big(\underbrace{Y(P_i/P)^{-\eta}}_{Q_i}\Big)^{\frac{1}{1-\alpha}}}_{L_i(Q_i)}$$

$$\pi = Y P^{\eta} P_i^{1-\eta} \quad - \quad w Y^{\frac{1}{1-\alpha}} P^{\frac{\eta}{1-\alpha}} P_i^{-\frac{\eta}{1-\alpha}}$$

F.O.C.:

$$\frac{\partial \pi}{\partial P_i} = 0 = (1-\eta) Y P^{\eta} P_i^{1-\eta-1} - \left(-\frac{\eta}{1-\alpha}\right) w Y^{\frac{1}{1-\alpha}} P^{\frac{\eta}{1-\alpha}} P_i^{-\frac{\eta}{1-\alpha}-1}$$

$$0 = (1-\eta) Y P^{\eta} P_i^{-\eta} + \frac{\eta}{1-\alpha} w Y^{\frac{1}{1-\alpha}} P^{\frac{\eta}{1-\alpha}} P_i^{\frac{-\eta-(1-\alpha)}{1-\alpha}}$$

$$-(1-\eta) Y P^{\eta} P_i^{-\eta} = \frac{\eta}{1-\alpha} w Y^{\frac{1}{1-\alpha}} P^{\frac{\eta}{1-\alpha}} P_i^{\frac{-\eta-(1-\alpha)}{1-\alpha}}$$

$$P_i^{-\eta} \left(P_i^{\frac{-\eta-(1-\alpha)}{1-\alpha}}\right)^{-1} = -\left(\frac{\eta}{1-\alpha}\right)\left(\frac{1}{1-\eta}\right) w Y^{\frac{1}{1-\alpha}} Y^{-1} P^{\frac{\eta}{1-\alpha}} P^{-\eta}$$

$$P_i^{\frac{-\eta(1-\alpha)+\eta+(1-\alpha)}{1-\alpha}} = \left(\frac{\eta}{\eta-1}\right)\left(\frac{1}{1-\alpha}\right) w Y^{\frac{1-(1-\alpha)}{1-\alpha}} P^{\frac{\eta-\eta(1-\alpha)}{1-\alpha}}$$

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Problem (cont.)

$$P_i^{\frac{-\eta+\eta\alpha+\eta+1-\alpha}{1-\alpha}} = \left(\frac{\eta}{\eta-1}\right) W \left(\frac{1}{1-\alpha}\right) Y^{\frac{\alpha}{1-\alpha}} P^{\frac{\eta-\eta+\eta\alpha}{1-\alpha}}$$

$$P_i^{\frac{1-\alpha+\eta\alpha}{1-\alpha}} = \left(\frac{\eta}{\eta-1}\right) W \left(\frac{1}{1-\alpha}\right) Y^{\frac{\alpha}{1-\alpha}} P^{\frac{\eta\alpha}{1-\alpha}}$$

$$P_i^{\frac{1+\alpha(\eta-1)}{1-\alpha}} = \ldots$$

$$P_i = \left(\left(\frac{\eta}{\eta-1}\right) W\right)^{\frac{1-\alpha}{1+\alpha(\eta-1)}} \left(\frac{1}{1-\alpha}\right)^{\frac{1-\alpha}{1+\alpha(\eta-1)}} Y^{\frac{\alpha}{1+\alpha(\eta-1)}} P^{\frac{\eta\alpha}{1+\alpha(\eta-1)}}$$

Check: for $\alpha = 0$ (matching baseline model) this should give

$$P_i = \frac{\eta}{\eta-1} W \quad \text{or} \quad \frac{P_i}{P} = \frac{\eta}{\eta-1} \frac{W}{P}$$

Yes!

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Problem (cont.)

2) Derive $P_i^*\left(\frac{W}{P}, P, Y\right)$

Taking equation from 1), multiply right-hand side by

$$\left(P^{-1}\right)^{\frac{1-\alpha}{1+\alpha(\eta-1)}} \; P^{\frac{1-\alpha}{1+\alpha(\eta-1)}} \quad \leftarrow \text{one}$$

gives

$$P_i = \left(\frac{\eta}{\eta-1}\frac{1}{1-\alpha}\right)^{\frac{1-\alpha}{1+\alpha(\eta-1)}} \left(\frac{W}{P}\right)^{\frac{1-\alpha}{1+\alpha(\eta-1)}} Y^{\frac{\alpha}{1+\alpha(\eta-1)}} P^{\frac{\eta\alpha}{1+\alpha(\eta-1)}} P^{\frac{1-\alpha}{1+\alpha(\eta-1)}}$$

$$P_i = \ldots \; P^{\frac{\eta\alpha+1-\alpha}{1+\alpha(\eta-1)}}$$

$$P_i = \ldots \; P^{\frac{1+\alpha(\eta-1)}{1+\alpha(\eta-1)}} \quad \leftarrow \text{one}$$

$$P_i = \left(\frac{\eta}{\eta-1}\frac{1}{1-\alpha}\right)^{\frac{1-\alpha}{1+\alpha(\eta-1)}} \left(\frac{W}{P}\right)^{\frac{1-\alpha}{1+\alpha(\eta-1)}} Y^{\frac{\alpha}{1+\alpha(\eta-1)}} P$$

Check again: for $\alpha = 0$, this should give

$$P_i = \frac{\eta}{\eta-1} W$$

Yes!

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Problem (cont.)

3) Derive $P_i^*(P, Y)$

We know

$$\frac{W}{P} = L_i^{\gamma-1} \quad \text{and} \quad H_i = L_i \quad \text{and} \quad H_i = Q_i^{\frac{1}{1-\alpha}}$$

and $Q = Y$ (output per firm = output per household)

so

$$\frac{W}{P} = H_i^{\gamma-1} = \left(Q_i^{\frac{1}{1-\alpha}}\right)^{\gamma-1} = \left(Y^{\frac{1}{1-\alpha}}\right)^{\gamma-1}$$

$$\Rightarrow \frac{W}{P} = Y^{\frac{\gamma-1}{1-\alpha}}$$

Substitute that into expression from 2),

$$P_i = \left(\frac{\eta}{\eta-1}\frac{1}{1-\alpha}\right)^{\frac{1-\alpha}{1+\alpha(\eta-1)}} \left(Y^{\frac{\gamma-1}{1-\alpha}}\right)^{\frac{1-\alpha}{1+\alpha(\eta-1)}} Y^{\frac{\alpha}{1+\alpha(\eta-1)}} P$$

$$P_i = \left(\frac{\eta}{\eta-1}\frac{1}{1-\alpha}\right)^{\frac{1-\alpha}{1+\alpha(\eta-1)}} Y^{\frac{\gamma-1}{1+\alpha(\eta-1)}} Y^{\frac{\alpha}{1+\alpha(\eta-1)}} P$$

$$P_i = \left(\frac{\eta}{\eta-1}\frac{1}{1-\alpha}\right)^{\frac{1-\alpha}{1+\alpha(\eta-1)}} Y^{\frac{(\gamma-1)+\alpha}{1+\alpha(\eta-1)}} P$$

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Problem (cont.)

4) Get "real rigidity equation"

Here,

$$p_i - p = \underbrace{\left(\frac{1-\alpha}{1+\alpha(\eta-1)}\right) \ln\left(\frac{\eta}{\eta-1}\frac{1}{1-\alpha}\right)}_{\text{constant}} + \left(\frac{(\gamma-1)+\alpha}{1+\alpha(\eta-1)}\right) y$$

Baseline:

$$p_i - p = \overset{\text{another}}{\text{constant}} + (\gamma-1)\, y$$

Is $\left(\frac{(\gamma-1)+\alpha}{1+\alpha(\eta-1)}\right) > (\gamma-1)$ <u>less</u> real rigidity

$\left(\qquad\right) < (\gamma-1)$ <u>more</u> real rigidity.

For parameter values $\frac{1}{(\gamma-1)} = 0.1$, $(\gamma-1) = 10$, $\eta = 5$, $\alpha = 1/3$,

$$\frac{10 + 1/3}{1 + 1/3 \cdot 4} = \frac{\frac{31}{3}}{\frac{7}{3}} = \frac{31}{7} \approx 4\tfrac{1}{2} < 10$$

this model has <u>more</u> real rigidity than baseline.