IMPERFECT FINANCIAL MARKETS

Bernanke, Gertler & Gilchrist (1999)

Embed asymmetric info, entrepreneurs' wealth effects into a NK macro model.

Illustrates debt deflation, financial accelerator.

__Households__: infinitely-lived, can borrow & lend at safe real interest rate R' (gross rate $= 1+r$)

Can't hold or use physical capital K

__Entrepreneurs__: can acquire physical capital & use it in production.

Can die: probability γ per period (so average lifetime $\frac{1}{1-\gamma}$)

Birth rate so that E's are fixed fraction of total popn.

Don't consume except when they die.

Produce output, sell in competitive "wholesale" mkt at mkt-clearing price

__Retailers__: buy undifferentiated wholesale output, turn it into differentiated goods to sell in monopoly markets. Subject to Calvo constraint in price adjustment. $X_t = \dfrac{\text{Avg price monopoly retail goods}}{\text{competitive wholesale price}}$

Note $1/X$ is "real marginal cost" for retailers.

Give monopoly profits to households.

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Overview (cont.)

Financial intermediaries

Borrows funds from h'holds from t to $t+1$, pays R_{t+1}. (Supply of funds from h'holds, Euler eqn)

Lends to many E's, equating expected return to R_{t+1}.

FI is analogue to Romer's "investor."

Labor & employment

No nominal wage rigidity.

E's supply all their labor inelastically.

H'holds supply labor to max U, with increasing marginal disutility.

E & H labor are different goods $L_t = H_t^{\Omega}(H_t^{e})^{1-\Omega}$ (4.6)

$$MPH = W_t = \text{Marginal disutility}$$

W_t: real wage for h'hold labor

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Overview (cont.)

Capital market

K_t Total supply of capital in t, predetermined, held by entrepreneurs, so changes in

Q_t (market price of capital / price of output [consumption]) (to be used next period)
affect E's wealth

E's use wealth + funds borrowed from FI to buy K to use next period (imagine selling & rebuying).

Q_t clears competitive market.

$$K_t^s = \Phi\left(\frac{I_{t-1}}{K_{t-1}}\right)K_{t-1} + (1-\delta)\,K_{t-1} \qquad (4.2)$$

(second-order adjustment cost like CEE)
$\Phi' > 0$, $\Phi'' < 0$, $\Phi(0) = 0$

"In equilibrium" (social planner or "competitive capital-producing firms):

$$0 = Q_t \cdot \partial K_{t+1}/\partial I_t - 1$$

(cost of one unit of Y_t)

profit from buying Y_t, using it as I_t, creating K_{t+1}

where $\partial K_{t+1}/\partial I_t = \Phi'(\ )\cdot \frac{1}{K_t}\cdot K_t = \Phi'(\ )$

hence $1 = Q_t\,\Phi(I_t/K_t)$

$$\Rightarrow Q_t = \left[\Phi'\left(\frac{I_t}{K_t}\right)\right]^{-1} \qquad (4.3)$$

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Asymm info, costly state verification

Much like Romer 10.2:

Realized return to capital controlled by entrepreneur:

Romer	Here
X	$\omega^j R^k$

(R^k:) economy-wide component, observed by everyone

(ω^j:) component idiosyncratic to firm/ent. j, observed only by ent. at no cost.

ω^j can be observed by "financial intermediary" if a cost is paid:

Romer	Here
c	$\mu \omega^j R^k Q K^j$

a fraction of realized return

where Q price ent. pays for capital

K^j capital installed by ent. j

Ent. must borrow cost of project minus his wealth:

Romer	Here
W	N

endogenous

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Asymm. info (cont.)

Financial intermediary's alternative is to earn riskless real return

Romer	Here
$1+r$	R

Unlike Romer, size of project (how much K) & cost is variable

Romer	Here
1	QK^j

Optimal loan contract

Like Romer, it says "ent. pays fin. int. $\bar{\omega}^j$, or ent. declares bankruptcy in which case fin. int. pays cost & gets the firm."

But $\bar{\omega}^j$ is not fixed sum of \$.
$\bar{\omega}^j$ is "indexed" to R^k, that is contract sets function or schedule $\bar{\omega}^j(R^k)$

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Asymm info (cont.)

At time t, E can purchase K^j_{t+1} to use in $(t+1)$

Cost: $Q_t K^j_{t+1}$

Realized return: $\omega^j R^k_{t+1} K^j_{t+1}$

$F(\omega)$ C.D.F. for ω^j $\quad E\{\omega^j\} = 1$

so R^k_{t+1} = average return across all businesses

$E\{R^k_{t+1} K^j_{t+1}\}$ expected return to business j

E must borrow $B^j_{t+1} = Q_t K^j_{t+1} - N^j_{t+1}$ (like $(1-W)$)

Verification cost $\mu \omega^j R^k_{t+1} Q_t K^j_{t+1}$ (like c)

As in Romer, show that (indexed) "debt contract" is optimal, and get an equation that determines total investment (capital purchases by E's).

Recall in Romer, investment

— risk-free rate (required return)

+ entrepreneurs' wealth

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Asymm info (cont.)

Total investment:

$$Q_t K^j_{t+1} = \psi(s_t) N^j_{t+1} \qquad \psi(1)=1, \psi'(\,)>0 \qquad (3.8)$$

$E[R^k_{t+1}/R_{t+1}]$, "expected discounted return to capital"

or

$$\frac{E\{R^k_{t+1}\}}{R_{t+1}} = s\left(\frac{N^j_{t+1}}{Q_t K^j_{t+1}}\right) \qquad s'(\cdot) < 0 \qquad (3.9)$$

share of capital financed by E's wealth

(Required expected return to capital) investment

Note: loglinear approximation of this is

$$E_t\{r^k_{t+1}\} - r_{t+1} = -\nu\left[n_{t+1} - (q_t + k_{t+1})\right] \qquad (4.17)$$

or

$$E_t\{r_{t+1}\}^k = r_{t+1} - \nu\left[n_{t+1} - (q_t + k_{t+1})\right]$$

Romer:

$$\gamma \geq (1+r) + A(\bar{w}, \ldots\ldots)$$

We'll get back to this.

Now, look at how it fits into NK model.

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Log-linear approximations around LRSS

Lower-case letters are log (percent) deviations from LRSS

Equations that look familiar

Output is sum of C+I+G+ observation costs)

$$y_t = \frac{C}{Y} c_t + \frac{I}{Y} i_t + \frac{G}{Y} g_t + \frac{C^e}{Y} c^e_t + \phi^y_t \qquad (4.14)$$

from dying E's

Euler eqn consumption!

$$c_t = -r_{t+1} + E_t\{c_{t+1}\} \qquad (4.15)$$

Agg. prodn fn! h'hold labor

$$y_t = a_t + \alpha k_t + (1-\alpha)\Omega h_t \qquad (4.20)$$

(Why don't you see E's labor? It's fixed, in the constant.)

Intratemporal F.O.C. (W/P)·MU_c = Marginal disutility labor!

$$\underbrace{\underbrace{y_t - h_t}_{\text{MPL}} - x_t}_{\text{real wage}} - c_t = \eta^{-1} h_t \qquad (4.21)$$

markup of retail over MPL

Marginal util of C, from log utility

Marg disutil labor

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Log-linear...

Familiar equations (cont.)

NK Phillips curve:

$$\pi_t = E_{t-1}\{K(-X_t) + \beta \pi_{t+1}\} \qquad (4.22)$$

(recall $\frac{1}{X_t}$ (in logs $-X_t$) is real MC for retailer

Change in capital stock:

$$k_{t+1} = \delta i_t + (1-\delta)k_t \qquad (4.23)$$

(From $k_{t+1} = \frac{I}{K} i + (1-\delta)k_t$ and $\frac{I}{K} = \delta$ in LRSS)

Central bank's interest-rate rule w/exogenous shocks:

$$r_t^n = \rho r_{t-1}^n + \xi \pi_{t-1} + \varepsilon_t^{rn} \qquad (4.25)$$

(nominal rate $= r_{t+1} + E\{p_{t+1} - p_t\}$

IS shocks (from govt. spending)

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \qquad (4.26)$$

Productivity shocks

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \qquad (4.27)$$

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Log-linear (conts.)

New equations

Price, value of capital

$$q_t = \varphi (i_t - k_t) \qquad (4.19)$$

From $Q_t = \left[\Phi'(I_t / K_t) \right]^{-1}$

Realized return to installed capital

$$r^k_{t+1} = (1-\varepsilon)\Big(\underbrace{y_{t+1} - k_{t+1}}_{Y/K} - \underbrace{x_{t+1}}_{\text{inverse of retailer markup}}\Big) + \varepsilon q_{t+1} - q_t \qquad (4.18)$$

shares in LRSS

Entrepreneur's wealth

$$n_{t+1} = \frac{\gamma R K}{N} (r^k_t - r_t) + r_t + n_t + \cdots \qquad (4.24)$$

LRSS ratio of capital holdings to entrepreneur's net wealth. Greater than one because e's borrow ("leverage")

E's wealth & investment

Take (3.9), aggregate across E's (j's) to get (4.5),

$$E_t\{r^k_{t+1}\} - r_{t+1} = -\nu \left[n_{t+1} - (q_t + k_{t+1}) \right] \qquad (4.17)$$

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Financial accelerator

Recall this means $r\uparrow \to Y\downarrow \to$ E's wealth $\downarrow \to I\downarrow \to Y\downarrow \ldots$

Here:

$r\uparrow \to Y\downarrow$

$$c_t = -r_{t+1} + E_t\{c_{t+1}\} \qquad (4.15)$$

$Y\downarrow \to$ wealth $\downarrow$

$$r^k_{t+1} = (1-\varepsilon)(y_{t+1} - k_{t+1} - x_{t+1}) + \ldots \qquad (4.18)$$

$$n_{t+1} = \frac{\gamma RK}{N}(r^k_t - r_t) + \ldots \qquad (4.24)$$

Wealth $\downarrow \to I\downarrow$

wealth$\downarrow$ raises required return to capital

$$E_t\{r^k_{t+1}\} = r_{t+1} - \nu\left[n_{t+1} - (q_t + k_{t+1})\right] \qquad (4.17)$$

which must lower k_{t+1} in (4.18)...

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Back to loan contract

For each possible value of mutually-observable R^k_{t+1}, contract specifies $\bar{\omega}^j$

- if $\omega^j < \bar{\omega}^j$ probability $F(\bar{\omega}^j)$

E says "can't pay," I gets $\omega^j R^k_{t+1} K^j_{t+1}$

- if $\omega^j \geq \bar{\omega}^j$ probability $1-F(\bar{\omega}^j)$

E pays $Z^j_{t+1} B^j_{t+1} = \bar{\omega}^j R^k_{t+1} Q_t K^j_{t+1}$

& E gets to keep $(1-\bar{\omega}^j) R^k_{t+1} Q_t K^j_{t+1}$

$\bar{\omega}^j$ is a function of R^k_{t+1} so that all risk from R^k variation is borne by risk-neutral E, none borne by risk-averse households

Means when R^k_{t+1} is low, schedule calls for high $\bar{\omega}^j$ (high Z^j)

Schedule $\bar{\omega}^j(R^k_{t+1})$ is such that

riskless rate (gross)

expected value of loan to investor $= R_{t+1} B^j_{t+1}$

$$= R_{t+1}(Q_t K^j_{t+1} - N^j_{t+1})$$

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A loan contract ignoring uncertainty in R^k

In realized states of ω^j where $\omega^j < \bar{\omega}^j$,

I gets $(1-\mu)\omega^j R^k_{t+1} Q_t K^j_{t+1}$

so $\underset{\substack{\uparrow \\ \text{probability} \\ \text{E can't pay}}}{F(\omega^j)}\ \underset{\substack{\uparrow \\ \text{expected value of what I gets} \\ \text{in the event E can't pay}}}{E\{(1-\mu)\omega^j \ldots / \omega^j < \bar{\omega}^j\}}$

$$= (1-\mu)\int_0^{\bar{\omega}^j} \omega R^k_{t+1} Q_t K^j_{t+1}\, dF(\omega) = (1-\mu) R^k_{t+1} Q_t K^j_{t+1} \int_0^{\bar{\omega}^j} \omega\, dF(\omega)$$

so expected return to I from loan is:

$$[1-F(\bar{\omega}^j)]\,\bar{\omega}^j R^k_{t+1} Q_t K^j_{t+1} + (1-\mu) \ldots \int_0^{\bar{\omega}^j} \omega\, dF(\omega)$$

Derivative of above is + for small $\bar{\omega}^j$

(−) for big $\bar{\omega}^j$

hence a value of $\bar{\omega}^j$ that maxes it. (Analogous to R^{MAX} in Romer.)

Hence if this maxed value $< R_{t+1}(Q_t K^j_{t+1} - N^j_{t+1})$

For <u>any</u> value of K^j_{t+1}, E can't borrow

"rationing"

Assume E <u>can</u> borrow.

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... ignoring uncertainty in R^k (cont.)

Setting

$$\left([1-F(\bar{\omega}^j)]\bar{\omega}^j + (1-\mu)\int_0^{\bar{\omega}} \omega \, dF(\omega) \right) R^k_{t+1} Q_t K^j_{t+1} = R_{t+1}(Q_t K^j_{t+1} - N^j_{t+1})$$

eqn. (3.5) in paper with a correction

$$\boxed{\text{analogue to Romer} \quad \frac{2\gamma - D}{2\gamma} D + \frac{D}{2\gamma}\left(\frac{D}{2} - c\right) = (1+r)(1-w)}$$

defines a function that gives required value of $\bar{\omega}^j$ for each capital quantity K^j_{t+1} an E might choose.

If N^j_{t+1} smaller, required $\bar{\omega}^j$ bigger for any given K^j_{t+1}.

Given this function, E chooses K^j_{t+1} to max expected value of profit.

Note above can also be written

$$[1-F(\bar{\omega}^j)]\bar{\omega}^j R^k_{t+1} Q_t K^j_{t+1} + \int_0^{\bar{\omega}} \omega \, dF(\omega) R^k_{t+1} Q_t K^j_{t+1}$$

$$= R_{t+1}(\quad) + \underbrace{\mu R^k_{t+1} Q_t K^j_{t+1} \int_0^{\bar{\omega}^j} \omega \, dF(\omega)}_{\text{expected value of verification cost. (Romer's } A)}$$

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..... ignoring uncertainty in R^k (cont.)

If $\omega > \bar{\omega}$ probability $[1-F(\bar{\omega}^j)]$

E gets to keep $\omega^j R^k_{t+1} Q_t K^j_{t+1} - \underbrace{\bar{\omega}^j R^k_{t+1} Q_t K^j_{t+1}}_{Z^j_{t+1} B^j_{t+1}}$

Probability that $\omega > \bar{\omega}$ so that E gets something, times expected value of return to business in that event before loan repayment

$$[1-F(\bar{\omega}^j)] E[\omega^j R^k_{t+1} Q_t K^j_{t+1} / \omega > \bar{\omega}] = \int_{\bar{\omega}}^{\infty} \omega R^k Q K \, dF(\omega)$$

Expected return to E is thus

$$\int_{\bar{\omega}}^{\infty} \omega R^k_{t+1} Q_t K^j_{t+1} \, dF(\omega) - [1-F(\bar{\omega})] \bar{\omega}^j R^k_{t+1} Q_t K^j_{t+1} \qquad (3.6)$$

[To allow for uncertainty in K^j_{t+1}, take expectation of this given distn. for K^j_{t+1}]

Using bottom of p. 13, above becomes...

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$$R^k_{t+1} Q_t K^j_{t+1} - \mu R^k_{t+1} Q_t K^j_{t+1} \int_0^{\bar{\omega}} \omega \, dF(\bar{\omega}) - R_{t+1} \left(Q_t K^j_{t+1} - N^j_{t+1} \right)$$

rearrangement of (3.7), ignoring uncertainty about R^k_{t+1}

[Romer analogue would be

$$Y - A - (1+r)(1-w)$$]

E chooses K^j_{t+1} to max expected value of this, given functional reln between $\bar{\omega}^j$ & K^j_{t+1} determined by top of p. 13

Results:

$$E\{R^k_{t+1}\} = s\left(\frac{N^j_{t+1}}{Q_t K_{t+1}}\right) R_{t+1} \quad \text{where } s'(\,) < 0 \qquad (3.9)$$

share of firm financed by E

or $\dfrac{E\{R^k_{t+1}\}}{R_{t+1}} = s(\quad)$

Loglinear approximation of this is

$$E_t\{r^k_{t+1}\} - r_{t+1} = -\nu\left[n_{t+1} - (q_t + k_{t+1})\right] \qquad (4.17)$$