KEYNESIAN DSGE

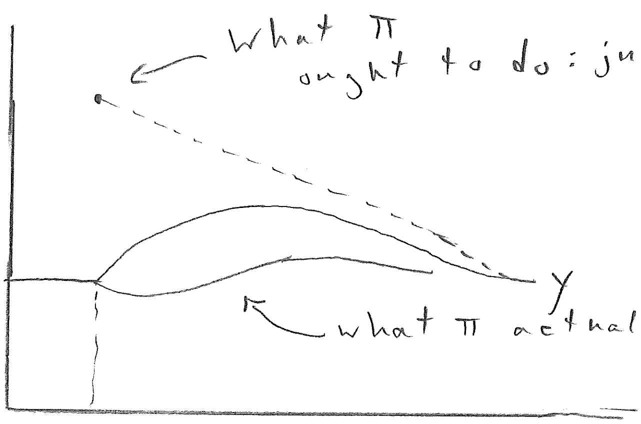
CEE Model

Phillips curve problem

original Calvo: $\pi_t = {}_t\pi^e_{t+1} + \beta y_t$ or $\underbrace{(\pi_{t+1} - \pi_t)^e = -\beta y_t}_{\text{Inflation must be slowing down as long as } y > 0}$

output gap

Implied behavior for π, given y path:



CEE's fix:

Make π more persistent (no jumps, lagged π affects current π)

Calvo: if you can't reset price, $P_{jt} = P_{jt-1}$

gives $\pi_t = {}_t\pi^e_{t+1} + \frac{\alpha^2}{1-\alpha}\phi y_t$

fraction adjusting in a period

$P^*_{jt} = P_t + \phi y_t$

desired p_i for a period

CEE: if you can't reset, $P_{jt} = \pi_{t-1} P_{jt-1}$ (8)

gives $\pi_t = \frac{1}{2}\pi_{t-1} + \frac{1}{2}{}_{t-1}\pi^e_{t+1} + \frac{\alpha^2}{1-\alpha}\phi y_t$

from CEE (32), translated:

their $\hat{\pi} \Rightarrow \pi - \pi^*$

$\beta \Rightarrow 1$

$(1-\xi_p) \Rightarrow \alpha$ — fraction adjusting

$\hat{s}_t \Rightarrow \phi y_t$

deviation of real mc from LRSS

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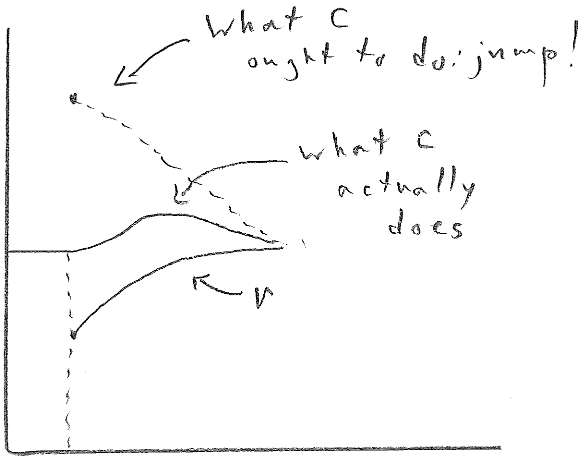
Consumption problem

Original Euler equation: $c_t = \underbrace{e^{\rho}}_{\beta\ \text{(CEE)}} E_t \left[c_{t+1} \frac{1}{1+r_{t+1}} \right]$

Implied behavior for C given y and r path:



As long as $r < r^*$, C must be growing slower than in LRSS, while boost to y means higher lifetime income.

CEE's Fix:

Make C more persistent through "habit formation"

Baseline: $\text{felicity} = u_c(c_t) + u_{1-l}(1-l)$

CEE: $= u_c(c_t - b c_{t-1}) + \ldots\ldots$ (III)

which means $\frac{\partial u}{\partial c_t} = u_c'(c_t - b c_{t-1})$ (recall $u_c'' < 0$

$$\frac{\partial^2 u}{\partial c_t \partial c_{t-1}} = u_c''(c_t - b c_{t-1})(-b)$$

MU of c_t is higher if I consumed a lot last period.
I won't shift lots of C to period just after shock, because c_{t-1} was relatively low; I'll boost c gradually...

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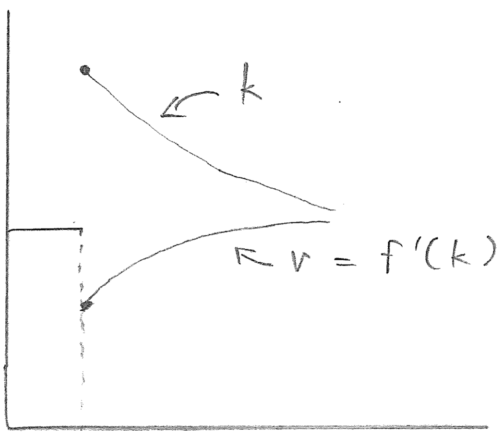
Investment problem

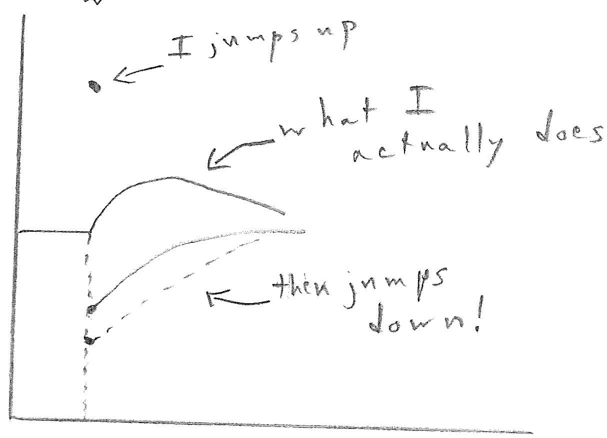
K/L

In baseline DSGE, firms (social planner) set $f'(k) = r + \delta$

Implied behavior for k given r path:

which means for investment:





CEE's fix:

Add costs of changing (raising or lowering) rate of investment from previous period

$$k_{t+1} = (1-\delta)\,k_t + \left(1 - S\left(\frac{i_t}{i_{t-1}}\right)\right) i_t \qquad (13, 20)$$

i is production (purchases) of output for investment

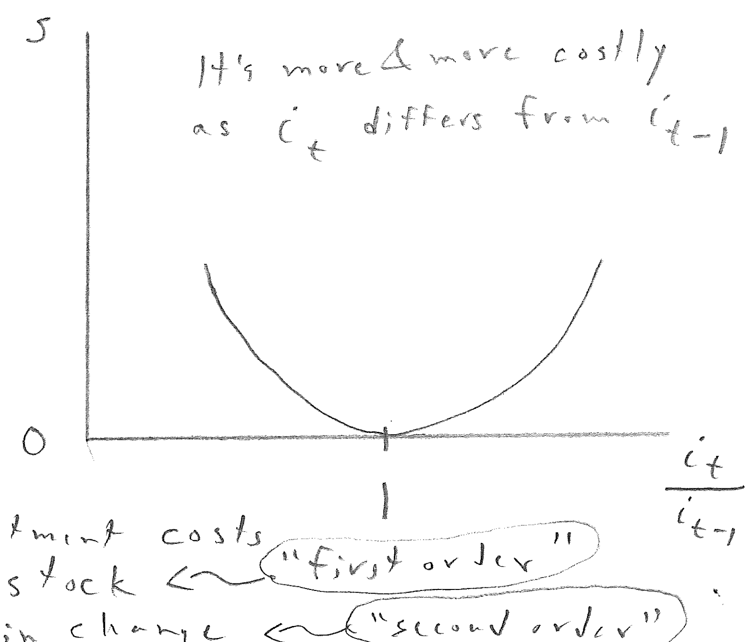
where S is a function

such that

$S(1) = S'(1) = 0$

$S''(1) = \eta > 0$

I won't jump up (or down), as that would be very costly



Note: most models assume adjustment costs apply to change in capital stock ← "first order"
rather than change in change ← "second order"

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Real Rigidity

Recall real rigidity in simple textbook model

$$p_{it}^* = p_t + \phi y_t$$

small ϕ means lots of real rigidity

ϕ is positive because labor market is perfectly competitive with upward-sloping, inelastic L^S & $mc = w$

($y\uparrow \to 2^0 \uparrow \to \frac{W}{P}\uparrow$; for given p_t, means $w\uparrow \to mc\uparrow$)

Romer showed that this means menu cost must be big to deter price adjustment, so he talks about complications to model that make ϕ smaller, increase real rigidity. (p. 282-85).

CEE also need to add real rigidity to their model.

Why?

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CEE

Real Rigidity

Why do CEE need more real rigidity?

Recall their NKPC with indexing:

$$\pi_t = \tfrac{1}{2}\pi_{t-1} + \tfrac{1}{2}\,{}_{t-1}\pi^e_{t+1} + \frac{\alpha^2}{1-\alpha}\,\phi\, y_t$$

real rigidity parameter

fraction of firms adjusting in one period

Their estimates say response of π_t to interest rate cut is small (Fig. 1), so they need coeff. on y_t to be small.

So they need ϕ to be small.

Why can't they make α small?

Because small α would be implausible.

α is observable in data! Sort of.

If model is true, one can infer α from average length of "price contracts," i.e. average lifetime of a price (+ indexation).

Short "contract length," big α.

And people believe contracts are < 1 year.

? How is contract length related to α?

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Real Rigidity

"Contract length" & price adjustment frequency

Translate to CEE notation: $\alpha \to 1 - \varepsilon_p$ or w

ε_p is probability price is not adjusted

ε_p^j Prob. price still stuck at $t+j$

$(1-\varepsilon)\varepsilon^{j-1}$ Prob. price stuck until $t+j-1$, then adjusted

$E[j]$ ("contract duration")

$$= 1(1-\varepsilon) + 2(1-\varepsilon)\varepsilon + 3(1-\varepsilon)\varepsilon^2 + \ldots$$

$$= 1 - \varepsilon + 2\varepsilon - 2\varepsilon^2 + 3\varepsilon^2 - 3\varepsilon^3 + \ldots$$

$$= 1 + \varepsilon + \varepsilon^2 + \varepsilon^3 + \ldots = \frac{1}{1-\varepsilon} \quad \left(\text{or } \frac{1}{\alpha}\right)$$

"Our point estimate of ε_w implies that wage contracts last, on average, 2.8 quarters. Our point estimate of ε_p ---- 2.5 quarters."

Plausible! But to get this, they had to make ϕ small.

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Devices to get real rigidity

1) Noncompetitive labor market, nominal wage rigidity

Make wagesetting work just like pricesetting.

Makes (average) W less sensitive to labor demand.

2) "Financing the wage bill."

Makes W ≠ MC of labor.

Makes MC of labor include W <u>and</u> interest rate, so that i↓ pulls down on MC labor, hence pulls down of MC of production

3) Variable capital utilization

Something that makes mc of capital less sensitive to demand for capital.

Wait a minute! Isn't capital a "fixed factor," i.e. one firm's capital stock is more or less fixed in "short run," i.e. $\frac{1}{4}$ to $\frac{1}{4}$?

No! Not in this model!

KEYNESIAN DSGE

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Real rigidity (cont.)

Tricky thing about capital

Usually capital is a "fixed factor:" a firm cannot adjust capital input at cyclical frequencies (i.e. ¼ to ¼)

Here, as in many DSGE models, a firm's capital input is as variable as labor, even in the "short run."

Total investment, slowed by "second-order" adjustment cost, determines quantity of capital, + capacity utilization determines supply of capital services.

A firm is free to hire as much capital service as it wants at market-clearing rental rate.

But household "investment" in k (to be rented out to firms) is slowed by adjustment cost.

So supply of capital doesn't change much when i↓

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CEE

Real rigidity

Financing wage bill

$$s_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\left(r_t^k\right)^{\alpha}\left(w_t R_t\right)^{1-\alpha} \qquad (7)$$

mc

rental cost of capital

actual wage payment to labor

1 + interest rate

Note: Y_{jt} isn't on RHS, and anything that reduces interest rate ← monetary policy loosening!

actual reduces mc at given w_t!

This reduces ϕ

makes real wage bigger when interest rate lower.

Capacity utilization

Recall investment, capital stock respond slowly to interest rate cut.

So K supply fixed (almost) as demand rises.

That tends to make capital rental rate r_t^k rise when interest rate cut & y rises, makes mc rise & real wage fall! Bad!

So fool with capacity utilization: make effective supply of K (stock • utilization rate) rise when interest rate cut. $k_t = u_t \bar{k}_t$

Huh? Recall King & Rebelo....

effective supply

utilization

physical stock

KEYNESIAN DSGE

CEE

Real rigidity

Capacity utilization (cont.)

Trick: cost of running $\bar{k}$ harder is not faster depreciation but immediate cost in current output (or income, or C).

Household's Bellman equation:

$$V(M_t, \text{other assets}) = \underset{u_t,\ \text{etc.}}{\text{Max}} \left\{ u(c, h, q) + E_t[V(M_{t+1}, \text{etc.})] \right\}$$

money holding; utilization; q: real money balance: "money in the utility function"

where

$$M_{t+1} = \text{------} + R_t^k u_t \bar{k}_t - P_t (i_t + c_t + a(u_t)\bar{k}_t)$$

R_t^k: nominal rental rate; P_t: price level

where $a'(u_t) > 0$, $a''(u_t) > 0$

F.O.C. with respect to u:

$$0 = E_t[V_M(M_{t+1})] \cdot \frac{\partial M_{t+1}}{\partial u_t}$$

hence

$$0 = \frac{\partial M_{t+1}}{\partial u_t} = R_t^k \bar{k}_t - P_t a'(u_t)\bar{k}_t$$

hence

$$a'(u_t) = \frac{R_t^k}{P_t} \qquad (22)$$

$\Leftarrow$ current real rental rate on capital

When central bank cuts i, output rises with $\bar{k}$ fixed so real rental rate rises, so $a'(u_t)$ must rise, so u_t must rise, tending to boost supply of capital services & dampen rise in rental rate

$\Rightarrow$ hold down hike in MC $\Leftarrow$ real rigidity

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Capacity utilization (cont.)

How would it be different if cost were depreciation↑ ?

$\delta'(\,)>0,\ \delta''(\,)>0$

Say $\bar{k}_{t+1} = [1-\delta(u_t)]\bar{k}_t + \ldots\ldots$

no $a(u_t)\bar{k}_t$

$$M_{t+1} = R_t[M_t \ldots\ldots]\ldots + R_t^k u_t \bar{k}_t + \ldots\ldots$$

$$M_{t+2} = R_{t+1}[M_{t+1}\ldots\ldots]\ldots + \underbrace{R_{t+1}^k u_{t+1}}_{\text{dollar value of a unit of installed } \bar{k}_{t+1}} \bar{k}_{t+1} + \ldots\ldots$$

F.O.C.: $$0 = E_t\left[V_M(M_{t+2})\right]\cdot\frac{\partial M_{t+2}}{\partial u_t}$$

$$\frac{\partial M_{t+2}}{\partial u_t} = R_{t+1}\frac{\partial M_{t+1}}{\partial u_t} + R_{t+1}^k u_{t+1}\frac{\partial \bar{k}_{t+1}}{\partial u_t}$$

$$0 = R_{t+1}R_t^k\bar{k}_t + R_{t+1}^k u_{t+1}\left(-\delta'(u_t)\bar{k}_t\right)$$

$$\Rightarrow \delta'(u_t) = R_{t+1}R_t^k\frac{1}{R_{t+1}^k u_{t+1}}$$

When central bank cuts interest rates,

$R_{t+1}\downarrow \quad R_t^k\uparrow$ effect on u_t ambiguous

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Real rigidity

Wage rigidity

In single-period utility function, increasing marginal disutility of labor:

$$u(c_t - b_{t-1}) - z(h_t) + v(q_t) \qquad (11)$$

where $$z(\ ) = \psi_0 \ h_t^2 \qquad (19)$$

If labor market were perfectly competitive, increase in output / labor demand would require higher real wage:

$$z'(h_t) = u'(c_t - b_{t-1}) \frac{\partial c_t}{\partial h_t}$$

$$\frac{\psi_0}{2} h = u'(\quad) \, w_t \leftarrow \text{real wage}$$

$$h_t = u'(\quad) \frac{2}{\psi_0} w_t$$

that means bigger ϕ in $p_{it}^* = p_t + \phi y_t$

so CEE

1) Make each household's labor a differentiated good: each household is a monopolist

2) Each household's wage set like a price, with Calvo + lagged inflation.

With this + flat mc, CEE don't really need overlapping-contract price rigidity.

KEYNESIAN DSGE

CEE Model

Monetary shock

It's actually shock to money-supply growth rate μ

$$\mu_t = \mu + \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots \quad (17)$$

LRSS money growth rate matching

π^* $\quad (\pi^* = \mu - g)$

g

so CEE must specify money demand in model. They put money in utility function:

$$E^j_{t-1} \sum_{\tau=0}^{\infty} \beta^{t-\tau} \left[u(c_{t+\tau} - bc_{t+\tau-1}) - z(h_{j,t+1}) + v(q_{t+\tau}) \right] \quad (11)$$

$$M \rightarrow \frac{Q_t}{P_t} \rightarrow q_{t+\tau}$$

KEYNESIAN DSGE

CEE Model

Composite "final consumption good"

In textbook model of monopolized economy,

$$u = C - \frac{1}{\gamma} L^{\gamma}$$

$$C = \left[\int_{i=0}^{1} c_i^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} \qquad \eta > 1$$

elasticity of product demand

"Dixit-Stiglitz"

CEE assume C is one good but it is "produced" by aggregating individual goods bought from monopoly producers

$$Y_t = \left(\int_0^1 Y_{jt}^{1/\lambda_f} \right) \qquad (3)$$

and producer of aggregate good charges a price equal to marginal cost:

$$P_t = \left[\int_0^1 P_{jt}^{1/1-\lambda_f} dj \right]^{1-\lambda_j} \qquad (5)$$

see how this is like "price index" in textbook model

$$P = \left(\int_{i=0}^{1} P_i^{1-\eta} di \right)^{\frac{1}{1-\eta}}$$

KEYNESIAN DSGE

CEE Model

Composite labor input to production

Like consumption good,
Each household sells its labor at it's
Calvo-with-indexation monopoly wage
to a labor aggregation that makes
"aggregate labor input" λ_w

$$L_t = \left(\int_0^1 h_{jt}^{1/\lambda_w} \, dj \right)$$

and sells it to goods-producers at a wage
equal to marginal cost of making L:

$$W_t = \left[\int_0 W_{jt}^{\frac{1}{1-\lambda_w}} \, dj \right]^{1-\lambda_w}$$

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Consumption insurance

Recall each household was a labor monopolist.

All households whose Calvo bells go off "reoptimize" to same wage, but across other households wages vary. So employment varies. So labor income varies.

So it seems like consumption, demand for capital (saving) would vary across households,

Hard to keep track of!

Easier to keep "representative agent" structure!

To do that, "state-contingent securities" "whose payoffs are contingent on whether it can reoptimize its wage decision" (p. 11)

i.e. consumption insurance that equalizes MU of C across households.

Recall $u(c_t - bc_{t-1}) - z(h_j) + v(q_t)$ (11)

Felicity is separable across c & h, so work hours don't affect MU of c, so equalizing MU c means equalizing C across households.

So all households consume & save same.