STAGGERED PRICE ADJUSTMENT

Fischer (predetermined prices) [Fischer, JPE 1977]

Taylor (fixed prices) [Taylor, AER 1979]

Both are "time-dependent" price adjustment.

Assumption for both models:

price level

$$p_i^* = c + p + \phi y$$

output gap

Desired, optimal price for firm i

or

can be normalized to zero to simplify notation

$$\underbrace{p_i^* - p}_{\text{Desired "real" or "relative" price for firm } i} = c + \phi y$$

Original justification (in original papers):

$p_i^* = c + w_i$ [Profit-maxxing price is markup over mc, mc determined by wage]

$$w_i^* = w - \beta(\text{Unemp. rate})$$

wage level

negatively related to y

desired wage at firm i

STAGGERED PRICE ADJUSTMENT

Assumptions for both models (cont.)

$$p_i^* - p = \phi y$$ small ϕ means "real rigidity"

$$y = m - p$$

$$\Rightarrow p_i^* - p = \phi(m - p)$$

$$p_i^* = \phi m + (1-\phi)p \qquad (6.63, 7.17)$$

"Law of iterated expectations"

$E_t x_{t+j}$ Time t's expected value for x_{t+j}

$$E_{t-\tau}\left[E_t x_{t+j}\right] = E_{t-\tau} x_{t+j}$$

Preset prices & staggering:

In period t, ½ of producers (group 1) set p_t^1 & p_{t+1}^1

In period t+1, others (group 2) set p_{t+1}^2 & p_{t+2}^2

so in each period, half of prices were set in the past

Producers set p_{it} to minimize:

$$(p_{it} - E[p_{it}^*])$$

Cheesy approximation:

$$p_t = \tfrac{1}{2}(p_t^1 + p_t^2)$$

price level

STAGGERED PRICE ADJUSTMENT

Predetermined Prices (Fischer model)

p^1_t need not equal p^1_{t+1} , p^2_{t+1} need not equal p^2_{t+2}

- Assumption about information for pricesetters

Set prices at "beginning" of period, before realization of m_t
Know how other pricesetters behave (they're like me)

Certainty equivalence then implies:

$$p^1_{it} = E_{t-1}[p^*_{it}] = E_{t-1}[\phi m_t + (1-\phi) p_t]$$

$$p^1_{it+1} = E_{t-1}[p^*_{it+1}] = E_{t-1}[\phi m_{t+1} + (1-\phi) p_{t+1}]$$

$$p^2_{it} = E_{t-2}[p^*_{it}] = E_{t-2}[\phi m_t + (1-\phi) p_t]$$

$$p^2_{it+1} = E_t[p^*_{it+1}] = E_t[\phi m_{t+1} + (1-\phi) p_{t+1}]$$

which means

$$p^1_{it} = \phi E_{t-1}[m_t] + (1-\phi)\tfrac{1}{2}(p^1_t + p^2_t) \qquad (7.20)$$

known because it was set before

known because I know how others behave

In equilibrium $p^1_{it} = p^1_t$ so (solving for p^1_t):

$$p^1_t = \frac{2\phi}{1+\phi} E_{t-1}[m_t] + \frac{1-\phi}{1+\phi} p^2_t \qquad (7.22)$$

taking expectation as of $t-2$:

$$E_{t-2}[p^1_t] = \frac{2\phi}{1-\phi} E_{t-2}[m_t] + \frac{1-\phi}{1+\phi} p^2_t \qquad (7.23)$$

known in $t-2$

because $E_{t-2}[E_{t-1}[m_t]] = E_{t-2}[m_t]$

Now find p_t^2:

$$p_{it}^2 = \phi E_{t-2}[m_t] + (1-\phi)\tfrac{1}{2}\left(E_{t-2}[p_t^1] + p_t^2\right) \qquad (7.21)$$

known because I know how others behave

Substituting for $E_{t-2}[p_t^1]$ from (7.23) & assuming $p_{it}^2 = p_t^2$ gives:

$$p_t^2 = E_{t-2}[m_t] \qquad (7.25)$$

Substituting this into (7.22) will give p_t^1:

$$p_t^1 = \frac{2\phi E_{t-1}[m_t] + (1-\phi)E_{t-2}[m_t]}{1+\phi}$$

Rearranging (add $[\phi E_{t-2}[m_t] - \phi E_{t-2}[m_t]]$ to top of fraction):

$$p_t^1 = E_{t-2}[m_t] + \frac{2\phi}{1+\phi}\left(E_{t-1}[m_t] - E_{t-2}[m_t]\right) \qquad (7.26)$$

Now, overall price level:

$$p_t = \tfrac{1}{2}(p_t^1 + p_t^2) = E_{t-2}[m_t] + \frac{\phi}{1+\phi}\left(E_{t-1}[m_t] - E_{t-2}[m_t]\right) \qquad (7.27)$$

$$y_t = m_t - p_t = \underbrace{\frac{1}{1+\phi}\left(E_{t-1}[m_t] - E_{t-2}[m_t]\right)}_{\text{information about } m_t \text{ that came in over period } t-1} + \underbrace{\left(m_t - E_{t-1}[m_t]\right)}_{\text{money (demand) surprise within this period}} \qquad (7.28)$$

effect of this depends on value of ϕ

$$p_{it}^* - p_t = c + \phi y_t$$

"real rigidity"

Staggering with Predetermined ...

Implications

1) "AD shifts that become anticipated after first prices are set affect output"

Example: m is random walk

$m_t = m_{t-1} + u_t$ $\qquad m_t = m_{t-2} + u_{t-1} + u_t$

means $E_t m_{t+j} = m_t$ for all j

hence $E_{t-2} m_t = m_{t-2}$

$E_{t-1} m_t = m_{t-1} = m_{t-2} + u_{t-1}$

AD shift from $t-2$ (when first prices are set)
to t $= u_{t-1} + u_t$

"AD shift that becomes anticipated after first prices are set" is u_{t-1} (becomes observed "anticipated in $t-1$")

$$y_t = \frac{1}{1+\phi}\left(E_{t-1} m_t - E_{t-2} m_t\right) + \left(m_t - E_{t-1} m_t\right)$$

$$= \frac{1}{1+\phi} u_{t-1} + u_t$$

(affects output)

Implications (cont'd)

2) "Interactions among price-setters can either increase or decrease the effects of price stickiness"

Depends on ϕ (real rigidity parameter)

"Price stickiness" means that prices are predetermined.
"Interactions among price setters" is coming from staggering. In any period, ½ of price setters are currently setting their price for that same period; ½ have their price for that period preset.

Does presence of preset prices make "the pricesetters that are free to adjust" hold their prices more rigid, hence keep overall price level more rigid, hence magnify effect of Δm on y? ("increase the effects of price stickiness" on y)?

or does presence of preset prices make "pricesetters that are free to adjust" respond a lot to Δm, hence make overall price level respond more to Δm, weaken effects on y?

What do we compare with? What would be a situation where there is price stickiness, but no interaction?
No staggering!

Staggering with predetermined...

Implications

2) Interactions (cont.)

What happens if prices preset, but no staggering, i.e. *everyone* sets price plan in period t, based on info from $t-1$, for t & $t+1$?

$$p_{it} = E_{t-1}\, p_{it}^{*} \qquad p_{it+1} = E_{t-1}\, p_{it+1}^{*}$$

everyone's like me

recall $p_i^{*} = \phi m + (1-\phi) p$

so $p_{it} = E_{t-1}\left[\phi m_t + (1-\phi) p\right]$ and $E_{t-1}[p] = p_{it}$

so $p_t = \phi E_{t-1} m_t + (1-\phi) p_t$ hence $p_t = E_{t-1} m_t$

Repeat for $t+1$, get $p_{t+1} = E_{t-1} m_{t+1}$

Example: m is random walk

$$m_t = m_{t-1} + u_t \qquad E_t m_{t+j} = m_t \text{ for all } j$$

gives

$$y_t = m_t - p_t = m_t - E_{t-1} m_t = m_t - m_{t-1} = u_t$$

$$y_{t+1} = m_{t+1} - p_{t+1} = m_{t+1} - E_{t-1} m_{t+1} = m_{t+1} - m_{t-1} = u_t + u_{t+1}$$

Now compare with Fischer model...

Staggering with predetermined...

Implications

2) Interactions (cont.)

No staggering: $y_t = u_t$

$$y_{t+1} = u_t + u_{t+1}$$

Staggering: $y_t = \frac{1}{1+\phi} u_{t-1} + u_t$

$$y_{t+1} = \frac{1}{1+\phi} u_t + u_{t+1}$$

What's effect of realized $u \neq 0$ on output history $\sum_{-\infty}^{\infty} y_t$?

For example, $u_t > 0$.

With staggering, $u_t > 0$ raises output in period it hits: $y_t = u_t$

and also raises output in following periods $y_{t+1} = \frac{1}{1+\phi} u_t$

Total effect on $\sum y = \left(1 + \frac{1}{1+\phi}\right) u_t$

Without staggering, effect of $u_t > 0$ depends on whether it hits in "odd" or "even" period.

If it hits in "odd," first half of the "year,"

it raises output $y_t = u_t$ and $y_{t+1} = u_t$

If it hits in "even," second half of year,

it raises output $y_{t+1} = u_{t+1}$

but has no effect on y_{t+2}

Staggering with predetermined ---

Implications

2) Interactions (cont.)

So without staggering, effect of $u \neq 0$

on $\sum y = u$ if it hit in odd

$\sum y = 2u$ if it hit in even

so **average** effect is

$$\tfrac{1}{2} u + \tfrac{1}{2} 2u = \left(1 + \tfrac{1}{2}\right) u$$

compare with $\left(1 + \frac{1}{1+\phi}\right) u$ ← staggering

If $\phi < 1$, staggering **magnifies** effect of u

$\left(\text{because } 1 + \frac{1}{1+\phi} > 1 + \frac{1}{2}\right)$

If $\phi > 1$, staggering weakens effect of u

Staggering with predetermined...

Implications

3) Monetary shocks' effect on Y is slightly more persistent than in LSF, but still not very persistent

Fischer: $y_t = \frac{1}{1+\phi}(E_{t-1}m_t - E_{t-2}m_t) + (m_t - E_{t-1}m_t)$

LSF: $y_t = \frac{b}{1+b}(m_t - E_{t-1}m_t)$

Example: m is random walk $m_t = m_{t-1} + u_t$

$E_t m_{t+j} = m_t$ for all j

so $m_t - E_{t-1}m_t = m_t - m_{t-1} = u_t$

$E_{t-1}m_t - E_{t-2}m_t = m_{t-1} - m_{t-2} = u_{t-1}$

so

Fischer: $y_t = \frac{1}{1+\phi}u_{t-1} + u_t$

LSF: $y_t = \frac{b}{1+b}u_t$

4) Systematic monetary policy can help stabilize output (not in Romer)

See assigned problem

STAGGERED PRICE ADJUSTMENT

Fixed Prices (Taylor Model)

$$\underbrace{p_t^1 = p_{t+1}^1}_{\text{group one}} \qquad \underbrace{p_{t+1}^2 = p_{t+2}^2}_{\text{group two}}$$

Assumption about information for pricesetters:

set prices <u>after</u> observing m_t

Additional notation:

$x_t = p_t^1 = p_{t+1}^1$ } price set by pricesetters who adjust in $t, t+2$, etc.

$x_{t-1} = p_t^2 = p_{t-1}^2$ } price set by pricesetters who adjust in $t-1, t+1$, etc.

Certainty equivalence (and no time-discounting)

$$x_{it} = \frac{1}{2}\left(p_{it}^* + E_t[p_{it+1}^*]\right)$$

$$x_{it} = p'_{it} = \frac{1}{2}\left((\phi m_t + (1-\phi)p_t) + (\phi E_t[m_{t+1}] + (1-\phi)E_t[p_{t+1}])\right) \quad (7.30)$$

price set at time t for t and $t+1$

Compare to predetermined prices (Fischer) model, which had:

$$p'_{it} = E_{t-1}[\phi m_t + (1-\phi)p_t]$$

price set at time $t-1$ for t

$\boxed{\text{Key difference}}$: If prices predetermined but not fixed, price set for period t depends on expected values of period t variables.

If prices fixed, price set for period t depends on expected value of p_{t+1}, which will in turn depend on expected value of p_{t+2} <u>ad</u> <u>infinitem</u>

Staggering with Fixed Prices (Taylor model) (cont.)

General solution for x_t (using "lag operators") gives:

$$x_t = \lambda x_{t-1} + \frac{\lambda}{A}\frac{1-2A}{2}\left(m_t + (1+\lambda)\left(E_t m_{t+1} + \lambda E_t m_{t+2} + \lambda^2 E_t m_{t+3} + \dots\right)\right) \quad (7.50)$$

where $\lambda = \dfrac{1-\sqrt{\phi}}{1+\sqrt{\phi}}$ Note: for $0<\phi<1$ ("real rigidity"), $0 < \lambda < 1$

$$A = \frac{1}{2}\,\frac{1-\phi}{1+\phi} \quad \text{and} \quad A + A\lambda^2 = \lambda$$

Rewrite (7.50):

price set at time t for t and t+1

$$x_t = \lambda x_{t-1} + \lambda \frac{\phi}{1-\phi}\left(m_t + (1+\lambda)\left(E_t m_{t+1} + \lambda E_t m_{t+2} + \lambda^2 E_t m_{t+3} + \dots\right)\right)$$

Note: price set today depends on beliefs about path of m (AD) into infinite future.

A weird implication: expected future $m \uparrow$ causes recession today

Recall $p_t = \frac{1}{2}(x_t + x_{t-1})$

$$y_t = m_t - p_t = m_t - \tfrac{1}{2}(x_t + x_{t+1})$$

Holding m_t fixed, higher levels for $E_t m_{t+1}$, $E_t m_{t+2}$ etc. raises today's price level, hence reduces today's output.

To determine p_t, y_t, must specify (beliefs about) process determining m

A simple case is random walk

$$m_t = m_{t-1} + u_t \quad \text{where } u \text{ is i.i.d., mean zero}$$

means

$$m_t = E_t m_{t+1} = E_t m_{t+2} = \dots$$

Staggering with Fixed Prices (Taylor..) (cont.)

Assuming m is random walk

Recall

$$x_t = \lambda x_{t-1} + \lambda \frac{\phi}{1-\phi}\left(m_t + (1+\lambda)\left(E_t m_{t+1} + \lambda E_t m_{t+2} + \lambda^2 E_t m_{t+3} + \dots\right)\right)$$

becomes

$$= \dots\dots \left(m_t + (1+\lambda)\left(m_t + \lambda m_t + \lambda^2 m_t + \lambda^3 m_t \dots\right)\right)$$

$$= \dots\dots \left(m_t + (1+\lambda)\left(\frac{1}{1-\lambda} m_t\right)\right)$$

$$= \lambda x_{t-1} + \lambda \frac{\phi}{1-\phi}\left(1 + \frac{1+\lambda}{1-\lambda}\right) m_t$$

$$= \lambda x_{t-1} \frac{\lambda}{A} \frac{1-2A}{2}\left(1 + \frac{1+\lambda}{1-\lambda}\right) m_t \qquad (7.51)$$

Recall $A + A\lambda^2 = \lambda$

means $(1+\lambda^2)A = \lambda \Rightarrow A = \frac{\lambda}{1+\lambda^2}$

Substitute that into (7.51) gives

$$x_t = \lambda x_{t-1} + (1-\lambda) m_t \qquad (7.35)$$

Recall $p_t = \frac{x_t + x_{t-1}}{2}$

hence

$$p_t = \frac{1}{2}\left[\lambda x_{t-2} + (1-\lambda) m_{t-1} + \lambda x_{t-1} + (1-\lambda) m_t\right]$$

$$= \lambda \frac{x_{t-1} + x_{t-2}}{2} + (1-\lambda)\frac{m_t + m_{t-1}}{2}$$

$$= \lambda \, p_{t-1} + (1-\lambda)\frac{m_t + m_{t-1}}{2}$$

Price level is "sticky," depends on lagged price level. "Inertia."

Staggering with... (Taylor)

Assuming m is random walk (cont.)

How does output behave?

Recall $y = m - p$

hence

$$y_t = m_t - \lambda p_{t-1} - (1-\lambda)\frac{m_t + m_{t-1}}{2}$$

Recall $m_t = m_{t-1} + u_t$

$$y_t = m_{t-1} + u_t - \lambda p_{t-1} - (1-\lambda)\frac{m_{t-1} + u_t + m_{t-1}}{2}$$

$$= \ldots\ldots\ldots - \frac{(1-\lambda)}{2}u_t - (1-\lambda)m_{t-1}$$

$$= m_{t-1} + u_t - \lambda p_{t-1} - \frac{(1-\lambda)}{2}u_t - m_{t-1} + \lambda m_{t-1}$$

$$= \left(1 - \frac{1-\lambda}{2}\right)u_t + \lambda(\underbrace{m_{t-1} - p_{t-1}}_{y_{t-1}})$$

$$= \lambda y_{t-1} + \frac{1+\lambda}{2}u_t \qquad (7.44)$$

A shock to m (AD) causes disturbances to y that last a <u>long</u> time: they don't die out after just one or two periods (like Fischer model).

Business cycles are more "persistent" than in Fischer.

Staggering with Fixed Prices (cont.)

Where did

$$x_t = \lambda x_{t-1} + (1-\lambda) m_t \quad \text{come from?} \qquad (7.35, 7.51)$$

$$\lambda = \frac{1-\sqrt{\phi}}{1+\sqrt{\phi}}$$

For random walk case

"Method of Undetermined Coefficients"

Use intuition to guess the answer, then check that it works

Recall:

$$p_t = \tfrac{1}{2}(x_t + x_{t-1})$$

$$p_{it}^* = \phi m + (1-\phi) p$$

$$x_t = \tfrac{1}{2}\left(p_{it}^* + E_t[p_{it+1}^*]\right)$$

$$E_t[m_{t+j}] = m_t$$ demand is a random walk

hence

$$x_t = \tfrac{1}{2}\Bigg(\overbrace{\Big[\phi m_t + (1-\phi)\underbrace{\tfrac{1}{2}(x_t + x_{t-1})}_{p_t}\Big]}^{p_{it}^*} + \overbrace{\Big[\phi \underbrace{m_t}_{E_t[m_{t+1}]} + (1-\phi)\underbrace{\tfrac{1}{2}\big(x_t + E_t[x_{t+1}]\big)}_{E_t[p_{t+1}]}\Big]}^{E_t[p_{it+1}^*]}\Bigg)$$

Solve for x_t gives

$$x_t = A\left(x_{t-1} + E_t[x_{t+1}]\right) + (1-2A) m_t \qquad (7.32)$$

$$A = \frac{1}{2}\,\frac{1-\phi}{1+\phi}$$

or

$$x_t = A x_{t-1} + A E_t[x_{t+1}] + (1-2A) m_t$$

problem: what's $E_t[x_{t+1}]$?

Staggering with Fixed Prices: Method of Undetermined Coefficients (cont.)

? What can we guess about

$$x_t = A x_{t-1} + A E_t[x_{t+1}] + (1-2A) m_t \qquad (7.32)$$

price set for t & $t+1$ — price set for $t-1$ & t — price set for $t+1$ & $t+2$

① $E_t[x_{t+1}]$ must be determined by x_{t-1}, x_t, & $E_t[m_{t+j}]$
(Those are the only variables in the model!)

② $E_t[m_{t+j}] = m_t$ (because m is random walk)

hence $E_t[x_{t+1}]$ is function of m_t, x_t, & x_{t-1}

③ Everything in model is linear, so maybe
$E_t[x_{t+1}]$ is a <u>linear</u> function of m_t, x_t & x_{t-1}

So maybe (7.32) is:

$$x_t = \mu + \lambda x_{t-1} + \nu m_t \qquad (7.33)$$

Another guess:

If $x_{t-1} = m_t$, x_t should equal m_t, because $E_t[m_{t+1}] = E_t[m_{t+j}] = m_t$
(seems like p^*_{it} & $E_t[p^*_{it+j}]$ would equal m_t)

hence

$$m_t = \mu + \lambda m_t + \nu m_t \qquad (7.34)$$

can only hold if $\lambda + \nu = 1$, $\mu = 0$.

Hence $\nu = 1 - \lambda$

Staggering with Fixed Prices
Method of Undetermined Coefficients (cont.)

From last page,

$$x_t = \lambda x_{t-1} + (1-\lambda) m_t \qquad (7.35)$$

Is there a value of λ that would make the model work & be consistent with rational expectations?

(7.35) implies $x_{t+1} = \lambda x_t + (1-\lambda) m_{t+1}$

Rational expectations implies

$E_t[m_{t+1}]$ for random walk

$$\begin{aligned} E_t[x_{t+1}] &= \lambda x_t + (1-\lambda) m_t \\ &= \lambda(\lambda x_{t-1} + (1-\lambda) m_t) + (1-\lambda) m_t \\ &= \lambda^2 x_{t-1} + (1+\lambda)(1-\lambda) m_t \end{aligned}$$

$$E_t[x_{t+1}] = \lambda^2 x_{t-1} + (1-\lambda^2) m_t \qquad (7.36)$$

substitute into (7.32)

$$\Rightarrow x_t = (A + A\lambda^2) x_{t-1} + [A(1-\lambda^2) + (1-2A)] m_t \qquad (7.37)$$

must match

$$x_t = \lambda \; x_{t+1} + (1-\lambda) \; m_t \qquad (7.35)$$

hence

$$A + A\lambda^2 = \lambda$$

$$\Rightarrow A + A\lambda^2 - \lambda = 0$$

quadratic equation in λ

which is solved by

$$\lambda = \frac{1 \pm \sqrt{1-4A^2}}{2A} \qquad (7.40)$$

$\frac{1}{2} \frac{1-\phi}{1+\phi}$

Staggering with Fixed Prices

Method of Undetermined Coefficients (cont.)

Implies two possible values for λ:

$$\lambda_1 = \underbrace{\frac{1-\sqrt{\phi}}{1+\sqrt{\phi}}}_{\text{less than one}} \qquad \lambda_2 = \underbrace{\frac{1+\sqrt{\phi}}{1-\sqrt{\phi}}}_{\text{more than one}}$$

Recall $x_t = \lambda x_{t-1} + (1-\lambda) m_t$

λ_2 wouldn't work (coefficient on x_{t-1} couldn't be > 1, coefficient on m_t couldn't be negative)

so $\lambda = \dfrac{1-\sqrt{\phi}}{1+\sqrt{\phi}}$!