LUCAS SUPPLY FUNCTION (IMPERFECT-INFORMATION MODEL)

Setting

Yeoman barbers

Each household i uses its own labor to produce one good $Y_i = L_i$, sells it for price P_i

Many goods

Demand for goods affected by M and idiosyncratic factors

$$Y_i^D = Y\, Z_i\, (P_i/P)^{-\eta}$$

Y Aggregate real output (income, real GDP)

P Price level (price index)

Z_i Idiosyncratic, good-specific demand shock

and $Y = M/P$ (How? We'll get back to this

so, in logs, $y_i^D = (m-p) + z_i - \eta(p_i - p)$ (6.79)

averages to zero across all i

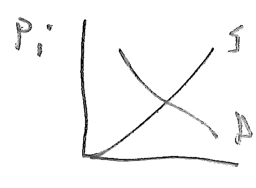
elasticity of demand

Perfect competition

Each household takes P_i as given, chooses $L_i = Y_i$

Must be many households producing each good, but to simplify notation_

so to clear market $L_i = Y_i^S = Y_i^D = (M/P)Z_i(P_i/P)^{-\eta}$



determines P_i

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Setting (cont.)

No saving, so no dynamic optimization

A household lives many periods, but cannot save or borrow, so in each period spends entire income $P_i Y_i$ on current consumption C_i

aggregate of all goods

$$P_i Y_i = P\, C_i$$

index of prices of all goods

Chooses $L_i\ (= Y_i)$ to maximize:

$$U_i = C_i - \frac{1}{\gamma} L_i^{\gamma} = \frac{P_i}{P} Y_i - \frac{1}{\gamma} Y_i^{\gamma} \qquad (6.74)$$

Why does $Y = M/P$?

1) M is \$AD, fixed at any point in time

2) M is money in the utility function

$$U_i = C_i^{\alpha} (M_i/P)^{1-\alpha} - \frac{1}{\gamma} L_i^{\gamma} \quad \text{where } PC_i + M_i = P_i L_i$$

Constraint gives $C_i = \dfrac{P_i L_i}{P} - \dfrac{M}{P}$

so

$$U_i = \left(\frac{P_i L_i}{P} - \frac{M}{P}\right)^{\alpha} (M/P)^{1-\alpha} - \frac{1}{\gamma} L^{\gamma}$$

F.o.C. $\dfrac{\partial U}{\partial (M/P)} = 0 = C^{\alpha}(1-\alpha)(M/P)^{-\alpha} + \alpha C^{\alpha-1}(-1)(M/P)^{1-\alpha}$

$$\Rightarrow C_i = \frac{\alpha}{1-\alpha} \frac{M_i}{P}$$

and Y is an index of C_i's

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Setting (cont.)

Imperfect information

When choosing L_i, household can see P_i but not P (or enough current information to infer P)

P^e Household's expected value for P, based on information set I ← a vector

Rational Expectations

$P^e(I) = E[P/I]$ derived from model

Why would this be true?

1) H'hold knows model, is rational, knows math

2) This is LRE of a stable economy where household has had many, many observations of P_t, I_t, and has figured out the pattern (e.g. by running regressions)

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Approximations we will use

P = Average of p_i's (6.80)

logs

y = Average of y_i's (6.81)

"Certainty equivalence":

1) Solve household's problem in case where it sees P, get $L_i(P_i/P)$

2) To describe case where it can't see P, just replace P with $P^e = E[P|I]$)

Is this OK?

Certainly, if

true $L_i(P_i/P^e) = L_i(P_i/P)$ + constant

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Something from stats: Bayes' Rule

How to deal with "noisy" information

X_t An unobservable variable σ^2

you have a prior distn. for X

mean $E[X]$ variance V_X

After X_t is realized, you observe a noisy signal of X_t:

$X_t^{sig} = X_t + \varepsilon$ (mean zero, variance V_ε

What is $E[X_t / X_t^{sig}]$ (after you've seen X_t^{sig}

if X & ε are normal?

$$E[X_t / X_t^{sig}] = \Theta X_t^{sig} + (1-\Theta) E[X]$$

where $\Theta = \dfrac{V_X}{V_X + V_\varepsilon}$

If you know X varies a lot (V_X big)

ε doesn't vary much (V_ε small)

revise your priors a lot in response to the signal

LSF (cont.)

How we'll solve model

1) Derive y_i hhold would choose if it knew p.
Under certainty equivalence, this tells us how hhold responds to p_i & $E[p]$.

2) Conjecture that p & $(p_i - p)$ will turn out to be normally dist'd & i.i.d.
Derive y_i under this conjecture.

3) Given y_i, get y, p. Confirm that p & $(p_i - p)$ are indeed.....

4) [Not in Romer] Rearrange things into form of a Phillips curve.

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1) Derive y_i if h'hold knew p

$$U_i = C_i - \frac{1}{\gamma} L_i^{\gamma}$$

and $C_i = (P_i Y_i)/P$ & $Y_i = L_i$

$$\Rightarrow U_i = \frac{P_i}{P} Y_i - \frac{1}{\gamma} Y_i^{\gamma} \qquad (6.74)$$

Take P_i & P as given, choose Y_i to max U_i, f.o.c. gives (converting to logs)

$$y_i^S = \frac{1}{\gamma - 1}(p_i - p) \qquad (6.77)$$

Given y_i^D above & $y = m - p$,

$$y_i^D = (m-p) + z_i - \eta(p_i - p) \qquad (6.79)$$

If h'hold knew p, we'd set $(6.77) = (6.79)$, solve for $(p_i - p)$. Then assume p = average p_i, average $z = 0$, & we'd be done.

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1) (cont.)

$$y_i^s = \frac{1}{\gamma - 1}(p_i - p)$$

call $p_i - p = r_i$

(available info)

"Certainty equivalence" means

$$y_i^s = \frac{1}{\gamma-1} E[r_i] = \frac{1}{\gamma-1} E[r_i / p_i] \quad (6.83)$$

2) Apply conjecture about p & r_i

that they're normal & i.i.d.

We know p_i & have a prior $E[p]$,

what's $E[r_i / p_i]$?

$$p_i = r_i + p = r_i + E[p] + (p - E[p])$$

$$\Rightarrow \underbrace{p_i - E[p]}_{\text{signal}} = \underbrace{r_i}_{X} + \underbrace{(p - E[p])}_{\text{noise}}$$

Conjecture r_i normal mean zero, V_r

p normal mean $E[p]$, V_p

hence

$$E[r_i/p_i] = \Theta(p_i - E[p]) + (1-\Theta)\underbrace{E[p_i - E[p]]}_{\text{zero}}$$

$$= \frac{V_r}{V_r + V_p}(p_i - E[p]) \quad (6.84)$$

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2) Apply conjecture (cont.)

so

$$y_i^s = \underbrace{\frac{1}{\gamma-1}\frac{V_r}{V_r+V_p}}_{\text{"}b\text{"}}(p_i - E[p]) \qquad (6.85)$$

Averaging across producers,

$$y = b(p - E[p]) \qquad (6.86)$$

Using $y = m - p$

$$m - p = b[p - E[p]]$$

$$\Rightarrow \quad p = \frac{1}{1+b} m + \frac{b}{1+b} E[p] \qquad (6.87)$$

$$y = \frac{b}{1+b} m - \frac{b}{1+b} E[p] \qquad (6.88)$$

If expectations are rational, then given $E[m]$ but prior to realization/observation of m,

$$E[p] = \frac{1}{1+b} E[m] + \frac{b}{1+b} E[p]$$

$$\Rightarrow E[p] = E[m]$$

so, again from (6.87),

$$p = \frac{1}{1+b} m + \frac{b}{1+b} E[m]$$

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3) Confirm conjecture

$$p = \frac{1}{1+b} m + \frac{b}{1+b} E[m]$$

Add $E[m] - E[m]$ to RHS, rearrange:

$$p = E[m] + \frac{1}{1+b}(m - E[m])$$

and $p - E[p] = p - E[m] = \frac{1}{1+b}(m - E[m])$

Look! If $(m - E[m])$ is normal with V_m,

then $(p - E[p])$ is normal with $V_p = \left(\frac{1}{1+b}\right)^2 V_m$!

That verifies *part* of conjecture.

But what about r_i? Is it normal, independent of p?

We'll check that. We'll write y_i^s & y_i^d as functions of r_i & solve for r_i.

Recall $y_i^s = b(p_i - E[p])$

$$= b((p_i - E[p]) + (p - E[p]) - (p - E[p]))$$

$$= b(p_i - p) + b(p - E(p))$$

Recall $y_i^d = y + z_i - \gamma(p_i - p)$ (6.78)

$$y = b(p - E[p]) \quad (6.85)$$

$$\Rightarrow y_i^d = b(p - E[p]) + z_i - \gamma(p_i - p) \quad \text{so....}$$

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Confirm conjecture (cont.)

Set $y_i^d = y_i^s$

$$b(p-E[p]) + z_i - \gamma r_i = b r_i + b(p-E[p])$$

$$\Rightarrow r_i = z_i/(\gamma+b)$$

Look! r_i determined by z_i. So if z_i uncorrelated with $(m-E[m])$, r_i uncorrelated with p.

And $V_r = \left(\frac{1}{\gamma+b}\right)^2 V_z$ & we can make z normal.

So, both conjectures confirmed.

$$b = \frac{1}{\gamma-1}\frac{V_r}{V_r+V_p} = \frac{1}{\gamma-1}\left[\frac{V_z}{V_z + \frac{(\gamma+b)^2}{(1+b)^2} V_m}\right]$$

this defines b, though you can't solve it.

Implications of LSF

From $y = \frac{b}{1+b} m - \frac{b}{1+b} E[p]$ (6.86)

& $E[p] = E[m]$ (6.88)

$$\Rightarrow y = \frac{b}{1+b}(m - E[m])$$

recall $p = E[m] + \frac{1}{1+b}(m - E[m])$

Expected, foreseeable movements in m affect only p, not y. Only "surprise" in m affects y.

So "systematic monetary policy cannot help stabilize output," unless "policymakers have information that is not available to private agents" (i.e. the firms/h'holds).

Any change in m set as function of publicly available info will be anticipated by public, affect E[m] hence p not y.

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Example of policy ineffectiveness

$$m_t = m^*_t + v_t$$ (other factors

(set by central bank

$$v_t = v_{t-1} + u_t$$ Random walk, $E_{t-1}[u_t] = 0$

$$E_{t-1}[v_t] = v_{t-1}$$

"Passive" monetary policy: $m^* = \bar{m}$ always

"Active" policy: counteract shocks to m as much as possible. Use m^* to counteract observable portion of v_t, which is v_{t-1}

$$m^*_{t-1} - m^*_{t-1} = -(v_{t-1} - v_{t-2}) = -u_{t-1}$$

so if policy is passive

$$m_t = \bar{m} + \sum_{\tau=0}^{\infty} u_{t-\tau} = \bar{m} + \underbrace{\sum_{\tau=1}^{\infty} u_{t-\tau}}_{v_{t-1}} + u_t$$

if policy is active

$$m_t = u_t$$

See the assigned problem!

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Expectations-augmented Phillips curve

Recall this is $\pi_t = {}_{t-1}\pi^e_t + \beta(y - \bar{y})_t$

Here, $y = b(p - E[p])$ (6.86)

Define $E[p] = E_{t-1}[p_t]$

$$y_t = b(p - p_{t-1} - E_{t-1}[p_t] - p_{t-1})$$

$$= b(\pi_t - E_{t-1}\pi_t)$$

or

$$\pi_t = E_{t-1}\pi_t + \frac{1}{b} y_t$$

Later developments:

Allow savings, dynamic optimization

Introduce labor market with wage W_t.

Households see W_t but not P_t.

When surprise $m\uparrow \rightarrow W\uparrow$? Is it $(W/P)\uparrow$?

Criticisms

1) Households can't see prices of things they buy?

2) With rational expectations, LSF implies that effect of a monetary shock on Y goes away as soon as people know that it happened.

Example: surprise slowdown in M growth rate.

In reality, effects of such shocks appears to be "persistent."