MONEY

What happens if we introduce money M, and
P Price level (price of "output") in \$ "Nominal"?

What is "money"?

An asset that is "medium of exchange" for transactions
(sales of output, employment of L & K, loan contracts)

In most macro models,

1) "Cash in advance constraint"

To buy/sell $P \cdot Q$ (output) someone must hold that quantity of money. So $M^D = P \cdot Q$ or $\frac{M^D}{P} = Q$

2) "Money in the utility function"

"Real balance" M/p held by an agent enters utility function similar to C, leisure

A shortcut meant to depict trades with money take less time than trades without money

Real interest rate r versus "nominal" rate i

$$r = \frac{\text{How much output asset pays next period}}{\text{" " " " costs this period}} - 1$$

$$i = \frac{\text{How many \$ asset pays next period}}{\text{" " " " costs this period}} - 1$$

MONEY

What is "money"? (cont.)

Relationship between r, i, P_t, P_{t+1}

For each unit of output I buy this period, I get $(1+r)$ units next period.

What is corresponding i?

$$i = \frac{P_{t+1} \cdot (1+r)}{P_t \cdot 1} - 1 = \frac{P_{t+1}}{P_t}(1+r) - 1$$

$$\frac{P_{t+1}}{P_t} = 1 + \pi \leftarrow \text{inflation rate}$$

$$i = (1+\pi)(1+r) - 1 \approx \pi + r \quad \text{for "small" } \pi \text{ and } r$$

Unrealistic assumptions in our models

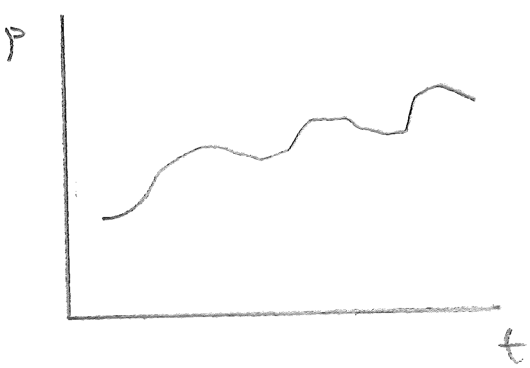
For money, $i = 0$

M^S is exogenous or at least fixed within a period

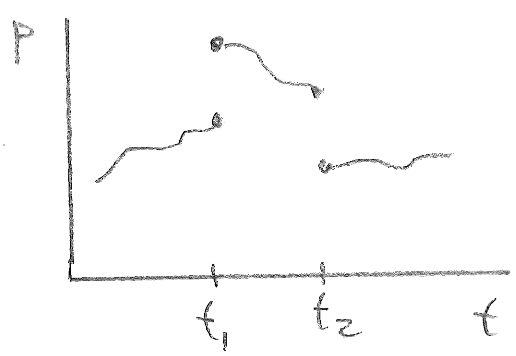
Money

How must P behave in a RBC type model?

In model where all markets always clear, P must sometimes "jump," big discontinuous change from one period to the next, so that at that moment $\pi = \infty$



Continuous change in P, so $\partial P/\partial t$ finite



Discontinuous changes in P, so $\partial P/\partial t = \infty$ at t_1, t_2

Keynesians believe this can't happen: mechanisms on "aggregate supply" side of economy make $\partial P/\partial t$ finite, price level is "sticky."

This gives Keynesian models, theory of business cycles.

MONEY

How must P behave... (cont.)

Examples (Romer 12.1)

$\bar{Y}$
$\bar{r}$ values from RBC type model. "Natural rates."

$i \approx r + \pi^e \leftarrow$ (expected π

$$\frac{M^D}{P} = L(\overset{-}{i}, \overset{+}{Y})$$ like from money-in-utility function

minus sign

given M^s,

$$\frac{M^s}{P} = L(\bar{i}, Y) = L(\bar{r} + \pi^e, \bar{Y})$$

We'll take paths for $\bar{Y}$, $\bar{r}$, M^s as given, see what P has to do. Note: π^e must match what is happening to P, unless something happens unexpectedly, as a surprise.

For extra simplicity, sometimes assume:

$$\frac{M^D}{P} = Ye^{-bi}$$ "semilog form"

means $m^d - p = y - bi$

$$\partial(m^d - p)/\partial y = 1 = \frac{\partial(M^D/P)/(M/P)}{\partial Y/Y} = 1$$ ("income elasticity of money demand"

[Note this means, if i fixed, rate of growth of $(m^D - p)$ equals rate of growth of y

Money

How must P behave? (cont'd)

Price adjustments given path for M

$\bar{Y}, \bar{r}$ fixed, $M = M_0 e^{mt}$ (M grows at rate m)

Path for P that solves the system:

P grows at rate m

$\pi^e = m$

$\bar{i} = \bar{r} + m$

$\overline{\left(\frac{M}{P}\right)} = L(\bar{r} + m, \bar{Y})$

P_0 price level at time zero determined by:

$$\frac{M_0}{P_0} = L(\bar{r} + m, \bar{Y}) \Rightarrow P_0 = \frac{M_0}{L(\bar{r} + m, \bar{Y})}$$

$\bar{Y}, \bar{r}$ fixed, M grows at rate m, M "jumps" down at time t unexpectedly

Path for P:

P grows at rate m before & after jump

$\pi^e = m$

$\overline{\left(\frac{M}{P}\right)} = L(\bar{r} + m, \bar{Y})$

P jumps down at time t to keep $\left(\frac{M}{P_t}\right) = \overline{\left(\frac{M}{P}\right)}$

Money

How must P...?

Price adjustments given path for M (cont.)

$\bar{Y}, \bar{r}$ fixed, M grows at rate m,

at time t m falls unexpectedly from m_0 to m_1

Path for P:

Before time t,

P grows at rate m_0

$\pi^e = m_0$ ← we don't expect change in m

$$\left(\frac{\bar{M}}{P}\right)_0 = L(\bar{r} + m_0, \bar{Y})$$

At time t,

$\pi^e = m_1$

$$\left(\frac{\bar{M}}{P}\right)_1 = L(\underbrace{\bar{r} + m_1}_{\bar{i}_1}, \bar{Y}) > L(\underbrace{\bar{r} + m_0}_{\bar{i}_0}, \bar{Y}) = \left(\frac{\bar{M}}{P}\right)_0$$

$\frac{M}{P}$ needs to jump up! But at time t, M hasn't increased,

so P must jump down

$$\frac{M}{P_t} \Big/ \frac{M}{P_{t-1}} = \frac{L(\bar{r} + m_1, \bar{Y})}{L(\bar{r} + m_0, \bar{Y})} > 1$$

$$\frac{P_t}{P_{t-1}} = \frac{L(\bar{r} + m_0, \bar{Y})}{L(\bar{r} + m_1, \bar{Y})} < 1$$

Note P jumps down before new, lower money growth rate has any effect on M!

Money

How must P ...

M^s fixed, $\bar{r}$ fixed, $\bar{Y}$ grows at g, semilog money demand

recall this means $\frac{\partial(M/P)/(M/P)}{\partial Y/Y} = 1$ ← income elasticity of money demand

For $\overline{\left(\frac{M}{P}\right)} = L(\bar{r} + \pi^e, \bar{Y})$ & any fixed π^e,

means $\overline{\left(\frac{M}{P}\right)}$ must grow at rate g

With M fixed that means $\bar{P}$ falls at rate g

hence $\pi^e = -g$

$\left(\frac{M}{\bar{P}}\right) = L(\bar{r} - g, \bar{Y})$

same, but g falls unexpectedly from g_0 to g_1 at time t

In old LRE, $\pi_0^e = -g_0$ $\quad \bar{i}_0 = \bar{r} - g_0$

In new LRE, $\pi_1^e = -g_1$ $\quad \bar{i}_1 = \bar{r} - g_1 > \bar{i}_0$ because $g_1 < g_0$

$\left(\frac{M}{P}\right)_t = L(\bar{r} - g_1, \bar{Y}_t) < \left(\frac{M}{P}\right)_{t-1} = L(\bar{r} - g_0, \bar{Y}_t)$

fixed M means

$\frac{M}{\bar{P}_t} < \frac{M}{\bar{P}_{t-1}} \qquad \bar{P}_t > \bar{P}_{t-1}$

P must jump up at time t

KEYNESIAN MACRO

LM Curve

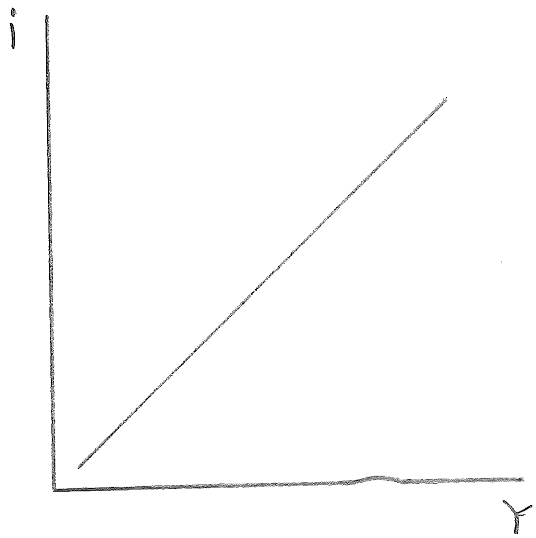
$\left(\frac{M}{P}\right)^D = L(\overset{-}{i}, \overset{+}{Y})$ e.g. $\left(\frac{M}{P}\right)^D = Ye^{-bi}$ "semilog"

"Money demand shocks"

$\left(\frac{M}{P}\right)^D = L(\overset{-}{i}, \overset{+}{Y}) + \varepsilon$

Say M^s fixed in a period, and take P as given

$\frac{M^s}{P} = L(\overset{-}{i}, \overset{+}{Y})$ gives LM curve



$M^s \uparrow \rightarrow$ shifts it out
$P \uparrow \rightarrow$ shifts it back
$\varepsilon \uparrow \rightarrow$ shifts it back

Example: from semilog

$m^s - p = y - bi + \varepsilon$

$i = \frac{1}{b}(-m^s + p + y + \varepsilon)$

More generally....

Keynesian macro

LM curve (cont.)

Why does LM slope up?

Upward slope means that $\partial i/\partial Y > 0$ for given $\frac{M}{P}$

How to show this is true:

- Differentiate L, then hold $\frac{M}{P}$ fixed.

$$\partial\left(\frac{M^D}{P}\right) = \frac{\partial L(i,Y)}{\partial i}\partial i + \frac{\partial L(i,Y)}{\partial Y}\partial Y = L_i\,\partial i + L_y\,\partial y$$

If $\partial\left(\frac{M^D}{P}\right) = 0$ (hold M/P fixed)

$$0 = L_i\,\partial i + L_y\,\partial Y$$

$$\frac{\partial i}{\partial Y} = -\frac{L_Y}{L_i} > 0$$

(+) (−)

$\boxed{\text{OR}}$

- "Implicit function theorem."
  If $F(X_1, X_2, X_3 \ldots) = 0$, $\quad \frac{\partial X_1}{\partial X_2} = -\frac{F_{X_2}}{F_{X_1}}$

hence for $L(i,Y) = \frac{M}{P}$ where $\frac{M}{P}$ fixed,
$L(i,Y) - \frac{M}{P} = 0$, $\quad \frac{\partial i}{\partial Y} = -\frac{L_Y}{L_i}$

Note: LM steeper ($\partial i/\partial Y$ bigger) when

L_Y big (money demand sensitive to income)

L_i small (money demand insensitive to i)

IS-LM

LM curve (cont.)

How does $\Delta(\frac{M}{P})$ shift curve?

Holding i fixed, what's $\partial Y / \partial (\frac{M}{P})$?

$$\partial\left(\frac{M}{P}\right) = L_i \, \partial i + L_Y \, \partial Y$$

$$= 0 + L_Y \, \partial Y$$

$$\frac{\partial Y}{\partial M/P} = \frac{1}{L_Y} > 0$$ hence $\frac{M}{P}\uparrow$ shifts LM out (or up)

Note: $P\uparrow$ equivalent to $M\downarrow$

Holding M fixed, changes in P shift LM curve

$P\uparrow \rightarrow \frac{M}{P}\downarrow \rightarrow$ LM shifts in.

Money demand shocks

$$\frac{M^D}{P} = L(i, Y) + \varepsilon$$ ← anything other than i, Y

$$\frac{M}{P} = L(i, Y) + \varepsilon$$

$$\frac{M}{P} - \varepsilon = L(i, Y)$$

Note: $\varepsilon\uparrow$ ← something increases $\frac{M^D}{P}$ given i, Y

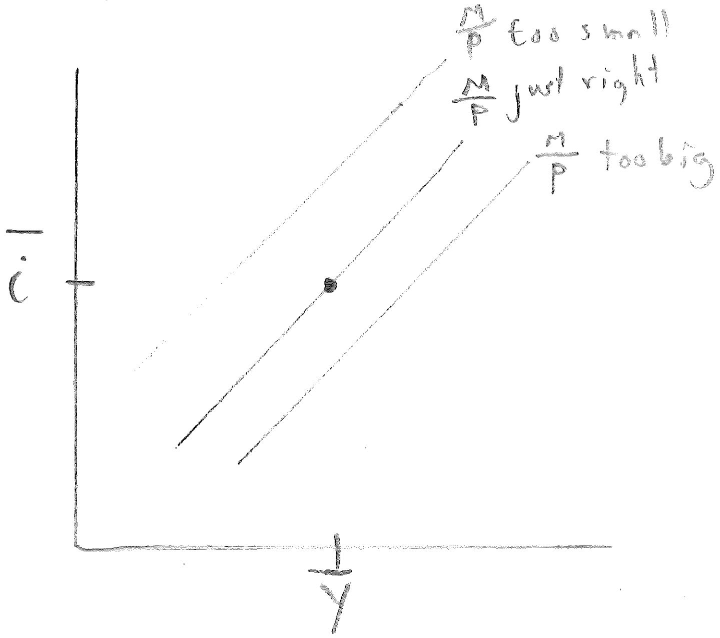
equivalent to $\frac{M}{P}\downarrow$

hence LM shifts back.

Money

LM curve & natural rates $\bar{r}$, $\bar{Y}$

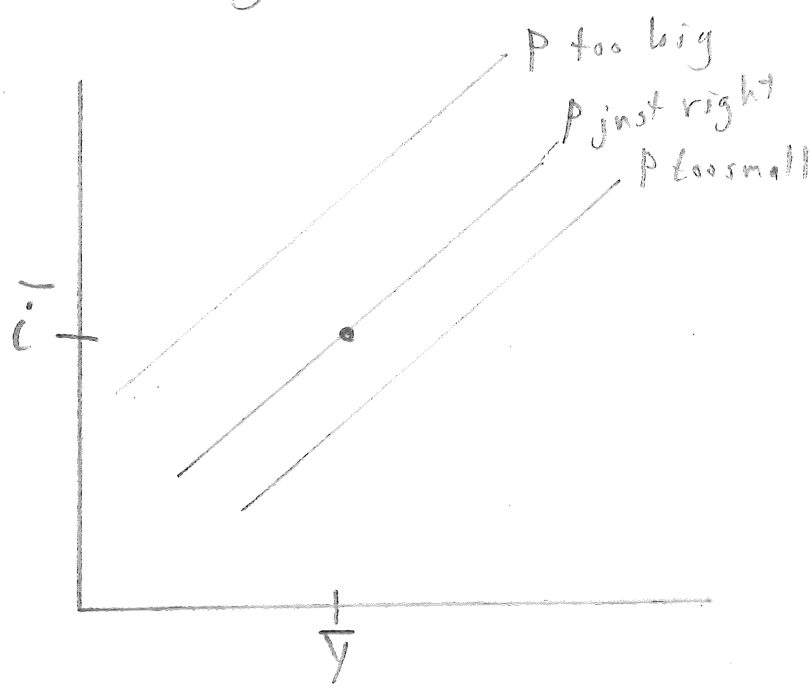
$\bar{i} = \bar{r} + \pi^e$ Nominal interest rate matching $\bar{r}$



For any $\bar{r}$, $\bar{Y}$, and π^e a value $\overline{\left(\frac{M}{P}\right)}$ for which

$$\overline{\left(\frac{M}{P}\right)} = L(\underbrace{\bar{i}}_{\bar{r}+\pi^e}, \bar{Y})$$

Taking M as given, $\overline{\left(\frac{M}{P}\right)}$ implies $\bar{P}$ for which



$$\frac{M}{\bar{P}} = L(\bar{i}, \bar{Y})$$

required price level

Note! $\bar{P}$ moves proportionately with M to make $\frac{M}{\bar{P}} = \overline{\left(\frac{M}{P}\right)}$

real balances consistent with $\bar{Y}$, $\bar{r}$, π^e