KEYNESIAN DSGE

NK IS/LM

Simplest case: no persistence in disturbances, interest-rate rule

Notation: y output gap

r real interest rate minus natural rate

$$y_t = {}_t y^e_{t+1} - s r_t + u^{IS}_t$$

govt. spending; can also be variation in prefs

Romer calls this $1/\theta$

$$\pi_t = {}_t\pi^e_{t+1} + \kappa y_t + u^{\pi}_t$$

very tricky. Not obvious what this could be. Controversial

plus interest-rate rule

$$r_t = \phi_\pi \pi_t + \phi_y y_t + u^{mp}_t$$

What is u^{mp}_t?

— Wild behavior by central bank

— Consequence of random measurement error in c.b.'s estimates of current y & π

$$r_t = \phi_\pi(\pi_t + e_t) + \ldots$$

$$= \phi_\pi \pi_t + \phi_y y_t + \underbrace{\phi_\pi e_t}_{u^{mp}_t}$$

Rational expectations: $${}_t x^e_{t+1} = E_t[x_{t+1}]$$

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Simplest case (cont.)

Persistent disturbances means

$$u_t^{IS} = \rho_{IS} u_{t-1}^{IS} + \varepsilon_t^{IS}$$

$$u_t^{\pi} = \rho_{\pi} u_{t-1}^{\pi} + \varepsilon_t^{\pi}$$

$$u_t^{mp} = \rho_{mp} u_{t-1}^{mp} + \varepsilon_t^{mp}$$

(i. i. d.

$0 < \rho < 1$

No persistence means $\rho_{IS} = \rho_{\pi} = \rho_{mp} = 0$

What we'll see:

1) Model is equivalent to OK IS/MP

2) Correlations between y, π, r depend on which disturbances (ε^{IS}, ε^{π} or ε^{mp}) are hitting.

Identification.

Can an observer see effects of an mp shock?

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How we'll solve

Conjecture that $E_t[y_{t+1}] = 0$

$$E_t[\pi_{t+1}] = 0$$

then verify.

$$y_t = -s r_t + \varepsilon_t^{IS}$$

$$\pi = \kappa y_t + \varepsilon_t^{\pi}$$

$$r = \phi_\pi \pi_t + \phi_y y_t + \varepsilon_t^{mp}$$

Substitute π from PC into IRR, then substitute r from IRR into IS

$$r = \phi_\pi \kappa y + \phi_\pi \varepsilon^{\pi} + \phi_y y + \varepsilon^{mp} = (\phi_\pi \kappa + \phi_y) y + \dots$$

$$y_t = -s(\phi_\pi \kappa + \phi_y) y - s\phi_\pi \varepsilon^{\pi} - s\varepsilon^{mp} + \varepsilon^{IS}$$

$$= \frac{1}{1 + s(\phi_\pi \kappa + \phi_y)} \left(-s\phi_\pi \varepsilon_t^{\pi} - s\varepsilon_t^{mp} + \varepsilon_t^{IS} \right)$$

See: $\varepsilon^{\pi} \uparrow \rightarrow y \downarrow$ $\quad \varepsilon^{mp} \uparrow \rightarrow y \downarrow$ $\quad \varepsilon^{IS} \uparrow \rightarrow y \uparrow$

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Simplest case...

Substitute y into PC, get:

$$\pi_t = \frac{\kappa}{1+s(\phi_\pi \kappa + \phi_y)}\left(-s\varepsilon_t^{mp} + \varepsilon_t^{IS}\right) + \left(\frac{1+s\phi_y}{1+s(\phi_\pi \kappa + \phi_y)}\right)\varepsilon_t^{\pi}$$

See: $\varepsilon^{mp}\uparrow \rightarrow \pi\downarrow$ $\quad \varepsilon^{IS}\uparrow \rightarrow \pi\uparrow$ $\quad \varepsilon^{\pi}\uparrow \rightarrow \pi\uparrow$

Check: is $E_t[\pi_{t+1}] = E_t[y_{t+1}] = 0$?

Yes!

$$E_t[\pi_{t+1}] = E_t\left[\ldots\ldots \varepsilon_{t+1}^{mp} + \varepsilon_{t+1}^{IS}) + \ldots\ldots\ldots \varepsilon_{t+1}^{\pi}\right]$$

If ε's are i.i.d., $E_t[\varepsilon_{t+1}] = 0$.

Same for y_{t+1}.

Now, substitute y & π into IRR get:

$$r_t = \frac{1}{1+s(\quad)}\left[(\phi_\pi \kappa + \phi_y)\varepsilon_t^{IS} + \phi_\pi \varepsilon_t^{\pi} + \varepsilon_t^{mp}\right]$$

See: $\varepsilon\uparrow \rightarrow r\uparrow$

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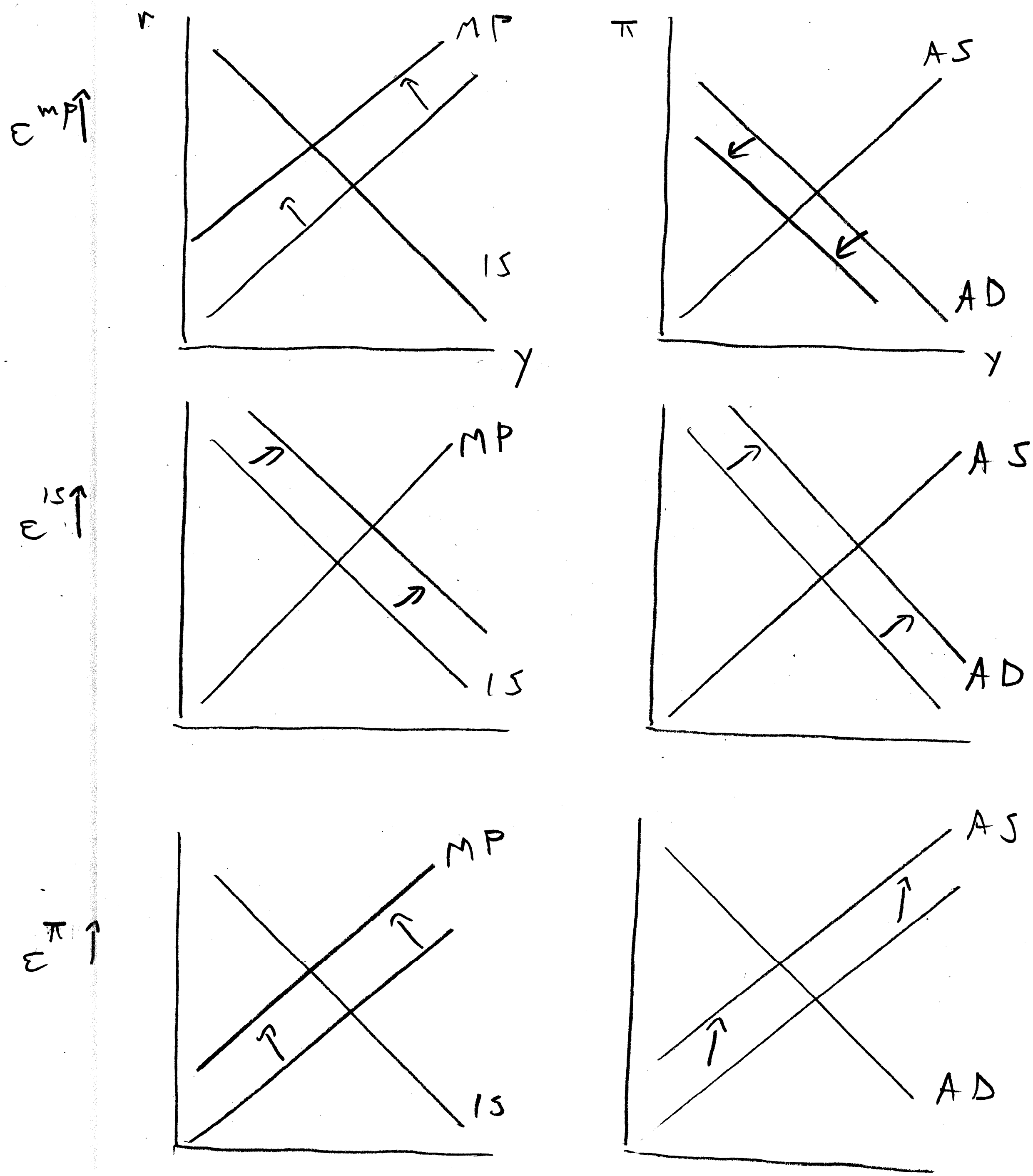
Simplest case (cont.)

In IS/MP graphs



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Simplest case (cont.)

Observable patterns, identification

$$y_t = -\frac{s}{Z}\varepsilon_t^{mp} - \frac{s\phi_\pi}{Z}\varepsilon_t^{\pi} + \frac{1}{Z}\varepsilon_t^{IS}$$

$$\pi_t = \frac{-sk}{Z}\varepsilon_t^{mp} + \frac{1+s\phi_y}{Z}\varepsilon_t^{\pi} + \frac{k}{Z}\varepsilon_t^{IS}$$

$$r_t = \frac{1}{Z}\varepsilon_t^{mp} + \frac{\phi_\pi}{Z}\varepsilon_t^{\pi} + \frac{\phi_\pi k + \phi_y}{Z}\varepsilon_t^{IS}$$

$$\text{where } Z = \frac{1}{1+s(\phi_\pi k + \phi_y)}$$

Recall structural equations:

$$y_t = -s r_t + \varepsilon_t^{IS}$$

$$\pi_t = k y_t + \varepsilon_t^{\pi}$$

$$r_t = \phi_\pi \pi_t + \phi_y y_t + \varepsilon_t^{mp}$$

Do data, realized variations in y, π, r reveal underlying structure?

Can you estimate values of parameters with regressions?

Identification.

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Observable (cont.)

If all disturbances are ε^{IS}

$\varepsilon^{IS}\uparrow \rightarrow y\uparrow, \pi\uparrow, r\uparrow$

Regress π on y: $\beta = \dfrac{\partial\pi/\partial\varepsilon^{IS}}{\partial y/\partial\varepsilon^{IS}} = \kappa$ ← coeff. in P.C.

reveals structure of PC

Regress y on r: $\beta = \dfrac{\partial y/\partial\varepsilon^{IS}}{\partial r/\partial\varepsilon^{IS}} = \dfrac{1}{\phi_\pi\kappa + \phi_y}$ ← not coeff. in IS!

Regress r on y & π: multicollinearity! won't work

Disturbances to IS equation reveal structure of PC, but not IS equation.

KEYNESIAN DSGE

If all disturbances are ε^{π}

$\varepsilon^{\pi}\uparrow \Rightarrow y\downarrow, \pi\uparrow, r\uparrow$

π on y: $\beta = -\dfrac{1+s\phi_y}{s\phi_\pi}$ (not coeff in PC!

y on r: $\beta = -s$ (coeff in IS

Regress r on y & π: won't work

Disturbances to PC reveal structure of IS, but not PC.

If disturbances are ε^{IS} & ε^{π}

π & y no longer collinear

Regression of r on y & π reveals ϕ_y, ϕ_π

but regressions of π on y, y on r results depend on relative magnitude of σ^2_{IS}, σ^2_{π}

KEYNESIAN DSGE

If all disturbances are ε^{mp}

$\varepsilon^{mp}\uparrow \rightarrow y\downarrow, \pi\downarrow, r\uparrow$

π on y reveals PC,

y on r reveals IS,

but r on π & y does not reveal IRR

Lesson

Observable variation in data reveals an underlying structural equation only if none of the disturbances creating variation in data come from that equation.

You need to understand what kind of disturbances you're dealing with.

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No persistence, monetary policy loss function

C.B. sets r_t to minimize

$$L = E\left[\tfrac{1}{2}(\pi - \pi^*)^2 + \tfrac{1}{2}(y - y^*)^2\right]$$

given info available to C.B.

To simplify, we'll say $\pi^* = 0$ ← desired π

If $y^* > 0$, C.B. is aiming to keep output above natural rate.

If $y^* = 0$, ...

Recall that in microeconomic model we used to derive these equations, natural rate is too low due to monopoly — higher y would boost utility of representative agent.

So $y^* > 0$ makes sense.

We'll consider various cases.

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loss fn. (cont.)

1) $y^* = 0$, C.B. can't see ε_t's when it sets r_t

As before, conjecture public's $y^e_{t+1} = \pi^e_{t+1} = 0$

then verify.

$$y_t = -s r_t + \varepsilon^{IS}_t$$

$$\pi_t = \kappa y_t + \varepsilon^{\pi}_t = -s\kappa r_t + \kappa \varepsilon^{IS}_t + \varepsilon^{\pi}_t$$

$$\min_{r_t} E\left[\tfrac{1}{2}\left(-s\kappa r_t + \kappa \varepsilon^{IS}_t + \varepsilon^{\pi}_t\right)^2 + \tfrac{1}{2}\left(-s r_t + \varepsilon^{IS}_t\right)^2\right]$$

Recall $E[X^2] = (E[X])^2 + \sigma^2_X$

When C.B. sets r_t, $E[\varepsilon^{IS}_t] = E[\varepsilon^{\pi}_t] = 0$

it knows variances σ^2_{IS} σ^2_{π}

$$\min_{r_t} \tfrac{1}{2}\Big[\underbrace{(-s\kappa r_t)^2}_{(E[\pi])^2} + \underbrace{\kappa^2 \sigma^2_{IS} + \sigma^2_{\pi}}_{\sigma^2_{\pi}} + \underbrace{(-s r_t)^2}_{(E[y])^2} + \underbrace{\sigma^2_{IS}}_{\sigma^2_{y}}\Big]$$

Take F.O.C.: $\dfrac{\partial L}{\partial r} = 0$

$$0 = (-s\kappa r^*_t)(-s\kappa) + (-s r^*_t)(-s) \quad \text{so } r^* = 0.$$

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1) cont.

Result: $y_t = \varepsilon_t^{IS}$

$\pi_t = \kappa \varepsilon_t^{IS} + \varepsilon_t^{\pi}$

Check: is $E_t[y_{t+1}] = E_t[\pi_{t+1}] = 0$? Yes!

Regress π on y: $\beta = \kappa$ (ε_t^{π} is residual)

Regress y on r: can't; no variation in r.

What if you regress y on real interest rate, rather than real interest rate minus natural rate?

As natural rate varies, real rate does too, so you can try this.

But coefficient is zero.

So is coefficient from regression of π on real interest rate.