SIMPLE NEW KEYNESIAN MODELS: INTRO

Structure of models

"Microfoundations"

Means we start with households maximizing utility, firms maximizing profits. E.g. Romer 6.1, 7.1

+ constraint on price and/or wage adjustment
  e.g. Taylor, Calvo, Rotemberg
  maybe with "indexation"

Solve model by:

— defining LRSS, including $\pi \leftarrow$ (LRSS π

— take loglinear approximations around LRSS path

— assume process generating stochastic shocks (e.g. "white noise," AR(1), ...)

— assume economy must go to LRSS, work backward to current period t using rational expectations

For simple NK models, assume $Y = L$
or fixed K so no saving, investment.

SIMPLE NK models ... (cont.)

Structure

"Core" equations

(following Romer (7.84)

1) NK IS: $y_t = {}_t y^e_{t+1} - s r_t + u^{IS}_t$

("natural" rate," or LRSS

y is log output gap: $\ln(Y_t) - \ln \bar{Y}$

r is real interest rate gap: $r_t - \bar{r}$

(Romer 7.85

2) NKPC

Most frequently Calvo $\pi_t = \beta \pi^e_{t+1} + \kappa y_t + u^{\pi}_t$

but recall this only works for $\bar{\pi} = 0$, so maybe indexing...

Together, 1) & 2) determine y_t & π_t

given y^e_{t+1}, π^e_{t+1}, u^{IS}_t, u^{π}_t, and r_t

$$\left(\pi_t = \pi^e_{t+1} + \kappa\left(y^e_{t+1} - s r_t + u^{IS}_t\right) + u^{\pi}_t \right)$$

Where does r_t come from?

SIMPLE NK...

Structure

Where does r_t come from?

3 options

1) Interest rate rule

as in IS/MP model $r(\overset{+}{\pi}, \overset{+}{y})$

Examples: $r_t = \phi_\pi \pi_t + \phi_y y_t + u_t^{MP}$ ← (simplest, usual)

$$r_t = \phi_\pi \pi_{t+1}^e + \phi_y y_{t+1}^e + u_t^{MP}$$ ← (Romer (7.86)

Makes "three equation model"

often called "New Keynesian IS/LM" but should be called "New Keynesian IS/MP"

If you assume u^{IS}, u^{π}, u^{MP} are "white noise" (i.i.d.)

so that $E_t u_{t+1} = 0$ for all t,

this is same as IS/MP model we did earlier.

SIMPLE NK ....

Structure...

Where does r_t ...? (cont.)

2) M^S & M^D

Include real money balances in utility function, derive $(m-p)^d =$ ______ $\sim$ (fn of y, i

add equation determining m^s e.g.

$$M_t = M + \varepsilon_t \quad \text{where } \varepsilon_t = \rho\varepsilon_{t-1} + e_t$$

constant

growth of log money supply

like Christiano, Eichenbaum, Evans (2005)

This is also called "NK IS/LM"

3) Central bank loss function

Assume central bank chooses r_t to minimize

$$E\left[\alpha_y y_t^2 + \alpha_\pi(\pi_t - \pi^*)^2\right]$$

or $$E\left[\sum_{\tau=0}^{\infty} \beta(\alpha_y y_t^2 + \alpha_\pi(\pi_t - \pi^*)^2\right]$$

Recall y is output gap, so this means central bank wants $y = \bar{y}$

π^* is desired or target π.

Note: $\bar{\pi}$ will equal π^*.

Monetary policy must support LRSS

Recall that to derive from microfoundations & to use core equations solving back from LRSS,

we assumed $\overline{\pi}$ (LRSS π must equal zero if Calvo w/o indexing)

Whatever we assume about monetary policy must be consistent with this.

That is, monetary policy must push π to $\overline{\pi}$ in LR.

Some monetary policies won't do this.

To see this, use NKPC with indexing to $\overline{\pi}$.

(You can make this Calvo by setting $\overline{\pi} = 0$.)

$$\pi_t = \overline{\pi} + \beta(\pi^e_{t+1} - \overline{\pi}) + \kappa y_t$$

Recall this is $r_t - \overline{r}$

With NKIS, gives:

$$\pi_t = \overline{\pi} + \beta(\pi^e_{t+1} - \overline{\pi}) + \kappa y^e_{t+1} - \kappa s r_t$$

Some types of behavior for r_t push π to $\overline{\pi}$, others don't.

SIMPLE NK...

Monetary policy must... (cont.)

$$\pi_t = \bar{\pi} + \beta\left(\pi^e_{t+1} - \bar{\pi}\right) + \kappa y^e_{t+1} - \kappa s r_t$$

In LRSS we want, $y^e = 0$, $\pi^e = \bar{\pi}$, $\pi_t = \bar{\pi}$

To make this happen, need:

1) When $\pi^e = 0$, $y^e = 0$, $r_t = 0$

2) When $\pi^e > \bar{\pi}$ and/or $y^e > 0$, $r_t > 0$
so that $\pi_t < \pi^e$, pushed down toward $\bar{\pi}$

3) When $\pi^e < \bar{\pi}$ and/or $y^e < 0$, $r_t < 0$
so that $\pi_t > \pi^e$, ...

Something that <u>won't</u> work: $r = 0$ always.

This would allow multiple equilibria ("sunspots"):
any π can be LRSS!

SIMPLE NK ...

Monetary policy condition for interest-rate rules:
"Taylor principle"

Recall "Taylor rule" = interest rate rule

Interest-rate rules can be in terms of r, like:

1) $r_t = \phi_\pi (\pi^e_{t+1} - \bar{\pi}) + \phi_y y^e_{t+1}$ (Romer (7.86) is this with $\bar{\pi} = 0$

2) $r_t = \phi_\pi (\pi_t - \bar{\pi}) + \phi_y y_t$

or in terms of i, like:

3) $i_t = \bar{i} + \phi_\pi (\pi_t - \bar{\pi}) + \phi_y y_t$ where $\bar{i} = \bar{r} + \bar{\pi}$

To satisfy monetary policy condition for LRSS,

1) must have $\phi_\pi > 0$, $\phi_y \geq 0$
obvious!

2) must have $\phi_\pi > 0$, $\phi_y \geq 0$
to rule out LRSS where $\pi \neq \bar{\pi}$

3) must have $\phi_\pi > 1$, $\phi_y \geq 0$
See: this makes $r_t > 0$ when $\pi_t > \bar{\pi}$, etc.

Condition that nominal interest rate rule must have $\phi_\pi > 1$ is called "Taylor principle."

SIMPLE NK ...

How to derive NKIS in terms of output gap

Usually NKIS is written in terms of output gap & interest-rate gap, e.g. Romer (7.84).

But using consumption Euler equation gives output or log output in terms of real interest rate & expected output.

How do you get to equation in terms of gaps?

Example.

$$U = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left(\frac{C_{t+\tau}^{1-\theta}}{1-\theta} - V(L_{t+\tau}^{\gamma}) \right)$$

note: Felicity separable in C, L, and across time Romer (6.2)

θ is "coefficient of relative risk aversion"

$\theta \to 1$ log utility

F.O.C. using budget constraint & "envelope theorem"

gives $C_t^{-\theta} = \beta(1+r_t) E_t C_{t+1}^{-\theta}$

Take logs:

$-\theta c_t = \ln \beta + \ln(1+r_t) + \ln E_t C_{t+1}^{-\theta}$

SIMPLE NK...

How to derive NKIS ... (cont.)

$$-\theta c_t = \ln\beta + \ln(1+r_t) + \ln E_t C_{t+1}^{-\theta}$$

Approximations:

1) $\ln E_t C_{t+1}^{-\theta} \approx -\theta E_t \ln C_{t+1}$

Not true: Jensen's inequality! But for some distributions for C_{t+1}, difference is a constant.

2) $\ln(1+r) \approx r$

OK for "small" r

gives $c_t = -\frac{1}{\theta}\ln\beta + c^e_{t+1} - \frac{1}{\theta} r_t$

No investment etc., so $y_t = c_t$;

$$y_t = -\frac{1}{\theta}\ln\beta + y^e_{t+1} - \frac{1}{\theta} r_t$$

Now, we want to put this in terms of $(y-\bar{y})_t$ $(r-\bar{r})_t$

log of natural rate of output, LRSS

SIMPLE NK....

How to derive NKIS.... (cont.)

$$y_t = -\frac{1}{\theta}\ln\beta + y^e_{t+1} - \frac{1}{\theta}r_t$$

In LRSS, y grows at rate g, so $\bar{y}_{t+1} = \bar{y}_t + g$

This defines $\bar{r}$:

$$y_t = -\frac{1}{\theta}\ln\beta + \bar{y}_t + g - \frac{1}{\theta}\bar{r}$$

$$\Rightarrow \bar{r} = -\ln\beta + \theta g$$

Now $r_t = (r_t - \bar{r}) + \bar{r} = (r_t - \bar{r}) - \ln\beta + \theta g$

Substitute into above:

$$y_t = -\frac{1}{\theta}\ln\beta + y^e_{t+1} - \frac{1}{\theta}\left((r_t - \bar{r}) - \ln\beta + \theta g\right)$$

$$= -g + y^e_{t+1} - \frac{1}{\theta}(r_t - \bar{r})$$

Subtract $\bar{y}_t$ from both sides:

$$y_t - \bar{y}_t = -g - \bar{y}_t + y^e_{t+1} - \frac{1}{\theta}(r_t - \bar{r})$$

$$= -(\bar{y}_t + g) + \text{....}$$

$$= -\bar{y}_{t+1} + \text{....}$$

$$(y - \bar{y})_t = (y - \bar{y})^e_{t+1} - \frac{1}{\theta}(r_t - \bar{r})$$

Done!

SIMPLE NK.....

What is NKIS disturbance term? Government

$$y_t = {}_t y^e_{t+1} - s r_t + u^{IS}_t$$ Romer (6.8)

$$y_t = {}_t y^e_{t+1} - s r_t$$ what we just derived

So what is u^{IS}_t?

One way to get it:
add government to model.
G_t Government purchases of output.

$$Y_t = C_t + G_t$$

$$C_t = Y_t - G_t = Y_t\left(1 - \frac{G_t}{Y_t}\right)$$ call this γ

$$c_t = y_t + \ln(1-\gamma_t)$$

If γ_t is small enough, $\ln(1-\gamma_t) \approx -\gamma_t$

Using $c_t = -\frac{1}{\theta}\ln\beta + c^e_{t+1} - \frac{1}{\theta} r_t$,

substitute in

$$c_t = y_t - \gamma_t$$

$$c^e_{t+1} = y^e_{t+1} - \gamma^e_{t+1}$$ gives...

SIMPLE NK ...

What is ... Government (cont.)

$$y_t = -\tfrac{1}{\theta}\ln\beta + y^e_{t+1} - \tfrac{1}{\theta} r_t + (\gamma_t - \gamma^e_{t+1})$$

Define $\bar{r}$ & $\bar{y}$ as before.

In LRSS $\gamma_t = \gamma^e_{t+1} = \bar{\gamma}$, so as before

$$\bar{r} = -\ln\beta + \theta g$$

$$\bar{y}_{t+1} = \bar{y}_t + g$$

gives

$$(y-\bar{y})_t = (y-\bar{y})^e_{t+1} - \tfrac{1}{\theta}(r_t - \bar{r}) + (\gamma_t - \gamma^e_{t+1})$$

$$y_t = + y^e_{t+1} - s r_t + u^{IS}_t$$

Note this is kind of like old-Keynesian IS curve, where $G_t \uparrow \Rightarrow$ shifts IS out.

Here, what shifts out IS is high G/Y relative to expected future G/Y.

SIMPLE NEW KEYNESIAN MODELS

Expectations of long-run future have big effect on present

This is "forward guidance puzzle," Romer 7.9.

Because behavior in model is "forward looking," current y_t, π_t affected by expectations of events/policies in distant future. "Absurd" implications. (Romer

NKIS curve

$$y_t = y^e_{t+1} - s r_t$$

working back from $t+\infty$ when $y_{t+\infty} = 0$, $r_{t+\infty} = 0$,

$$y_{t+\infty-1} = -s r_{t+\infty-1}$$

$$y_{t+\infty-2} = \underbrace{-s r_{t+\infty-1}}_{y^e} - s r_{t+\infty-2}$$

$$y_{t+\infty-3} = \underbrace{-s r_{t+\infty-1} - s r_{t+\infty-2}}_{y^e} - s r_{t+\infty-3}$$

Going back to t,

$$y_t = -s \sum_{\tau=0}^{\infty} r^e_{t+\tau}$$

SIMPLE NK MODELS

Expectations of LR future...

$$y_t = -s \sum_{\tau=0}^{\infty} r^e_{t+\infty}$$

If $r_t = \rho r_{t-1} + \varepsilon_t$ ~ (i.i.d.

this looks reasonable:

$$y_t = -s \sum_{\tau=0}^{\infty} \rho^\tau r_t = -s \frac{1}{1-\rho} r_t$$

But what if central bank gives "forward guidance" that it will cut r_t at future time $t+T$?

Say $r = 0$ before & after $t+T$,
$r = -1$ at $t+T$

looking forward from $t+T$, $r = 0$ always

Then $y_{t+T+1} = 0$

$y_{t+T} = -s(-1) = s$ (all periods from t through T

$y_{t+T-1} = s$

$= = = =$

$y_t = s$

"impact ... on output today is the same regardless of how far in the future the change occurs." (p. 357)

"the <u>cumulative</u> effect on output is proportional to how far in advance the reduction is announced."

SIMPLE NEW KEYNESIAN MODELS

Expectations ....

NKPC

Using indexing to $\bar{\pi}$,

$$(\pi-\bar{\pi})_t = \beta(\pi-\bar{\pi})^e_{t+1} + \kappa y_t$$

Working back from $t+\infty$ when $y_{t+\infty}=0$, $\pi_\infty-\bar{\pi}=0$,

$$y_{t+\infty-1} = \kappa y_{t+\infty-1}$$

$$y_{t+\infty-2} = \beta \kappa y_{t+\infty-1} + \kappa y_{t+\infty-2}$$

$$y_{t+\infty-3} = \beta^2 \kappa y_{t+\infty-1} + \beta \kappa y_{t+\infty-2} + \kappa y_{t+\infty-3}$$

Going back to t,

$$(\pi-\bar{\pi})_t = \kappa \sum_{\tau=0}^{\infty} \beta^\tau y^e_{t+\tau}$$

or $\pi_t = \bar{\pi} + \kappa \sum_{\tau=0}^{\infty} \cdots$

If $y_t = \rho y_{t-1} + \varepsilon_t$, this looks reasonable!

$$\pi_t = \bar{\pi} + \kappa \sum_{\tau=0}^{\infty} \beta^\tau \rho^\tau y_t = \bar{\pi} + \kappa \frac{1}{1-\beta\rho} y_t$$

but....

SIMPLE NK MODELS....

Expectations....

NKPC (cont.)

What if central bank gives "Forward guidance" as before?

$$\pi_t = \bar{\pi} + K \sum_{\tau=0}^{T} \beta^{\tau} s$$

See: if $T = 1$, small effect.
if T = big, big effect.

"the impact... on inflation is larger
when the delay ... is longer"

As $T \rightarrow \infty$,

$$\pi_t \rightarrow \bar{\pi} + K \sum_{\tau=0}^{\infty} \beta^{\tau} s = \bar{\pi} + K \frac{1}{1-\beta} s$$

"the implied impact of a reduction of 1 percentage point for 1 quarter in the far-off future is enormous...." (p. 358)